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A Bayesian Statistics Approach for Terrain Based Navigation and its Terrain Generation Through Second Order Gauss-Markov Process

Mr. Bhatt Ajay Vipul*, ,
*M. Tech Avionics Engineering,
School of Aeronautical Science, Hindustan University,
Chennai, India

Mr. Ravi Kumar K**
**Research Supervisor, Scientist 'D'
DARE-DRDO, Bangalore, India

Mr. P. S. B. Kirubakaran***
***Research Supervisor, Assistant Professor,
School of Aeronautical Science, Hindustan University,
Chennai, India

***Abstract*— An algorithm related to Bayesian statistics using second order Gauss-Markov process for terrain generation is demonstrated in this research paper for solving non-linear state estimation problem called terrain based navigation. UAV or manned aircraft fixes its position using GPS, but at times when GPS information is not available, which may be intentional or unintentional aircraft's navigating capability is affected. At that time, terrain based navigation is very much useful in order to estimate location of travelling aircraft by continuously measuring terrain heights below it and comparing it with stored digital elevation map of proposed area over which it is flying.**

***Keywords*— Terrain Based Navigation (TBN), Second order Gauss-Markov process, Van-Loan method, Bayesian statistics**

I. INTRODUCTION

Terrain based navigation also known as terrain reference navigation will tally the height information of the terrain area with the on-board digital terrain map to provide positional estimation of aircraft. The aircraft altitude above mean sea level is measured with a barometric altimeter and distance between aircraft and direct terrain below it is measured by radar altimeter. So underneath terrain height is calculated by taking the difference of barometric reading with radar altimeter reading. The obtained terrain height is compared with stored elevation map for determining aircraft position over a certain area.

Basically terrain based navigation is differentiated in two ways: 1) Batch based TBN. 2) Sequential TBN. The batch based algorithm gathers terrain heights over a period of time and then matches with stored elevation map to estimate location through post processing. Sequential based approach continuously estimates position of aircraft for each terrain height evaluated during flying. However since it is a non-linear estimation problem to be solved during prolonged flying over non-linear variation in terrain heights, we employed Bayesian statistics based approach which converts the expected states of aircraft into probability mass function. So that to keep track of aircraft flying, through posteriori distribution by considering belief that the indices obtained at which posteriori distribution achieves its peak is the best estimate for state of aircraft over the area. Other approaches related to the non-linear problem is faced by stochastic linearization [2], bank of Kalman Filters [3, 4] and Unscented Kalman filter (UKF) [5].

II. SECOND ORDER GAUSS-MARKOV PROCESS FOR TERRAIN GENERATION

A stationary Gaussian process that has an exponential autocorrelation function is called a Gauss-Markov process. The process is non-deterministic, so a typical sample time function would show no deterministic structure and would look like typical noise. The exponential autocorrelation function indicates that sample values of the process gradually become less and less correlated as the time separation between samples increases. So this process becomes appropriate for terrain generation in the sense that as samples increases with time terrain heights variation can be seen to a large extent. The Gauss-Markov process is an important process in applied work because:
1) It seems to fit a large number of physical processes with reasonable accuracy.
2) It has a relatively simple mathematical description.

The second order process has a power spectral density function (PSD) of the form:

$$S_x(j\omega)= \frac{b^2\sigma^2}{\omega^4+\omega_0^4} \tag{1}$$

Where, σ^2 = Mean square value or position variance parameter, ω_0 = Natural frequency parameter (in rad/s), $b^2 = 2\sqrt{2}\,\omega_0^3$. Since the approach is 2-Dimensional, in order to determine location error in terms of X and Y direction, leads to continuous time state model:

$$\begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1\\ -\omega_0^2 & -\sqrt{2}\,\omega_0\end{bmatrix}}_{G}\begin{bmatrix}X\\ Y\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ b\sigma\end{bmatrix}}_{F}u \tag{2}$$

Also, $$GWG^T = \begin{bmatrix} 0 & 0 \\ 0 & b^2\sigma^2 \end{bmatrix} \quad (3)$$

Where, W = 1 (unity white noise), Δt = 1s, d = Column matrix containing pseudorandom number from standard normal distribution.

The required inputs for the Van-Loan method [7] are F, G, W and Δt for calculating $\emptyset_k$ (transition matrix) and Q_k (covariance matrix). Since Q_k is usually not diagonal, let C be a linear transformation matrix obtained by transposing the matrix obtained by Cholesky factorization of Q_k. Using $\emptyset_k$ and Q_k terrain heights are prepared varying between 35m to 80m over 5Km×5Km area along X and Y direction by implementing following equation:

$$h_{terrain\ (t+1)} = \emptyset_k . h_{terrain\ (t)} + \text{C.d} \quad (4)$$

III. USING BAYESIAN STATISTICS

Here the goal of Bayesian statistics is to estimate the state of an aircraft during flying over varying terrain. Bayes rule here is utilized as follows:

$$\text{p(H/O)=p(H)p(O/H)/p(O)} \quad (5)$$

Where, H = Likely state or hypothesis over which aircraft is supposed to be present, O = Observation of height of terrain through sensors, p(H/O) = Posteriori estimate of state, p(H) = Prior estimate of state, p(O/H) = Likelihood function, p(O) = Normalized by probability of data in general.

The scheme will be to convert prior estimation into posteriori estimation through observation provided in terms of estimated state of aircraft by relating terrain height with stored elevation data. The process of this conversion is governed by likelihood function p(O/H) which is a Gaussian distribution. Let N(μ, P) denotes Gaussian distribution with mean vector μ and covariance matrix P,

$$N(\mu, P) = \frac{1}{\sqrt{(2\pi)^2|P|}} \exp(-0.5(x-\mu)^T P^{-1}(x-\mu)) \quad (6)$$

Where, x = Expected state of aircraft after getting an observation.

So with each new terrain evaluated, prior distribution is multiplied relating to likelihood of the measurement taken. Posteriori distribution generated in this way is scaled up or down associated with the likelihood of data obtained. So as new observations are available, process becomes iterative and posteriori distribution of previous observation will become prior distribution of recently taken observation.

IV. SYSTEM MODEL

Our simulation deals with 2-Dimensional case, so here we have to deal with two position state. We describe the process and measurement models according to the following:
Process model:

$$x_{k+1} = x_k + u_k + w_k \quad (7)$$

Measurement model:

$$\underbrace{h_{baro} - h_{radalt}}_{Z_k} = h(x_k) + v_k \quad (8)$$

Where x_k , u_k, w_k denotes recent position of aircraft along X and Y direction, forward movement, a white Gaussian process noise respectively. The terrain height Z_k i.e. an observation is equal to terrain height according to recent aircraft position over the stored map with a white Gaussian process noise v_k. Note that w_k and v_k are mutually independent white noise.

V. ALGORITHM

The algorithm corresponding to simulation is as follow:

(A) First, we need to generate a representative terrain profile. Use a second order Gauss–Morkov model with σ = 20m and ω = 0.01rad/m in (1). Note that the independent variable in this random process model is now space instead of time, hence the units of rad/m for ω. Generate a profile with a discrete space interval of 1m over the range of matrix 5Km×5Km.

(B) Next, generate the rotorcraft's motion profile. Use a second order Gauss-Markov model for the horizontal and another for the vertical axis to generate a bounded random process model. Use a sampling interval of Δt = 1s, and the parameters σ = 10m and ω = 0.51rad/s in (1). Then add this random process to nominal motion that moves at 140m/s as visualize in Fig.1 but remains at a fixed altitude, starting at coordinates (10m, 10m, 200m) that is at 200m. Generate the profile for a 50s-duration.At each sampling time, compute the radar altimeter measurement by taking difference of the appropriate terrain height from the rotorcraft altitude. The terrain height required will, in general need to be computed by cubic spline (may go for linear interpolation but cubic spline interpolation gives better result) interpolation between samples generated in Part (A). To each radar altimeter measurement, add a measurement noise random sample from a Gaussian distribution with zero mean and a sigma of 1 m.

(C) For each interpolated value of radar altimeter, find where is the possibility of getting same measurement within a interval of [-1m, 1m] i.e. error in radar altimeter measurement, within the matrix along X and Y direction. The output will be rows and columns where the position of aircraft is expected after getting observation. Prepare a state space window of relatively smaller interval duration where the aircraft is expected to be. Do the computational efforts to translate this window along with aircraft translational movement in order to reduce time required to pin point the estimated state of aircraft.

(D) Initialize with uniform prior and posteriori. Turn the priori and posteriori into a probability mass function by dividing by the sum. Pull out the indices at which posteriori achieves its maximum to start. Keep these as the best estimate to start. Take a covariance matrix of [25 0;0 25] and generate posteriori distribution by multiplying prior distribution with likelihood function . After each iteration of the above process normalize this distribution to make it proper probability distribution that will be equal to number of rows and columns as expected state of aircraft, store the posteriori to the prior distribution . Pull out the indices at which posteriori achieves its maximum and that will be the estimated states of aircraft for a given observation.

(E) Do the entire process for 50-s sequence. Plot the X-axis and Y-axis position error over time.

VI. SIMULATION RESULTS

The simulation of terrain based navigation was performed for 50s flight trajectory, which starts from [10m, 10m] along X and Y axis as shown in Fig.1. by discontinuous line. Since the approach is for two state estimation Fig.2. depicts a plot concerning to X state estimation error and Fig.3. depicts a plot concerning to Y state estimation error. Due to rigorous computational working required to generate even 5Km×5Km, the simulation could be performed for just 50s but still it gives broad idea how a terrain reference navigation system works and to experience its state estimating capability.

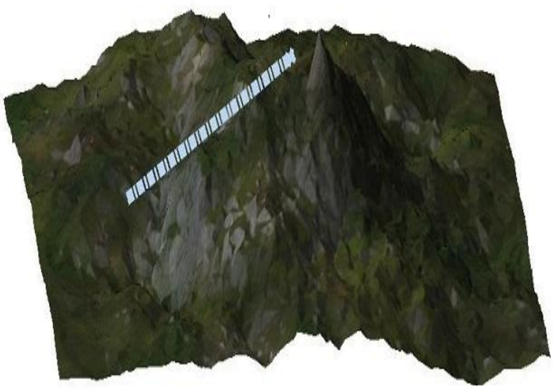

Fig.1. Simulated terrain produce by second order Gauss-Markov process over 5Km×5Km area and dotted line indicates the aircraft flight path.

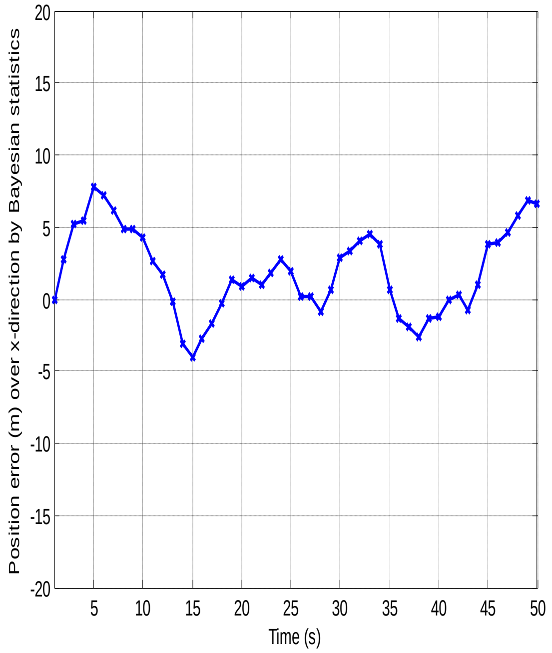

Fig.2. Position error in meters along X-direction over a flight of 50s

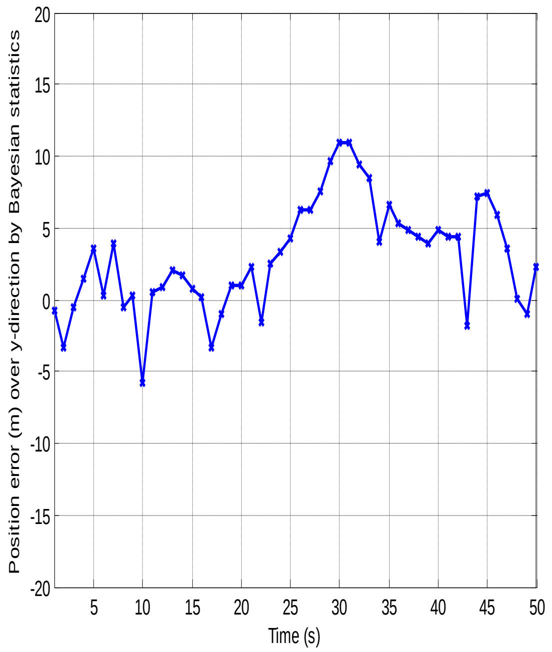

Fig.3. Position error in meters along Y-direction over a flight of 50s

VII CONCLUSION

The simulated results show error along X-axis to be in the range [8m,-5m] and along Y-axis to be in the range [12m,-8m]. However performance of terrain based navigation greatly depends on variation of terrain gradient in the area over which aircraft is flown. Nearly equal height terrain helps to a very less amount for terrain based navigation while varying terrain to a greater extent helps for this navigation technique a lot. Also the present technique for estimating the state greatly depends on how the system estimates the prior belief and puts the mass on nearly true state at the time of initialization and if can, then all the posteriori belief coming after it will have nearly true state output. But if prior belief of state estimation during initialization is itself faulty then entire chain of posteriori belief following it will be wrong.

VIII FUTURE WORK

This paper is concerned related to solving terrain based navigation as a non-linear recursive estimation problem with Bayesian statistics but other well known techniques for non-linear estimation problem are extended Kalman filter (EKF), particle filter based approach using extended Kalman filter for local linearization, unscented Kalman filter(UKF) can be implemented effective in order to visualize individual methods position error giving characteristics which can be further kept for comparison purpose. Also in terms of application basis that is after estimating current state of aircraft, potential obstacles coming within path of aircraft can be known since the terrain elevations of the area over which aircraft is flown is known. Owing to this knowledge advance terrain avoidance cueing (ATAC) technique can be implemented to signify which path is better suited to deviate in order to get rid of obstacle.

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