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A Combined Approach for Solving Periodic Vehicle Routing Problem with Fleet and Driver Scheduling

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Abstract

This paper develops a model for the optimal management of periodic deliveries of a given commodity called Periodic Vehicle Routing Problem (PVRP). The goal is to schedule the deliveries according to feasible combinations of delivery days and to determine the scheduling of fleet and driver and routing policies of the vehicles. The objective is to minimize the sum of the costs of all routes over the planning horizon. We propose a combined approach of heuristic algorithm and exact method to solve the problem.

Keywords: *Vehicle routing problem, scheduling, combined approach*

1. Introduction

In the distribution of goods or services, there exists a problem that is known as the vehicle routing problem. In fact there is a wide variety of situations and therefore the problem is not unique but a vast class of problems, each one with its own characteristics and constraints. Vehicle Routing Problem is one of the important issues that exist in transportation system. Many researchers have been working in this area to discover new methodologies in selecting the best routes in order to find the better solutions.

The vehicle routing problem (*VRP*) can be defined as follows: vehicles with a fixed capacity *Q* must deliver order quantities q_i $(i=1,\dots,n)$ of goods to n customers from a single depot (*i* = 0). Knowing the distance d_{ij} between customers *i* and *j* $(i, j=1,\dots,n)$, the objective of the problem is to minimize the total distance traveled by the vehicles in a way that only one vehicle handles the deliveries for a given customer and the total quantity of goods that a single vehicle delivers is not larger than *Q* [1].

In PVRP, Each customer $i \in I = \{1,2,\dots,i,\dots,n\}$ specifies a set $k(i)$ of combinations, and the visit days are assigned to the customer by selecting one of these combinations. Thus, the vehicles must visit the customer *i* on the days belonging to the selected combination.

Early formulations of the PVRP were developed by Beltrami and Bodin [2] and by Russell and Igo [11] who proposed heuristics applied to waste collection problems. Tan and Beasley [14] use the idea of the generalized assignment method proposed by Fisher and Jaikumar [9] and assign a visiting schedule to each vertex. Eventually a heuristic for the VRP is applied to each day. Russell and Gribbin [12] developed a heuristic organized in four phases. Solution methods in these papers have focused on two-stage (construction and improvement) heuristics. Cordeau et al. [6] present another algorithm: The solution algorithm is a TS heuristic which, differently from the above heuristics, may allow infeasible solutions during the search process. Similarly, good results were obtained in the more recent work of Ha. Dji constantinuo and Baldacci [8], Angelelli and Speranza [1], and Blakeley et al. [3] who provide specific practical applications of the PRVP.

Francis et al. [10] introduce the Periodic Vehicle Routing Problem with Service Choice (PVRP-SC) which allows service levels to be determined endogenously. The PVRP-SC is defined as follows:

Given: A set of nodes with known demand and minimum visit frequency requiring service over the planning period; afleet of capacitated vehicles; a set of service schedules with headways and service benefits; and a network with travel times.

Find: An assignment of nodes to service schedules and a set of vehicle routes for each day of the planning period with the objective

Minimize: the total routing cost incurred net of the service benefit accrued.

Francis et al. [10] develop an integer programming formulation of the PVRP-SC with exact and heuristic solution methods. Due to the computational complexity of the problem, solutions to the discrete PVRP-SC are limited by instance size. Continuous approximation models are better suited for large problem instances, yet the use of continuous approximation models for periodic routing problems has been limited.

Daganzo [7] presents modeling techniques for distribution problems with varying service requirements. Smilowitz and Daganzo [13] develop continuous approximation models for distribution network design with multiple service levels. These references show that continuous approximations can be powerful tools for strategic and tactical decisions when service choice exists. In continuous approximation models, aggregated data are used instead of more detailed inputs.

This paper concerns with PVRP with fleet and driver scheduling (PVRFDSP). The basic framework of the vehicle routing part can be viewed as a Heterogeneous Vehicle Routing Problem with Time Windows (HVRPTW) in which a limited number of heterogeneous vehicles, characterized by different capacities are available and the customers have a specified time

windows for services. We propose a mixed integer programming formulation to model the problem. A feasible neighbourhood heuristic search is addressed to get the integer feasible solution after solving the continuous model of the problem.

Section 2 reviews the integer programming formulation of the PVRP with Service Choice from Francis et al. [10]. Section 3 describes the mathematics formulation of the (PVRFDSP). Feasible neighbourhood heuristic search is given in Section 4. The conclusions are described in Section 5.

2. Models Of The PVRP- SC

In this section, we present the discrete formulations of the PVRP-SC from Francis et al. [10]. In the PVRP-SC, customers are visited a preset number of times over the period with a schedule that is chosen from a menu of schedule options. Let S denote this menu of schedules, and T denote the set of days in the period. The parameter a_{sd} links schedules to days, where if $a_{sd}=1$ day $d\in T$ is in schedule $s\in S$ and $a_{sd}=0$ otherwise. Each schedule $s\in S$ has an associated visit frequency γ^s measured by number of days in the schedule: $\gamma^s = \sum_{d\in T} a_{sd}$. For given schedule option, the headway between visits is defined in terms of the visit frequency as $H^s = 1/\gamma^s$. Each schedule has an associated benefit α^s related to the cost benefit of more frequent service which is assumed to be stationary over the time period.

2.1Discrete Formulation Of The PVRP-SC

The discrete formulation of the PVRP-SC is defined for a set of nodes, N_0, which consists of customers nodes, N, and a depot, $i=0$, and a set of arcs connecting nodes, $A=\{(i,j): i,j\in N_0\}$. Each customer node $i\in N$ has a known daily demand, W_i, and a minimum service frequency, F_i, measured in days per period. The demand accumulated between visits, w_i^s, is a function of the schedule $s\in S$ and the daily demand of the node. The stopping time at a node, τ_i^s, is a function of the frequency of the schedule since more items accumulate with less frequent service and, therefore, require more time to load/unload. Associated with each arc $(i,j)\in A$ is a known travel cost, c_{ij}. There is a set K of vehicles, each with capacity C. The following allocation and routing variables define the solution to the discrete formulation.

$$y_{ik}^s = \begin{cases}1 & \text{if node } i\in N \text{ is visited by vehicle } k\in K \text{ on schedule } s\in S\\ 0 & \text{otherwise}\end{cases}$$

$$x_{ijk}^s = \begin{cases}1 & \text{if vehicle } k \text{ travels the arc } (i,j)\in A \text{ on day} \in T\\ 0 & \text{otherwise}\end{cases}$$

The discrete formulation for PVR-SC developed in Francis et al. [10] is:

$$Z^* = \min \sum_{k\in K}\Big[\sum_{d\in D}\sum_{(i,j)\in A} c_{ij}\, x_{ijk}^d + \sum_{s\in S}\sum_{i\in N} (\gamma^s \tau_i^s - W_i \alpha^s)\, y_{ik}^s \qquad (1)$$

Subject to

$$\sum_{s\in S}\sum_{k\in K}\gamma^s\, y_{ik}^s \geq F_i \quad \forall\, i\in N \qquad (2)$$

$$\sum_{s\in S}\sum_{k\in K} y_{ik}^s \leq 1 \quad \forall\, i\in N \qquad (3)$$

$$\sum_{s\in S}\sum_{i\in N} w_i^s a_{sd}\, y_{ik}^s \leq C \quad \forall\, k\in K, d\in T \qquad (4)$$

$$\sum_{j\in N_0} x_{ijk}^d = \sum_{s\in S} a_{sd}\, y_{ik}^s \quad \forall\, i\in N,\ k\in K, d\in T \qquad (5)$$

$$\sum_{j\in N_0} x_{ijk}^d = \sum_{j\in N_0} x_{jik}^d \quad \forall\, i\in N_0,\ k\in K, d\in T \qquad (6)$$

$$\sum_{ij\in Q} x_{jik}^d \leq |Q|-1 \quad \forall\, Q\subseteq N, k\in K, d\in T \qquad (7)$$

$$y_{ik}^s \in \{0,1\} \quad \forall\, i\in N, k\in K, s\in S \qquad (8)$$

$$x_{ijk}^d \in \{0,1\}\ \forall\, (i,j)\in A, k\in K, d\in T \qquad (9)$$

The objective function (1) balances arc travel times, stopping times and demand weighted service benefit. Constraints (2) enforce the minimum frequency of visits for each node. Constraints (3) ensure that one schedule and one vehicle are chosen for each demand node. Constraints (4) represent vehicle capacity constraints. Constraints (5) link the x and y variables for the demand nodes. Constraints (6) ensure flow conservation at each node. Constraints (7) are the subtour elimination constraints and ensure that all tours contain a visit to the depot. Constraints (8) and (9) define the binary variables for allocation and routing, respectively.

3. Mathematical Formulation Of PVRFDSP

. We denote the planning horizon by T and the set of drivers by D. The set of workdays for driver $l \in D$ is denoted by $T_l \subseteq T$. The start working time and latest ending time for driver $l \in D$ on day $t \in T$ are given by g_l^t and h_l^t, respectively. Let D_I and D_E denote the set of the internal and external drivers ($D = D_I \cup D_E$). For each internal driver $l \in D_I$, let H denote the maximum weekly working duration. We denote the maximum elapsed driving time without break by F and the duration of a break by G (according to the EU driver legislation).

Let K denote the set of vehicles. For each vehicle $k \in K$, let Q_k and P_k denote the capacity in weight and in volume, respectively. We assume the number of vehicles equals to the number of drivers. Denote the set of n customers (/nodes) by $N = \{1, 2, \ldots, n\}$. Denote the depot by $\{0, n+1\}$. Each vehicle starts from $\{0\}$ and terminates at $\{n+1\}$. Each customer $i \in N$ specifies a set of days to be visited, denoted by $T_i \subseteq T$. On each day $t \in T_i$, customer $i \in N$ requests service with demand of q_i^t in weight and p_i^t in volume, service duration d_i^t and time window $[a_i, b_i]$. Note that, for the depot $i \in \{0, n+1\}$ on day t, we set $q_i^t = p_i^t = d_i^t = 0$. Denote the set of preferable vehicles for visiting customer i by K_i ($K_i \in K$) and the extra service time per pallet by e if a customer is not visited by a preferable vehicle. The travel time between customer i and j is given by c_{ij}. Denote the cost coefficients of the travel time of the internal drivers by A and the working duration of the external drivers by B.

We define binary variable x_{ijk}^t to be 1 if vehicle k travels from node i to j on day t, binary variable wt i to be 1 if customer i is not visited by a preferred vehicle on day t. Variable v_{ik}^t is the time that vehicle k visits node i on day t. Binary variable z_{ik}^t indicates whether vehicle k takes a break after serving customer i on day t. Variable u_{ik}^t is the elapsed driving time for vehicle k at customer i after the previous break on day t. Binary variable y_{lk}^t is set to 1 if vehicle k is assigned to driver l on day t. Variables r_l^t and s_l^t are the total working duration and the total travel time for driver l on day t, respectively.

This notations used are given as follows :

Set:

T	The set of workdays in the planning horizon,
D_I	The set of internal drivers,
D_E	The set of external drivers,
D	The set of drivers $D = D_I \cup D_E$,
T_l	The set of workdays for driver $l \in D$,
K	The set of vehicles,
N	The set of customers,
N_0	The set of customers and depot $N_0 = \{0, n + 1\} \cup N$,
K_i	The set of preferable vehicles for customer $i \in N$,
T_i	The set of days on which customer $i \in N$ orders,

Parameter:

Q_k	The weight capacity of vehicle $k \in K$,
P_k	The volume capacity of vehicle $k \in K$,
c_{ij}	The travel time from node $i \in N_0$ to node $j \in N_0$,
$[a_i, b_i]$	The earliest and the latest visit time at node $i \in N_0$,
d_i^t	The service time of node $i \in N_0$ on day $t \in T_i$,
q_i^t	The weight demand of node $i \in N_0$ on day $t \in T_i$,
p_i^t	The volume demand of node $i \in N_0$ on day $t \in T_i$,
e	The extra service time per pallet when a non-preferable vehicle is used,
[g_l^t, h_l^t]	The start time and the latest ending time of driver $l \in D$ on day $t \in T$,
H	The maximum working duration for each internal driver over the planning horizon,
F	The maximum elapsed driving time without break,
G	The duration of the break for drivers,

K_1 The cost factor on the total travel time of internal drivers,

K_2 The cost factor on the total working duration of the external drivers,

Variables:

x_{ilk}^t Binary variable indicating whether vehicle $k \in K$ travels from node $i \in N_0$ to $j \in N_0$ on day $t \in T$,

w_i^t Binary variable indicating whether customer $i \in N_0$ is visited by a non-preferable vehicle on day $t \in T$,

v_{ik}^t The time at which vehicle $k \in K$ starts service at node $i \in N_0$ on day $t \in T$,

z_{ik}^t Binary variable indicating whether vehicle $k \in K$ takes break after serving node $i \in N_0$ on day $t \in T$,

u_{ik}^t The elapsed driving time of vehicle $k \in K$ at node $i \in N_0$ after the previous break on day $t \in T$,

y_{lk}^t Binary variable indicating whether vehicle $k \in K$ is assigned to driver $l \in D$ on day $t \in T$,

r_l^t The total working duration of driver $l \in D$ on day $t \in T$,

s_l^t The total travel distance of driver $l \in D$ on day $t \in T$.

The mathematical formulation for this problem is presented as follows:

$$\min K_1.\sum_{l\in D_I}\sum_{t\in T_i} s_l^t + K_2.\sum_{l\in D_E}\sum_{t\in T_i} r_l^t + Z' \qquad (10)$$

Subject to:

$$\sum_{k\in K}\sum_{j\in N_0} x_{ijk}^t = 1 \qquad \forall\, i \in N, t \in T_i \qquad (11)$$

$$\sum_{k\in K\setminus K_i}\sum_{j\in N_0} x_{ijk}^t = w_i^t \quad \forall\, i \in N, t \in T_i \qquad (12)$$

$$\sum_{i\in N}\sum_{j\in N_0} q_i^t x_{ijk}^t \le Q_k \quad \forall\, k \in K, t \in T \qquad (13)$$

$$\sum_{i\in N}\sum_{j\in N_0} p_i^t x_{ijk}^t \le P_k \quad \forall\, k \in K, t \in T \qquad (14)$$

$$u_{jk}^t \ge u_{ik}^t + c_{ij} - M(1 - x_{ijk}^t) - Mz_{ik}^t \quad \forall\, i,j \in N_0,\ k \in K, t \in T \qquad (15)$$

$$u_{jk}^t \ge c_{ij} - M(1 - x_{ijk}^t)\ \forall\, i,j \in N,\ k \in K, t \in T \qquad (16)$$

$$u_{ik}^t + \sum_{j\in N_0} c_{ij} x_{ijk}^t - F \le Mz_{ik}^t\ \forall\, i, \in N_0,\ k \in K, t \in T \qquad (17)$$

$$v_{jk}^t \ge v_{ik}^t + d_i^t + e.p_i^t.w_j^t + c_{ij} + G.z_{ik}^t\ M(1 - x_{ijk}^t)\ \forall\, i,j \in N_0,\ k \in K, t \in T \qquad (18)$$

$$b_i \ge v_{ik}^t \ge a_i \qquad \forall\, i \in N, k \in K, t \in T_i \qquad (19)$$

$$v_{0k}^t \ge \sum_{l\in D}(g_l^t.y_{lk}^t)\ \forall\, k \in K, t \in T \qquad (20)$$

$$v_{n+1,k}^t \le \sum_{l\in D}(h_l^t.y_{lk}^t)\ \ \forall\, k \in K, t \in T \qquad (21)$$

$$s_l^t \ge \sum_{i\in N_0}\sum_{j\in N_0} c_{ij} x_{ijk}^t - M(1 - yikt\ \ \forall\, l \in DI, k \in K, t \in Ti \qquad (22)$$

$$r_l^t \ge v_{n+1,k}^t - g_l^t - M(1 - y_{lk}^t)\ \forall\, l \in D_I, k \in K, t \in T_i \qquad (23)$$

$$\sum_{t\in T_i} r_l^t \le H \quad \forall\, l \in D_I \qquad (24)$$

$$x_{ijk}^t, w_i^t, z_{ik}^t, y_{lk}^t \in \{0,1\}\ \forall\, i,j \in N_0, l \in D,\ k \in K, t \in T \qquad (25)$$

$$v_{ijk}^t, u_{ik}^t, r_l^t, s_l^t \ge 0\ \forall\, i,j \in N_0, l \in D,\ k \in K, t \in T \qquad (26)$$

The objective function (10) minimizes weighted sum of the travel time of the internal drivers and the working duration of the external drivers over the planning horizon.

Constraints (11) state that each customer must be visited by one vehicle on each of its delivery days. Constraints (12) define whether each customer is visited by a preferable vehicle. Constraints (13-14) guarantee that the vehicle capacities are respected in both weight and volume.

Constraints (15-16) define the elapsed driving time. More specifically, for the vehicle (k) travelling from customer i to j on day t, the elapsed driving time at j equals the elapsed driving time at i plus the driving time from i to j (i.e., $u_{jk}^t \ge u_{ik}^t + c_{ij}$) if the vehicle does not take a break at customer i (i.e., $z_{ik}^t = 0$); Otherwise, if the vehicle takes a break at customer i (i.e., $z_{ik}^t = 1$), the elapsed driving time at j will be constrained by (10) which make sure it is greater than or equal to the travel time between i and j (i.e., $u_{jk}^t \geq c_{ij}$). Constraints (17) guarantee that the elapsed driving time never exceeds an upper limit F by imposing a break at customer i (i.e., $z_{ik}^t = 1$) if driving from customer i to its successor results in a elapsed driving time greater than F.

Constraints (18) determine the time to start the service at each customer. If j is visited immediately after i, the time v_{jk}^t to start the service at j should be greater than or equal to the service starting time v_{ik}^t at i plus its service duration d_i^t, the extra service time $e \cdot p_i^t$ if i is visited by an inappropriate vehicle (i.e., $w_j^t = 1$), the travel time between the two customers cij, and the break time G if the driver takes a break after serving I (i.e., $z_{ik}^t = 1$). Constraints (19) make sure the services start within the customers' time window.

Constraints (20-21) ensure that the starting time and ending time of each route must lie between the start working time and latest ending time of the assigned driver. Constraints (22) calculate the total travel time for each internal driver. Constraints (23) define the working duration for each driver on every workday, which equals the time the driver returns to the depot minus the time he/she starts work. Constraints (24) make sure that the internal drivers work for no more than a maximum weekly working duration, referred to as 37 week-hour constraints. Constraints (25-26) define the binary and positive variables used in this formulation. The overall model should contain constraints (2-7) from PVRP-SC.

4. Feasible Neighborgood Heuristic Search

While a straightforward brand-and-bound approach could be adopted, for many classes of large-scale problems such a procedure would be prohibitively expensive in terms of total computing time. We have adopted the approach of examining a reduced problem in which most of the integer variables are held constant and only a small subset allowed varying in discrete steps.

This may be implemented within the structure of a program by marking all integer variables at their bounds at the continuous solution as nonbasic and solving a reduced problem with these maintained as nonbasic.

The procedure may be summarized as follows:

Step 1 : Solve the problem ignoring integrality requirements.

Step 2 : Obtain a (sub-optimal) integer feasible solution, using heuristic rounding of the continuous solution.

Step 3 : Divide the set I of integer variables into the set I_1 at their bounds that were nonbasic at the continuous solution and the set I_2, $I = I_1 + I_2$.

Step 4 : Perform a search on the objective function, maintaining the variables in I_1 nonbasic and allowing only discrete changes in the values of the variables in I_2.

Step 5 : At the solution obtained in step 4, examine the reduced costs of the variables in I_1. If any should be released from their bounds, add them to set I_2 and repeat from step 4, otherwise terminate.

The above summary provides a framework for the development of specific strategies for particular classes of problems. For example, the heuristic rounding in step 2 can be adapted to suit the nature of the constraints, and step 5 may involve adding just one variable at a time to the set I_2.

At a practical level, implementation of the procedure requires the choice of some level of tolerance on the bounds on the variables and also their integer infeasibility. The search in step 4 is affected by such considerations, as a discrete step in a super basic integer variable may only occur if all of the basic integers remain within the specified tolerance of integer feasibility.

In general, unless the structure of the constraints maintains integer feasibility in the integer basic variables for discrete changes in the superbasic, the integers in the set I_2 must be made superbasic. This can always be achieved since it is assumed that a full set of slack variables is included in the problem.

5. Conclusions

This paper was intended to present a solution for one of the most important problems in Supply Chain Management, Distribution problems. The aim of this paper was to develop a model of Periodic vehicle Routing with Fleet and Driver Scheduling Problems This problem has additional constraint which is the limitation in the number of vehicles. The proposed algorithm employs nearest neighbor heuristic algorithm for solving the model. This algorithm offers appropriate solutions in a very small amount of time.

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