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A Comparison of Extended and Unscented Kalman Filters for the State Estimation of Induction Motor Drives

D.Sleeva Reddy[1] K.Ankalamma[2] M.Vijaya Kumar[3]

[1]Loyola Institute of Technology and Management Dhulipalle-522412.Guntur.A.P.India

[2&3] J.N.T.U.A College of Engineering Ananatapur. A.P.India .

Abstract-This paper investigates the application of Unscented Kalman Filter (UKF) for induction motor (IM) sensorless drives and compares the general UKF with Extended Kalman Filter (EKF) in detuned conditions. The speed and rotor resistance estimation results are compared. Simulation results for Unscented Kalman Filter are presented and compared with those of Extended Kalman Filter. It evaluates the very low speed performance of general UKF.Only General UKF is presented as it provides the best performance compared to other UKFs (basic,simplex,spherical) and compared with respect to EKF.It is concluded that the UKF provides more robust performance than the conventional EKF.

Keywords-Extended Kalman Filter, general Unscented Transformation, state estimation.

1.INTRODUCTION

Estimation in nonlinear systems is extremely important because almost all practical systems involve nonlinearities of one kind or another**.** In estimation theory, the Extended Kalman Filter (EKF) is the nonlinear version of the Kalman filter which linearizes about an estimate of the current mean and covariance. In case of well defined transition models, the EKF has been considered the de facto standard in the theory of nonlinear state estimation and navigation systems.Unlike its linear counterpart, the Extended Kalman Filter in general is not an optimal estimator (of course it is optimal if the measurement and the state transition model are both linear, as in that case the extended Kalman Filter is identical to the regular one). In addition, if the initial estimate of the state is wrong, or if the process is not modeled correctly, the filter may quickly diverge, owing to its linearization. Another problem with the Extended Kalman Filter is that the estimated covariance matrix tends to underestimate the true covariance matrix and therefore risks becoming inconsistent in the statistical sense without the addition of "stabilising noise”.

Although the EKF maintains the elegant and computationally efficient update form of KF,it suffers from number of serious limitations[1].EKF is difficult to tune, the jacobian can be hard to derive,and it can only handle limited amount of nonlineariy. A nonlinear Kalman Filter which shows promise as an improvement over the EKF is the Unscented Kalman Filter (UKF). In the UKF, the probability density is approximated by a deterministic sampling of points which represent the underlying distribution as a Gaussian. The nonlinear transformations of these points are intended to be an estimation of the posterior distribution, the moments of which can then be derived from the transformed samples. The transformation is known as the Unscented Transform(UT).The UKF tends to be more robust and more accurate than the EKF in its error estimation .

The paper is organized as follows. In section II, the general UT concept is discussed. In section III, the UKF and the discrete UKFs are discussed. The IM model and observer are described in section IV. Section V provides the simulation results.Conclusions and scope for future work are given in section VI.

II.GENERAL U.T

The Unscented Transformation is a method for calculating the statistics of a random variable, while undergoes a nonlinear transformation[2].The UT was developed to address the deficiencies of linearization by providing a more direct and explicit mechanism for transforming mean and covariance information.

Throughout the paper, the superscript (i) is used to denote the i_{th} sigma point. Suppose the mean m_x and covariance P_x of a n × 1 stochastic vector x are known and moreover *if one is* interested in the mean and covariance of the output of a known nonlinear function *y= h (x)*, A set of sigma points $\sigma_x^{(i)}$, i = 0, 1, 2,……N, with the same mean and covariance as the vector can be selected. Then, the sigma points are transformed through the known nonlinear function *h (x)* to obtain a set of projected sigma points $\sigma_x^{(i)}$, i = 0, 1, 2,……N. The weighted sample mean and sample covariance of the projected sigma points give a good approximation of the true mean and covariance of the output *y*. If the weight associated with the i^{th} sigma point is denoted by $W^{(i)}$ then the approximate mean and covariance of the output are calculated as

$$m_y = \sum_{i=0}^{N} W^{(i)} \sigma_y^{(i)} \quad (1)$$

.

$$P_y = \sum_{i=0}^{N} W^{(i)} \left(\sigma_y^{(i)} - m_y\right)\left(\sigma_y^{(i)} - m_y\right)^T \quad (2)$$

The general UT [5] uses a set of 2*n*+1 sigma points which lie on the$\sqrt{\boldsymbol{n}}^{\boldsymbol{th}}$ covariance contou[6]. This is taken as the general UT, since it includes the basic UT as a special case. The general UT uses the 2*n*+1 sigma points

$$\sigma_x^{(0)} = m_x \quad (3)$$

$$\sigma_x^{(0)} = m_x + \tilde{P}_x^{(i)} \qquad i = 1, \dots, 2n \quad (4)$$

$$\tilde{P}_x^{(i)} = \left(\sqrt{(n+k)P}\right)_{(i)}^T \qquad i = 1, \dots, n \quad (5)$$

$$\tilde{P}_x^{(n+i)} = -\left(\sqrt{(n+k)P}\right)_{(i)}^T \quad i = 1, .., n \quad (6)$$

Unlike the basic UT, the sigma points are assigned unequal weights in the calculation of the mean and covariance as follows:

$$W^0 = \frac{k}{n+k} \quad (7)$$

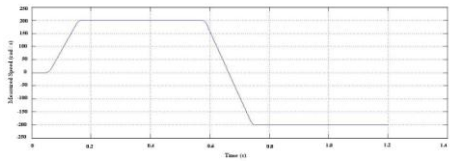
$$W^{(i)} = \frac{1}{2(n+k)} \qquad i = 1, \dots, 2n \quad (8)$$

The design parameter *k* determines the degree of emphasis on point $\sigma_x^{(0)}$, and reduces the higher-order approximation errors. For *k* = 0, the general UT reduces to basic UT.

III UNSCENTED KALMAN FILTER

The Kalman filter (KF) was originally developed for linear systems [7] but later applied to nonlinear systems using the linearized or extended KF (EKF) [8]. Although the performance of the EKF is poor in some situations, its performance is acceptable if the system nonlinearity is not severe. Its simplicity, together with the popularity of the KF, makes it the most widely applied nonlinear state estimator. Nonlinearly mapping an input random variable typically results in a complex distribution with a large number of associated parameters. Hence, optimal nonlinear state estimation requires knowledge of higher order statistics and the exact estimation of the states of a nonlinear system is often impossible in practice [9].

The Unscented Kalman Filter belongs to a bigger class of filters called Sigma-Point Kalman Filters or Linear Regression Kalman Filters,which are using the statistical linearization technique[3,4].



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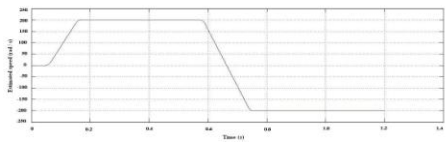

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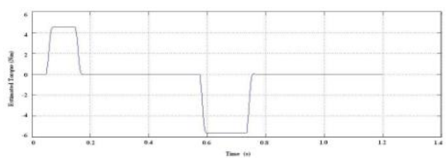

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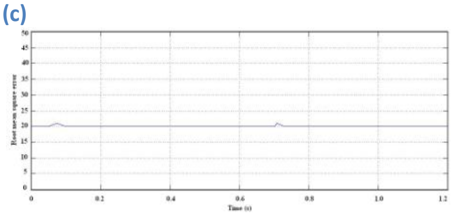

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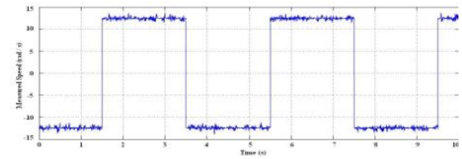
Fig.1-Simulation results for high speed operation of IM with general UKF with speed of 200rad(a) estimated speed, (b) measured speed,(c)estimated torque,(d)estimated stator and rotor flux magnitudes



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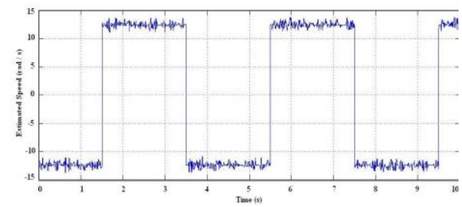

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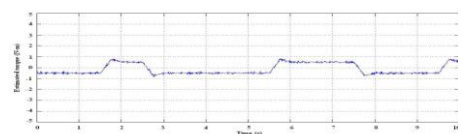

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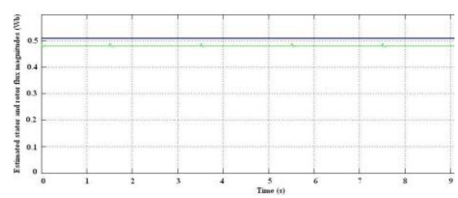

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Fig.2-Simulation **r**esults for low speed operation of IM with square speed reference of 12 rad/s and general UKF(a)estimated speed (b)measured speed (c)estimated torque(d)estimated stator and rotor flux magnitudes

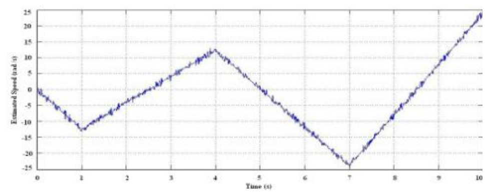
The discrete KF uses the first two statistical moments and updates them with time. This is the key idea when combining the UT and KF to obtain UKF. The UKF is basically the discrete KF in which an UT is used to obtain the mean and covariance updates. The UKF as presented here is a simplified UKF which is suitable for estimation of IM states. In general, the observation model can also be nonlinear and all parameters and functions can be time-varying. Moreover the UKF can be extended to the case of non-additive noise [8].

Given the discrete-time nonlinear system

and a linear measurement model

$$z_x = H.x_k + v_k \tag{10}$$

Where x_k is an n×1 state vector, z_k is an m×1 measurement vector, H is the measurement matrix (m×m) and f (x_k, u_k) is a known nonlinear state transition vector. It is assumed that the process noise w_k is white and zero mean with covariance matrix Q and measurement noise v_k is also white and zero mean with covariance matrix R. Estimates of the initial state $\hat{\boldsymbol{x}}_0^+$ and the initial error covariance matrix $\boldsymbol{P}_0^+$ are available. The iterations in the classic KF consist of a prediction



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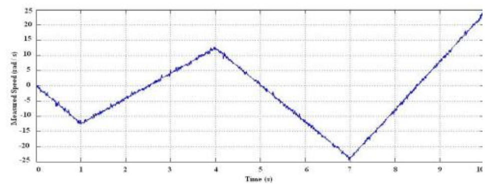

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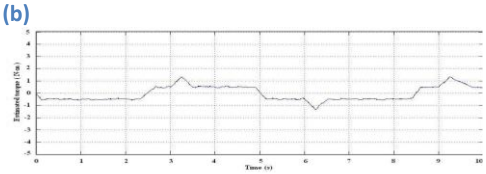

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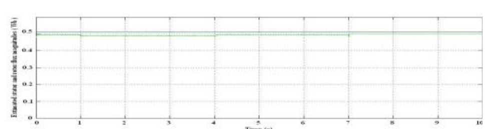

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Fig-Simulation results for high speed operation of IM with speed of 200 rad/s(a)estimated speed with EKF ,(b)measured speed with EKF,(c)estimated speed with general UKF,(d)measured speed with general UKF

step followed by a correction step. For the correction step the discrete KF equations are used.

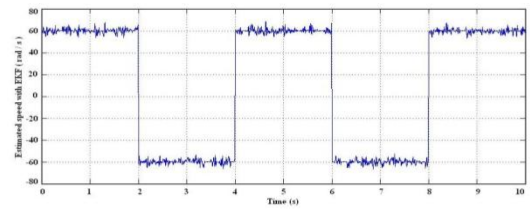
$$K_k = P_k^-.H^T.(H.P_k^-.H^T + R)^{-1} \quad (11)$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_k.(z_k - H.\hat{x}_k^-) \quad (12)$$

$$P_k^+ = (I - K_k.H).P_k^- \quad (13)$$

Where k_k is the Kalman gain.

The prediction step in the KF is the projection of the mean $\hat{\boldsymbol{x}}_{\boldsymbol{k}}^+$ and covariance $\boldsymbol{P}_{\boldsymbol{k}}^+$ in time using the



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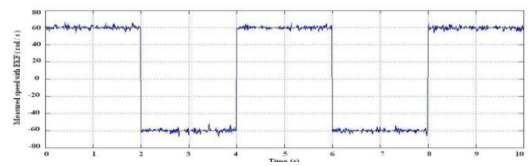

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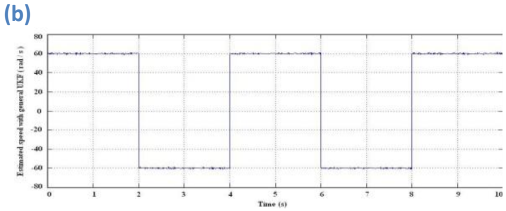

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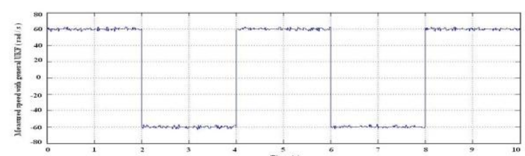

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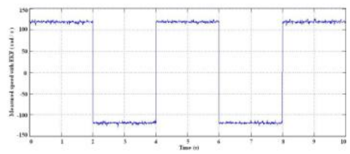
Fig.4: Simulation results for low speed operation of IM with speed of 60 rad/s(a)estimated speed with EKF(b)measured speed with EKF(c)estimated speed with general UKF(d)measured speed with general UKF

state equation (recall that the KF estimates are unbiased). For the nonlinear system of (9), the state equation is a nonlinear transformation of a stochastic input $\boldsymbol{x}_{\boldsymbol{k}}$. Hence, the UT can be used to obtain the mean $\hat{\boldsymbol{x}}_{\boldsymbol{k+1}}^+$ and covariance $\boldsymbol{P}_{\boldsymbol{k+1}}^-$ of its output.

The mean$\hat{\boldsymbol{x}}_{\boldsymbol{k}}^+$ and the covariance $\boldsymbol{P}_{\boldsymbol{k}}^+$ of the stochastic input $\boldsymbol{x}_{\boldsymbol{k}}$ are used to obtain a set of sigma points$\boldsymbol{\sigma}_{\hat{\boldsymbol{x}}_{\boldsymbol{k}}^+}^{(\boldsymbol{i})}$, (i= 0, 1, 2,…., N) and the corresponding weights ($W^{(i)}$, i=0,1, ….., N). Then the sigma points are projected in time using the non linear transformation(9).



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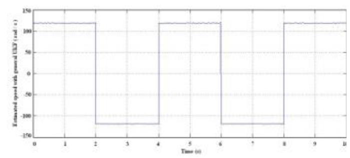

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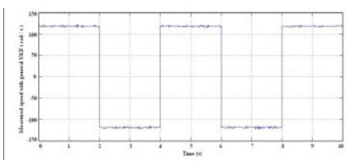

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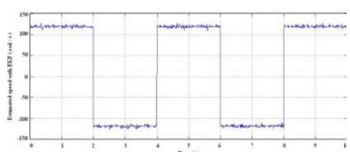

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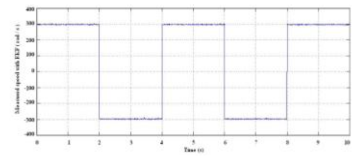
Fig:5- Simulation results for low speed operation of IM with speed of 120 rad/s (a)estimated speed with EKF(b)measured speed with EKF(c)estimated speed with general UKF(d)measured speed with general UKF

Given the projected sigma points $\boldsymbol{\sigma}^{(i)}_{\hat{x}^-_{k+1}}$, the predicted mean $\hat{x}_{k+1}$ using (1) and the predicted error covariance P^-_{k+1} using the following modified version of (2) are calculated.

$$P^-_{k+1} = \sum_{i=0}^{N} \left\{ W^{(i)} \left(\boldsymbol{\sigma}^{(i)}_{\hat{x}^-_{k+1}} - \hat{\boldsymbol{x}}^-_{k+1} \right) \left(\boldsymbol{\sigma}^{(i)}_{\hat{x}^-_{k+1}} - \hat{\boldsymbol{x}}^-_{k+1} \right)^T \right\} + Q \qquad (15)$$

Although the UKF is computationally costly, its computational load is acceptable for modern microprocessors. The most costly operations are the Cholesky factorization and outer products in obtaining the covariance of the projected sigma points. While the latter is an inevitable costly operation, Cholesky factorization can be simplified then the error covariance matrix is



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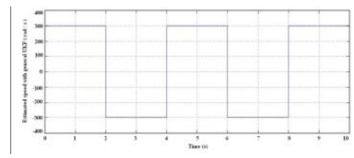

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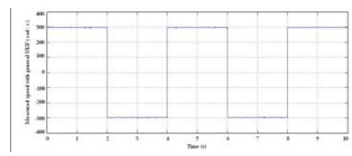

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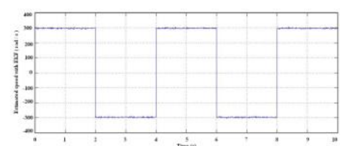

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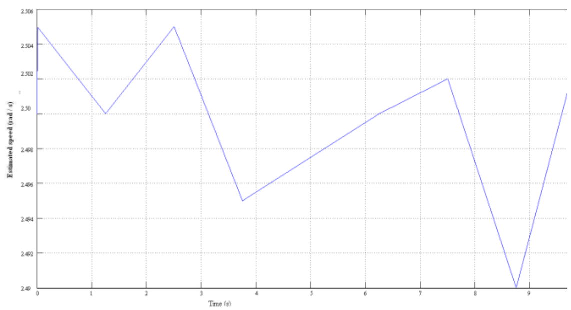
Fig:6-Simulation results for low speed operation with speed of 300rad/s(a)estimated speed with EKF (b)measured speed with EKF(c) estimated speed with general UKF (d)measured speed with general with general UKF

sparse. Since the load torque T_L appears only in the swing equation, its cross-correlation with both currents and both fluxes is negligible and it can be considered independent of both the currents and the fluxes. In addition, the orthogonality of the axes implies that $\varphi_{s\alpha}$ and $\varphi_{s\beta}$ are independent, as are $i_{s\alpha}$ and $i_{s\beta}$. Consequently, the covariance matrix is sparse. For application to the IM, symbolic manipulation can be used to simplify the expressions off-line and thereby significantly reduce the computational load.



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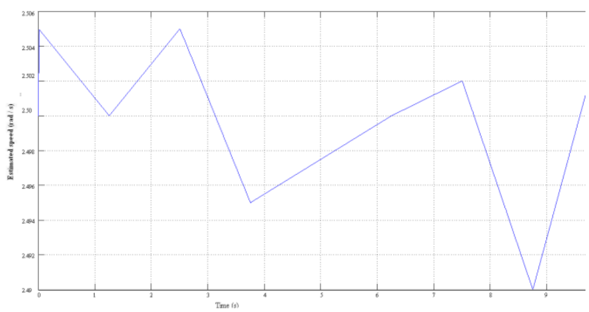

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Fig.7-Simulation results for rotor resistance

(a) rotor resistance with EKF (b)rotor resistance with UKF

IV INDUCTION MOTOR MODEL

The IM state space model in the stator reference frame is

$$\frac{d\underline{\psi}_s}{dt} = -R_s \underline{i}_s + \underline{u}_s \qquad (16)$$

$$\frac{d\underline{i}_s s}{dt} = -\left(\frac{1}{T_r\sigma} + \frac{1}{T_s\sigma} - j\omega_r\right)\underline{i}_s + \frac{1}{L_s\sigma}\left(\frac{1}{T_r} - j\omega_r\right)\underline{\psi}_s + \frac{1}{L_s\sigma \underline{u}_s} \qquad (17)$$

$$\frac{d\omega_r}{dt}\frac{3}{2J}p^2(\psi_{s\alpha}i_{s\beta} - \psi_{s\beta}i_{s\alpha}) - \frac{T_L}{J}p \qquad (18)$$

$$\frac{dT\ L}{dt} = 0 \qquad (19)$$

Where φ_s is stator flux space vector, i_s is stator current vector, p is the pole pairs, J is the inertia, Rs and R_r are the stator and rotor resistances, L_s and L_r are the stator and rotor inductances, and L_m is magnetizing inductance. $\boldsymbol{T_s = \frac{L_s}{R_s}, T_r = \frac{L_r}{R_r}, \sigma = \frac{L_sL_r - L_m^2}{L_sL_r}}$. ω_r is the rotor speed, and $u_s = u_{s\alpha} + j\,u_{s\beta}$ is the stator voltage vector, which is the system input. The load torque is T_L, it is usually unknown, and in this model is assumed constant. The choice of stationary reference frame results in a simpler mathematical model and a simpler UKF design. However the UT is applicable to any state-space representation of the IM, in any reference frame, and is not limited to the one chosen here.

To use the discrete KF, the IM model is discretized by solving the system's state equation to determine the states at the sampling instants. To avoid cross-coupling problems, the forward Euler method is used which provides an acceptable approximation of the system dynamics for a short sampling period t_s. The resulting system is

$$\underline{\psi}_s(k+1) = \underline{\psi}_s(k) - t_s.R_s.\underline{i}_s(k) + t_s.\underline{u}_s(k) \qquad (20)$$

$$\underline{i}_s(k+1) = \underline{i}_s(k) - t_s\left(\frac{1}{T_r\sigma} + \frac{1}{T_s\sigma} - j\omega_r(k)\right)\underline{i}_s(k) + \frac{t_s}{L_s\sigma}\left(\frac{1}{T_r} - j\omega_r(k)\right)\underline{\psi}_s(k) + \frac{t_s}{L_s\sigma}\underline{u}_s(k) \qquad (21)$$

$$\omega_r(k+1) = \omega_r(k) - \frac{t_s}{J}pT_L(k) + \frac{3t_s}{2J}p^2(\psi_{s\alpha}(k).i_{s\beta}(k) - \psi_{s\beta}(k).i_{s\alpha}(k)) \qquad (22)$$

$$T_L(k+1) = T_L(k) \qquad (23)$$

The state vector is chosen as

$$x = [\varphi_{s\alpha} \quad \varphi_{s\beta} \quad i_{s\alpha} \quad i_{s\beta} \quad \omega_r \quad T_L]^T \qquad (24)$$

The stator current is the measured output and the measurement is

$$H = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad (25)$$

Thus, the measurement model is linear.

V.SIMULATION RESULTS

General UKF is a good choice in terms of performance and computational load. Only the speed estimation and rotor resistance results are compared. For an unbiased comparison, the same parameter values are used for the general UKF and EKF in each simulation. The simulation results for general UKF are shown in Fig.1 where thirteen sigma points are used and k=1.The speed estimation errors are low during both transients and steady-state operation, and its overall performance in terms of torque and noise is excellent. The observer's low speed performance is tested with square wave speed reference.The low speed operation of general UKF is shown in fig.2.

Fig.3 shows the estimation results for general UKF in slow reversal operation. Figs-4,5,6 shows the comparison of EKF and UKF at different speeds. Fig .7 shows the rotor resistance of IM with EKF and UKF.

VI.Conclusions

This paper proposes the general unscented Kalman Filters and Extended Kalman filter for IM drive's state estimation.The comparison includes results for low speed operation of the drive. The results of extended kalman filter and unscented kalman filter are compared for speed and rotor resistance estimation.

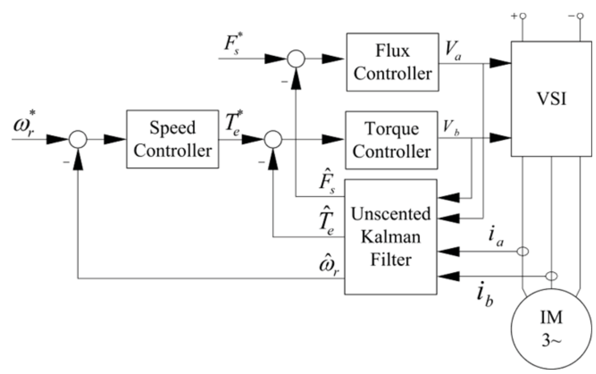
The variation of rotor resistance is negligible as the thermal model is not used. If the thermal model is used the variation is upto 50% due to temperature variations.

The general UKF which provides the best performance was compared with the conventional EKF under detuned conditions. It is concluded that the UKF provides more robust performance than the conventional EKF. The simulation prove that UKF has the capability to deliver superior performance under various operating conditions compared to EKF.

APPENDIX

The IM name plate data and parameters are:
P_N=0.75hp,U_{SN}=240V,f_{SN}=60HZ,n_N=1725rpm ,p=2,T_{eN}=3.1Nm,R_s=2.3Ω,R_r=2.5Ω,L_s=L_r=0.25H and L_m =0.24H.

The noise covariance matrices used are $Q=diag\{1.3x\ 10^{-6}, 1.3x\ 10^{-6}, 1.4x10^{-5}, 1.4x10^{-5}, 5.17x10^{-7}\}$ and $R=diag\{120,120\}$ and the initial setting is selected as a zero-mean vector with an identity error covariance matrix.



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