Published by :
http://www.ijert.org

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

A Discrete Time Two Commodity Inventory System

P. Senthil Kumar
School of Mathematics
Madurai Kamaraj University
Madurai - 625021, India

Abstract— **In this article, we consider a two-commodity inventory system under discrete time review. The demands for each commodity - $i(i=1,2)$ arrive according to a independent Bernoulli process. The maximum inventory level for the $i^{th}(i=1,2)$ commodity is fixed as $S_i, i=1,2$ and the reorder level as $s_i, i=1,2$. The ordering policy is defined as, when both the inventory levels are less than or equal to their respective reorder levels, we place an order for $Q_i(=S_i-s_i), i=1,2$ units . The lead time distribution is assumed to be geometric. The demands that occur during the stock-out period are considered to be lost. Some system performance measures in the steady state are derived and the total expected cost rate under a suitable cost structure is calculated. The results are illustrated numerically.**

Keywords —Discrete time; two -commodity; inventory system; joint order policy;

I. INTRODUCTION

One of the factors that contribute the complexity of the present day inventory system is the multitude of items stocked and this necessitated the multi-commodity systems. In dealing with such systems, in the earlier days models were proposed with independently established reorder points. But in situations were several product compete for limited storage space or share the same transport facility or are produced on (procured from) the same equipment (supplier) the above strategy overlooks the potential savings associated with joint ordering and, hence, will not be optimal. Thus, the coordinated, or what is known as joint replenishment, reduces the ordering and setup costs and allows the user to take advantage of quantity discounts.

Inventory system with multiple items have been subject matter for many investigators in the past. Such studies vary from simple extensions of EOQ analysis to sophisticated stochastic models. References may be found in [15, 2, 22, 24, 27, 17] and the references therein.

Kalpakam and Arivarignan [9] have introduced (s, S) policy with a single reorder level s defined in terms of the total number of items in the stock. This policy avoids separate ordering for each commodity and hence a single processing of orders for both commodities has some advantages in situation where in procurement is made from the same supplies, items are produced on the same machine, or items have to be supplied by the same transport facility.

Krishnamoorthy and Varghese [10] have considered a two commodity inventory problem without lead time and with Markov shift in demand for the type of commodity namely ‘‘commodity-1’’, ‘‘commodity-2’’ or ‘‘both commodity’’, using the direct Markov renewal theoretical results. Anbazhagan and Arivarignan [4, 5, 6, 7] have analyzed two commodity inventory system under various ordering policies. Yadavalli et al. [26] have analyzed a model with joint ordering policy and varying order quantities. Yadavalli et al. [27] have considered a two-commodity substitutable inventory system with Poisson demands and arbitrarily distributed lead time.

A two-commodity inventory system under continuous review is analyzed by Sivakumar [21]. They assumed that both the commodities are substitutable in the sense that at the time of zero stock, the other commodity is used to meet the demand. During the stock-out of both commodities an arriving demand entered the orbit of infinite size. They assumed constant retrial policy with exponential retrial time.

In all the above models, the authors assumed that the time axis is continuous. But, the discrete time systems, are more appropriate than their continuous time counterparts for modelling diverse productive processes, since the basic units in these systems are digital. In discrete time setting, it is assumed that the time axis is calibrated into epochs by small units and that all the events are deemed to occur only at these epochs. With the advent of fast computing devices and efficient transaction reporting facilities, such epochs with small gaps can be conveniently assumed so that events can occur at these epochs.

The analysis of discrete time queueing models has received considerable attention in the literature over the past years, in view of its applicability in the study of many computer and communication systems in which time is slotted ([23], [25]). An important application stems from the secondary and tertiary sector since, for example the current production systems of numerous factories operate on a discrete time basis where events can only happen at regularly spaced epochs.

In the case of inventory modelling under discrete times, the first paper was by Bar-Lev and Perry [8], who assumed that demands are non-negative integer valued random variables and items have constant life times. Lian and Liu [12] developed a discrete time inventory model with geometrically distributed inter-demand times, bulk demands and constant life time for items. They assumed $(0, S)$ ordering policy, with instantaneous supply which clears any backlog and restores the stock to the maximum capacity S. This assumption helped them to have fixed life time for all items. They derived the limiting distribution of inventory level through matrix-analytic method.

Lian et al. [13] developed a discrete time inventory system with discrete PH-renewal process for (batch) demand time points and assumed discrete-PH-distribution for life time of items. They also assumed zero lead time and that unsatisfied demand were completely backlogged.

Abboud [1] studied a discrete inventory model for production inventory systems with machine breakdowns. They assumed that the demand and production rates were constant and that the failure and repair times of each item were independently distributed as geometric.

In this paper we have considered a discrete time two commodity inventory system with independent reorder levels where a joint order for both commodities is placed only when the levels of both commodities are less than or equal to their respective reorder levels. The rest of the paper is organized as follows: In section 2, the mathematical model is described. Section 3 is the central one. The steady state analysis of the model is presented in section 4. Somekey system performance measures are derived and the total expected cost rate is calculated in section 5. In section 6, some numerical results are presented to illustrate the effect of the parameters on several performance characteristics.

II . Model Description

We consider a two-commodity inventory system where the time axis is divided into intervals of equal length, called slots. The end points of the slots are called slot boundaries. The system is monitored at the slot boundaries. The maximum storage capacity for the commodity - i is $S_i (i = 1,2)$ which is used to meet the demands. The demand is for single item per customer. We assume the following:

• The demand for the commodity - 1 arrives according to a Bernoulli process with probability a_1. Thus a_1 is the probability that a demand occurs at a slot and $\bar{a}_1 (= 1 - a_1)$, is the probability that a demand does not occur in a slot.

• The demand for the commodity - 2 arrives according to a Bernoulli process with probability a_2. Thus a_2 is the probability that a demand occurs at a slot and $\bar{a}_2 (= 1 - a_2)$, is the probability that a demand does not occur in a slot.

• The reorder level for the commodity - i is fixed as $s_i (1 \le s_i < S_i)$ with an ordering quantity for the commodity - i is $Q_i (= S_i - s_i > s_i + 1)(i = 1,2)$ items when both inventory levels are less than or equal to their respective reorder levels. The requirement $Q_i > s_i + 1 (i = 1,2)$ ensures that after the replenishment the inventory levels of both commodities are above their respective reorder levels; otherwise it may not be possible to reorder (according to this policy) which leads to perpetual shortage. The lead time is assumed to be distributed as geometric distribution with parameter $b (> 0)$.

• The demands that occur during stock-out periods are considered to be lost.

We assume that all the above activities are occurred in the slot. Therefore more than one event may occur in the same slot. For mathematical clarity, we need to define the order of the events to be occurred, here, first the replenishment of order, then the demand for the commodity - 1 and finally the demand for the commodity - 2 will be satisfied.

3 Analysis

Let X_t denote the inventory level of commodity - 1 and Y_t denote the inventory level of commodity - 2 at time t. From the assumptions made on the input and output processes, it can be shown that the stochastic process $\{(X_t, Y_t) : t = 0,1,2,\ldots,\}$ is a discrete time Markov chain with state space given by

$$E = \{(i,k) : i = 0,1,2,\ldots, S_1, k = 0,1,2,\ldots, S_2\}.$$

The transition probability function is defined as for $(i,k),(j,l) \in E$,

$$p((i,k),(j,l)) = Pr[X_{t+1} = j, Y_{t+1} = l \mid X_t = i, Y_t = k].$$

The transition probability matrix P of this process,

$$P = (\ p((i,k),(j,l))\ ), \quad (i,k),(j,l) \in E$$

Hence, we have

$$p((i,k),(j,l)) =$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

$$
\left\{
\begin{array}{lll}
\bar{a}_1\bar{a}_2, & j=i, & i=s_1+1,s_1+2,\ldots,S_1 \\
 & l=k, & k=1,2,\ldots,S_2 \\
 & (or) & \\
 & j=i, & i=1,2,\ldots,s_1 \\
 & l=k, & k=s_2+1,s_2+2,\ldots,S_2 \\
 & & \\
\bar{a}_1 a_2, & j=i, & i=s_1+1,s_1+2,\ldots,S_1 \\
 & l=k-1, & k=1,2,\ldots,S_2 \\
 & (or) & \\
 & j=i, & i=1,2,\ldots,s_1 \\
 & l=k-1, & k=s_2+1,s_2+2,\ldots,S_2 \\
 & & \\
a_1\bar{a}_2, & j=i-1, & i=s_1+1,s_1+2,\ldots,S_2 \\
 & l=k, & k=1,2,\ldots,S_2 \\
 & (or) & \\
 & j=i-1, & i=1,2,\ldots,s_1 \\
 & l=k, & k=s_2+1,s_2+2,\ldots,S_2 \\
 & & \\
a_1 a_2, & j=i-1, & i=s_1+1,s_1+2,\ldots,S_1 \\
 & l=k-1, & k=1,2,\ldots,S_2 \\
 & (or) & \\
 & j=i-1, & i=1,2,\ldots,s_1 \\
 & l=k-1, & k=s_2+1,s_2+2,\ldots,S_2 \\
 & & \\
\bar{a}_1, & j=i, & i=s_1+1,s_1+2,\ldots,S_1 \\
 & l=k, & k=0 \\
 & & \\
a_1, & j=i-1, & i=s_1+1,s_1+2,\ldots,S_1 \\
 & l=k, & k=0 \\
 & & \\
\bar{a}_2, & j=i, & i=0
\end{array}
\right.
$$

$$
\left\{
\begin{array}{lll}
\bar{b}a_1, & j=i-1, & i=1,2,\ldots,s_1 \\
 & l=k, & k=0 \\
 & & \\
\bar{b}\bar{a}_2, & j=i, & i=0 \\
 & l=k, & k=1,2,\ldots,s_2 \\
 & & \\
\bar{b}a_2, & j=i, & i=0 \\
 & l=k-1, & k=1,2,\ldots,s_2 \\
 & & \\
\bar{b}\bar{a}_1\bar{a}_2, & j=i, & i=1,2,\ldots,s_1 \\
 & l=k, & k=1,2,\ldots,s_2 \\
 & & \\
\bar{b}\bar{a}_1 a_2, & j=i, & i=1,2,\ldots,s_1 \\
 & l=k-1, & k=1,2,\ldots,s_2 \\
 & & \\
\bar{b}a_1\bar{a}_2, & j=i-1, & i=1,2,\ldots,s_1 \\
 & l=k, & k=1,2,\ldots,s_2 \\
 & & \\
\bar{b}a_1 a_2, & j=i-1, & i=1,2,\ldots,s_1 \\
 & l=k-1, & k=1,2,\ldots,s_2 \\
 & & \\
b\bar{a}_1\bar{a}_2, & j=i+Q_1, & i=0,1,2,\ldots,s_1 \\
 & l=k+Q_2, & k=0,1,2,\ldots,s_2 \\
 & & \\
b\bar{a}_1 a_2, & j=i+Q_1, & i=0,1,2,\ldots,s_1 \\
 & l=k+Q_2-1, & k=0,1,2,\ldots,s_2 \\
 & & \\
ba_1\bar{a}_2, & j=i+Q_1-1, & i=0,1,2,\ldots,s_1 \\
 & l=k+Q_2, & k=0,1,2,\ldots,s_2 \\
 & & \\
ba_1 a_2, & j=i+Q_1-1, & i=0,1,2,\ldots,s_1
\end{array}
\right.
$$

We define $<i>=((i,0),(i,1),\ldots,(i,S_2)), i=0,1,2,\ldots,S_1.$ By ordering the set of states as $(<0>,<1>,\ldots,<S_1>)$, the transition probability matrix P of the discrete time Markov chain can be conveniently expressed in a block partitioned form with entries,

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

$$[P]_{<i><j>} = \begin{cases} A_1, & j = i-1 & i = s_1+1, s_1+2, \ldots, S_1 \\ A_2, & j = i & i = s_1+1, s_1+2, \ldots, S_1 \\ A_3, & j = i-1 & i = 1,2,3,\ldots, s_1 \\ A_4, & j = i & i = 1,2,3,\ldots, s_1 \\ A_5, & j = i & i = 0 \\ C_1, & j = i+Q_1-1 & i = 0,1,2,\ldots, s_1 \\ C_2, & j = i+Q_1 & i = 0,1,2,\ldots, s_1 \\ 0, & otherwise. \end{cases}$$

where

$$[A_1]_{kl} = \begin{cases} a_1\bar{a}_2, & l = k & k = 1,2,3,\ldots, S_2 \\ a_1a_2, & l = k-1 & k = 1,2,3,\ldots, S_2 \\ a_1, & l = k & k = 0 \\ 0, & otherwise. \end{cases}$$

$$[A_2]_{kl} = \begin{cases} \bar{a}_1\bar{a}_2, & l = k & k = 1,2,3,\ldots, S_2 \\ \bar{a}_1a_2, & l = k-1 & k = 1,2,3,\ldots, S_2 \\ \bar{a}_1, & l = k & k = 0 \\ 0, & otherwise. \end{cases}$$

$$[A_3]_{kl} = \begin{cases} a_1\bar{a}_2, & l = k & k = s_2+1, s_2+2, \ldots, S_2 \\ a_1a_2, & l = k-1 & k = s_2+1, s_2+2, \ldots, S_2 \\ a_1\bar{a}_2\bar{b}, & l = k & k = 1,2,3,\ldots, s_2 \\ a_1a_2\bar{b}, & l = k-1 & k = 1,2,3,\ldots, s_2 \\ a_1\bar{b}, & l = k & k = 0 \\ 0, & otherwise. \end{cases}$$

$$[A_4]_{kl} = \begin{cases} \bar{a}_1\bar{a}_2, & l = k & k = s_2+1, s_2+2, \ldots, S_2 \\ \bar{a}_1a_2, & l = k-1 & k = s_2+1, s_2+2, \ldots, S_2 \\ \bar{a}_1\bar{a}_2\bar{b}, & l = k & k = 1,2,3,\ldots, s_2 \\ \bar{a}_1a_2\bar{b}, & l = k-1 & k = 1,2,3,\ldots, s_2 \\ \bar{a}_1\bar{b}, & l = k & k = 0 \\ 0, & otherwise. \end{cases}$$

$$[A_5]_{kl} = \begin{cases} \bar{a}_2, & l = k & k = s_2+1, s_2+2, \ldots, S_2 \\ a_2, & l = k-1 & k = s_2+1, s_2+2, \ldots, S_2 \\ \bar{a}_2\bar{b}, & l = k & k = 1,2,3,\ldots, s_2 \\ a_2\bar{b}, & l = k-1 & k = 1,2,3,\ldots, s_2 \\ \bar{b}, & l = k & k = 0 \\ 0, & otherwise. \end{cases}$$

$$[C_1]_{kl} = \begin{cases} a_1\bar{a}_2b, & l = k+Q_2 & k = 0,1,2,\ldots, s_2 \\ a_1a_2b, & l = k+Q_2-1 & k = 0,1,2,\ldots, s_2 \\ 0, & otherwise. \end{cases}$$

$$[C_2]_{kl} = \begin{cases} \bar{a}_1\bar{a}_2b, & l = k+Q_2 & k = 0,1,2,\ldots, s_2 \\ \bar{a}_1a_2b, & l = k+Q_2-1 & k = 0,1,2,\ldots, s_2 \\ 0, & otherwise. \end{cases}$$

It may be noted that the matrices $A_1, A_2, A_3, A_4, A_5, C_1$ and C_2 are square matrices of size S_2+1.

4 Steady State Analysis

It can be seen from the structure of the transition probability matrix P, that the discrete time Markov chain $\{(X_t, Y_t), t = 0,1,2,\ldots,\}$ on the finite state space (E) is irreducible. Hence, the limiting distribution, exits and it is defined as

$$\phi_{(i,k)} = \lim_{t\to\infty} Pr[X_t = i, Y_t = k \mid X_0, L_0]$$

exists and is independent of the initial state. We group the probabilities $\phi_{(i,k)}$ as follows:

$$\phi_{(i)} = \left(\phi_{(i,0)}, \phi_{(i,1)}, \ldots, \phi_{(i,S_2)}\right), i = 0,1,\ldots, S_1$$
$$and \quad \Phi = \left(\phi_{(0)}, \phi_{(1)}, \ldots, \phi_{(S_1)}\right)$$

Then, the limiting probability distribution Φ satisfies the following equations

$$\Phi P = \Phi$$
$$and \quad \Phi \mathbf{e} = 1.$$

Published by :
http://www.ijert.org

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

The first equation of the above yields the following set of equations:

$$\phi_{(i+1)}A_3 + \phi_{(i)}A_5 = \phi_{(i)}, \quad i = 0$$

$$\phi_{(i+1)}A_3 + \phi_{(i)}A_4 = \phi_{(i)}, \quad i = 1,2,\ldots,s_1 - 1$$

$$\phi_{(i+1)}A_1 + \phi_{(i)}A_4 = \phi_{(i)}, \quad i = s_1$$

$$\phi_{(i+1)}A_1 + \phi_{(i)}A_2 = \phi_{(i)}, \quad i = s_1 + 1, s_1 + 2, \ldots, Q_1 - 2$$

$$\phi_{(i+1)}A_1 + \phi_{(i)}A_2 + \phi_{(i-Q_1+1)}C_1 = \phi_{(i)}, \quad i = Q_1 - 1$$

$$\phi_{(i+1)}A_1 + \phi_{(i)}A_2 + \phi_{(i-Q_1+1)}C_1 + \phi_{(i-Q_1)}C_2 = \phi_{(i)},$$
$$i = Q_1, Q_1 + 1, \ldots, S_1 - 1$$

$$\phi_{(i)}A_2 + \phi_{(i-Q_1)}C_2 = \phi_{(i)}, \quad i = S_1$$

The equations (expect (7)) can be recursively solved to get

$$\phi_{(i)} = \phi_{(Q_1-1)}\Omega_i, \quad i = 0,1,\ldots,S_1.$$

where

$$\Omega_i = \begin{cases} D_i, \quad i = 0,1,\ldots,s_1, \\ \left[A_1(I - A_2)^{-1}\right]^{Q_1-1-i}, \\ i = s_1 + 1, s_1 + 2, \ldots, Q_1 - 2 \\ I, \quad i = Q_1 - 1, \\ \left\{[(I - A_2)A_1^{-1}] - D_0C_1A_1^{-1}\right\}, \quad i = Q_1 \\ \left\{ \begin{array}{l} [(I - A_2)A_1^{-1}]^{i-(Q_1-1)} \\ - \sum_{j=0}^{i-Q_1} D_jC_1A_1^{-1}[(I - A_2)A_1^{-1}]^{i-(Q_1+j)} \end{array} \right. \\ \left. - \sum_{k=0}^{i-Q_1-1} D_kC_2A_1^{-1}[(I - A_2)A_1^{-1}]^{i-(Q_1+1+j)} \right\}, \\ i = Q_1 + 1, Q_1 + 2, \ldots, S_1. \end{cases}$$

with

$$D_0 = \left[A_1(I - A_2)^{-1}\right]^{s_1-1}\left[A_1(I - A_4)^{-1}\right]\left[A_3(I - A_4)^{-1}\right]^{s_1-1}$$
$$\left[A_3(I - A_5)^{-1}\right] and$$

$$D_i = \left[A_1(I - A_2)^{-1}\right]^{s_1-1}\left[A_1(I - A_4)^{-1}\right]\left[A_3(I - A_4)^{-1}\right]^{s_1-i},$$
$$i = 1,2,\ldots,s_1$$

and $\phi_{(Q_1-1)}$ can be obtained by solving

$$\phi_{(Q_1-1)}\left\{\left[[(I - A_2)A_1^{-1}]^{s_1+1} - \sum_{j=0}^{s_1} D_jC_1A_1^{-1}[(I - A_2)A_1^{-1}]^{(s_1-j)} \right.\right. \quad (3)$$
$$\left. - \sum_{k=0}^{s_1-1} D_kC_2A_1^{-1}[(I - A_2)A_1^{-1}]^{(s_1-1-k)}\right](I - A_2) \quad (4) \quad (5)$$
$$\left. - [A_1(I - A_2)^{-1}]^{s_1-1}[A_1(I - A_4)^{-1}]C_2 \right\} = 0 \quad (6)$$

and

(7)

$$\phi_{(Q_1-1)}\left\{ \begin{array}{l} D_0 + \sum_{i=1}^{s_1} D_i + \sum_{i=s_1+1}^{Q_1-2} [A_1(I - A_2)^{-1}]^{Q_1-1-i} + I \\ + \left([(I - A_2)A_1^{-1}] - D_0C_1A_1^{-1}\right) \end{array} \right. \quad (8)$$

(9)

$$+ \sum_{i=Q_1+1}^{S_1} \left(\begin{array}{l} [(I - A_2)A_1^{-1}]^{i-(Q_1-1)} \\ - \sum_{j=0}^{i-Q_1} D_jC_1A_1^{-1}[(I - A_2)A_1^{-1}]^{i-(Q_1+j)} \end{array} \right. \quad (10)$$

$$\left. \left. - \sum_{k=0}^{i-Q_1-1} D_kC_2A_1^{-1}[(I - A_2)A_1^{-1}]^{i-(Q_1+1+j)} \right) \right\} \mathbf{e} = 1$$

5 System Performance Measures

In this section, we derive some importance system performance measures.

5.1 Expected Inventory level

Let η_{I_1} denote the expected inventory level for the commodity - 1 in the steady state.

$$\eta_{I_1} = \sum_{i=1}^{S_1}\sum_{k=0}^{S_2} i\phi_{(i,k)}.$$

Let η_{I_2} denote the expected inventory level for the commodity - 2 in the steady state.

$$\eta_{I_2} = \sum_{i=0}^{S_1}\sum_{k=1}^{S_2} k\phi_{(i,k)}.$$

5.2 Expected Reorder Rate

Let η_R denote the expected reorder rate in the steady state.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

$$\eta_R = \sum_{i=0}^{s_1} a_2\phi_{(i,s_2+1)} + \sum_{k=0}^{s_2} a_1\phi_{(s_1+1,k)} + a_1a_2\phi_{(s_1+1,s_2+1)}.$$

5.3 Expected Shortage Rate

Let η_{SR_1} denote the expected shortage rate of commodity - 1 in the steady state.

$$\eta_{SR_1} = \sum_{k=0}^{S_2} a_1\phi_{(0,k)}.$$

Let η_{SR_2} denote the expected shortage rate of commodity - 2 in the steady state.

$$\eta_{SR_2} = \sum_{i=0}^{S_1} a_2\phi_{(i,0)}.$$

5.4 Total Expected Cost Rate

The long-run total expected cost per unit time for this system in the steady state is given by

$$TC(S_1,S_2,s_1,s_2) = c_{h_1}\eta_{I_1} + c_{h_2}\eta_{I_2} + c_s\eta_R + c_{r_1}\eta_{SR_1} + c_{r_2}\eta_{SR_2}$$

where

c_{h_1}	The inventory carrying cost of commodity - 1 per unit item per unit time.
c_{h_2}	The inventory carrying cost of commodity - 2 per unit item per unit time.
c_s	Setup cost per order.
c_{r_1}	Shortage cost of commodity - 1 per unit item per unit time.
c_{r_2}	Shortage cost of commodity - 2 per unit item per unit time.

By putting the values of η s' from the above measures of system performance, we obtain $TC(S_1,S_2,s_1,s_2)$ as

$$TC(S_1,S_2,s_1,s_2) = c_{h_1}\left(\sum_{i=1}^{S_1}\sum_{k=0}^{S_2} i\phi_{(i,k)}\right) + c_{h_2}\left(\sum_{i=0}^{S_1}\sum_{k=1}^{S_2} k\phi_{(i,k)}\right)$$

$$+ c_s\left(\sum_{i=0}^{s_1} a_2\phi_{(i,s_2+1)} + \sum_{k=0}^{s_2} a_1\phi_{(s_1+1,k)} + a_1a_2\phi_{(s_1+1,s_2+1)}\right)$$

$$+ c_{r_1}\left(\sum_{k=0}^{S_2} a_1\phi_{(0,k)}\right) + c_{r_2}\left(\sum_{i=0}^{S_1} a_2\phi_{(i,0)}\right)$$

Due to the complex form of the limiting distribution, it is difficult to discuss the qualitative behaviour of the cost function TC analytically. Hence a detailed computational study of the expected cost rate function is carried out in the next section. (13)

A. 6 Numerical Analysis

To study the behaviour of the model developed in this work, several examples were performed and the set of representative results are shown here. Although we have not shown the convexity of $TC(s_1,s_2)$, our experience with considerable numerical examples indicate that the function $TC(s_1,s_2)$, is convex. (14)

A three dimensional plot of $TC(s_1,s_2)$ is presented in Figure 1. We use simple numerical search procedure to get the optimal values of TC, s_1 and s_2 (say TC^*, s_1^* and s_2^* respectively). The minimum (15) value of $TC = 0.829224$ is obtained at $(s_1^*, s_2^*) = (12,4)$.

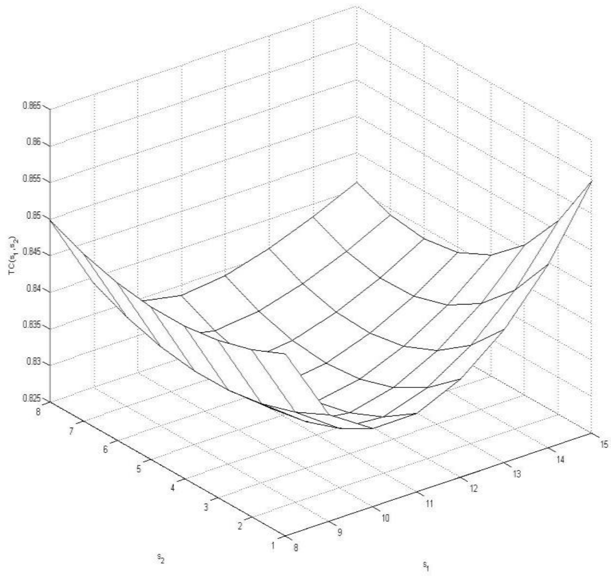

Figure 1: A three dimensional plot of total cost rate per unit time.

We have studied the effect of varying the costs and other system parameters on the optimal values and some of our results are presented in Tables 1 to 12. The lower entry in each cell corresponds to the total optimal cost value TC^* and the upper entries correspond to the local optima S_1^* and S_2^* respectively.

Example 1: We start by examining the effect of the system parameters namely, demand for commodity-1 with the probability, a_1,demand for commodity- 2 with the probability, a_2 and lead time distribution's success probability, b, on the optimal values (S_1^*, S_2^*) and the corresponding optimal cost TC^*. From tables 1 to 3, we observe the following:

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

• The total expected cost rate increases when each of a_1 and a_2 increase and it decreases with the increase in b.

• As is to be expected, S_1^* increase and S_2^* decrease with the increase in a_1. Similarly, S_1^* decrease and S_2^* increase with the increase in a_2. This is because, if a_1 and a_2 increases, then more demands occur. To avoid frequent stock out we have to maintain larger inventory.

• S_1^* and S_2^* decrease with the increase in b. Because if the lead time is small, a smaller stock would be preferable.

Example 2: Next, we study the impact of costs, namely, the holding cost of first commodity c_{h_1}, the holding cost of second commodity c_{h_2}, the setup cost c_s, the shortage cost of first commodity c_{r_1} and the shortage cost of second commodity c_{r_2} on the optimal values (S_1^*, S_2^*) and the corresponding total expected cost rate TC^*. From tables 4 to 12, we observe the following:

• The total expected cost rate increases with the increase in each of the costs $c_{h_1}, c_{h_2}, c_s, c_{r_1}$ *and* c_{r_2}.

• As each of holding costs c_{h_1} and c_{h_2} increases, the optimal values S_1^* and S_2^* are decreasing. This is because if the holding cost is high, more cost is required to maintain the large inventory. To avoid this, we would prefer to maintain small amount of stock.

• As the setup cost c_s increases, the optimal values S_1^* and S_2^* are increasing. If the setup cost is high, we should hold more items in the stock.

• As the shortage cost c_{r_1} increases, the optimal value S_1^* increases and the S_2^* decreases. Similarly the shortage cost c_{r_2} increases, the optimal value S_1^* decreases and the S_2^* increases.

REFERENCES

[1] Abboud, N. E., (2001), A Discrete-Time Markov Production-Inventory Model With Machine Breakdowns, *Computers & Industrial Engineering*, **39,** 95 - 107.

[2] Agarwal, V., (1984), Coordinated order cycles under joint replenishment multi-item inventories, *Naval Logistic Research Quarterly*, 131–136.

[3] Alfa, A. S., (2002), Discrete Time Queues and Matrix-Analytic Methods, *Top*, **10(2),** 147 - 185.

[4] Anbazhagan, N. and Arivarignan, G., (2000), Two-Commodity continuous review inventory system with coordinated reorder policy, *International Journal of Information and Management Sciences*, **11(3),** 19- 30.

[5] Anbazhagan, N. and Arivarignan, G., (2001), A two-commodity coordinated inventory system with renewal demands, *Electronnoe modelirovanie,* **23(2),** 5–16 (in Russian).

[6] Anbazhagan, N. and Arivarignan, G., (2001), Analysis of Two -Commodity Markovian Inventory system with lead time, *The Korean Journal of Computational and Applied Mathematics*, **8(2),** 427 - 438.

[7] Anbazhagan, N. and Arivarignan, G., (2003), Two-Commodity Inventory system with Individual and Joint Ordering Policies, *International Journal of Management and Systems,* **19(2),** 129 - 144.

[8] Bar-Lev, S. K. and Perry, D. (1989), A Discrete Time Markovian Inventory Model for Perishable Commodities, *Stochastic Analysis and Applications*, **7**, 243 - 259.

[9] Kalpakam, S. and Arivarignan, G., (1993), A coordinated multi-commodity (s, S) inventory system, *Mathematical and Computer Modelling*, textbf18, 69–73.

[10] Krishnamoorthy, A. and Varghese, T. V., (1994), A two commodity inventory problem, *International Journal of Information and Management Sciences*, **3,** 55 - 70.

[11] Latouche, G. and Ramaswami, V., (1999), *Introduction to matrix analytic methods in stochastic modeling,*. ASA SIAM, Philadelphia.

[12] Lian, Z. and Liu, L., (1999), A Discrete-Time Model for Perishable Inventory Systems, *Annals of Operations Research*, **87**, 103 - 116.

[13] Lian, Z., Liu, L. and Neuts, M. F., (2005), A Discrete-Time Model For Common Lifetime Inventory Systems, *Mathematics of Operations Research*, **30(3)**, 718 - 732.

[14] Liu, L. and Yang, T., (1999), An (s,S) random lifetime inventory model with a positive lead time, *European Journal of Operations Research*, **113**, 52 - 63.

[15] Miller, B. L., (1971), A multi-item inventory model with joint probability back-order criterion, *Operations Research*, 1467–1476.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

Table 1: Effect of a_1 and a_2 on the optimal values

a_1 / a_2	0.6		0.62		0.64		0.66		0.68		0.7	
0.5	36	32	36	31	37	31	38	31	39	31	40	31
	0.596968		0.602735		0.608424		0.614051		0.619615		0.625116	
0.52	35	32	36	32	37	32	38	32	38	31	39	31
	0.601644		0.607352		0.613006		0.618606		0.624057		0.629384	
0.54	35	33	36	33	36	32	37	32	38	32	39	32
	0.606411		0.612075		0.617655		0.623039		0.628377		0.633667	
0.56	35	34	35	33	36	33	37	33	38	33	38	32
	0.611276		0.616877		0.62224		0.627569		0.63286		0.638019	
0.58	35	35	35	34	36	34	37	34	37	33	38	33
	0.616228		0.621672		0.626976		0.632251		0.63737		0.642355	
0.6	35	35	35	34	36	34	37	34	37	34	38	34
	0.620631		0.62609		0.631466		0.636824		0.641995		0.64691	

Table 2: Effect of a_1 and b on the optimal values

a_1 / b	0.64		0.66		0.68		0.7		0.72		0.74	
0.2	37	35	37	34	38	34	39	34	40	34	41	34
	0.637157		0.642385		0.647428		0.652465		0.6575		0.662533	
0.3	36	34	37	34	37	34	38	34	38	33	39	33
	0.631466		0.636824		0.641995		0.64691		0.651736		0.656385	
0.4	35	34	36	34	37	34	38	34	38	33	39	33
	0.627004		0.632242		0.637487		0.642732		0.647956		0.653081	
0.5	35	34	36	34	37	34	37	34	37	33	38	33
	0.622057		0.62754		0.633048		0.638513		0.643441		0.648374	
0.6	35	34	35	34	36	34	36	33	37	33	38	33
	0.617222		0.622643		0.627774		0.632791		0.63789		0.643072	
0.7	34	34	34	33	35	33	36	33	37	33	38	33
	0.611996		0.617102		0.622171		0.627354		0.632645		0.63804	

Table 3: Effect of a_2 and b on the optimal values

a_2 / b	0.55		0.56		0.57		0.58		0.59		0.6	
0.1	39	36	38	36	38	36	38	37	38	37	38	38
	0.649383		0.652975		0.65673		0.660404		0.664113		0.667944	
0.2	36	34	36	34	36	35	36	35	35	35	35	35
	0.615055		0.617307		0.619912		0.621936		0.624798		0.626548	
0.3	35	33	35	34	35	34	35	35	35	35	35	35
	0.608965		0.611276		0.613486		0.616228		0.618129		0.620631	
0.4	34	33	34	33	34	34	34	34	34	34	34	35
	0.603423		0.605971		0.60882		0.610994		0.613707		0.616184	
0.5	34	33	34	33	34	33	34	34	33	34	33	34
	0.597483		0.600673		0.603083		0.605841		0.608789		0.611194	
0.6	33	32	33	33	33	33	33	33	33	34	33	34
	0.591711		0.594621		0.597293		0.600607		0.603158		0.60599	

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

Table 4: Effect of c_{h_1} and c_{h_2} on the optimal values

c_{h_1} / c_{h_2}	0.01		0.011		0.012		0.013		0.014		0.015	
0.01	39	36	38	35	38	35	36	34	36	34	36	34
	0.658962		0.671693		0.684347		0.695961		0.706759		0.717556	
0.011	38	35	38	35	37	34	36	34	35	33	35	33
	0.681807		0.694461		0.706773		0.718256		0.728977		0.739367	
0.012	37	34	37	34	37	34	35	33	35	33	35	33
	0.704464		0.716751		0.729038		0.74038		0.750771		0.761162	
0.013	37	34	36	33	36	33	35	33	35	33	34	32
	0.726728		0.738965		0.750891		0.762175		0.772565		0.782552	
0.014	36	33	36	33	36	33	34	32	34	32	34	32
	0.7488		0.760726		0.772652		0.783876		0.793861		0.803847	
0.015	36	33	35	32	35	32	34	32	34	32	33	31
	0.770561		0.782485		0.794057		0.80517		0.815156		0.824974	

Table 5: Effect of c_{h_1} and c_s on the optimal values

c_{h_1} / c_s	0.01		0.011		0.012		0.013		0.014		0.015	
6	34	30	33	30	32	29	32	29	32	29	31	28
	0.559854		0.572914		0.583524		0.594086		0.604649		0.615116	
7	36	32	34	31	34	31	33	30	33	30	32	30
	0.5878		0.600346		0.611571		0.622726		0.633614		0.644131	
8	37	33	36	33	35	32	35	32	34	32	33	31
	0.613436		0.625831		0.637443		0.649015		0.660361		0.670101	
9	37	34	37	34	36	33	36	33	35	33	34	32
	0.63714		0.649427		0.661682		0.673608		0.684192		0.694508	
10	39	36	38	35	38	35	36	34	36	34	36	34
	0.658962		0.671693		0.684347		0.695961		0.706759		0.717556	
11	40	37	39	36	38	36	37	35	37	35	37	35
	0.679541		0.692795		0.70546		0.717021		0.728226		0.739431	

Table 6: Effect of c_{h_1} and c_{r_1} on the optimal values

c_{h_1} / c_{r_1}	0.011		0.012		0.013		0.014		0.015		0.016	
0.7	37	36	36	36	36	36	35	36	34	35	34	35
	0.652792		0.663201		0.672823		0.682189		0.690589		0.698903	
0.8	38	36	38	36	37	36	36	35	35	35	35	35
	0.665074		0.676875		0.687695		0.697956		0.707688		0.716889	
0.9	39	36	38	35	37	35	37	35	36	35	35	34
	0.674766		0.687933		0.699703		0.711105		0.721956		0.732028	
1.0	39	35	38	35	38	35	37	35	36	34	36	34
	0.682774		0.696613		0.709516		0.722362		0.733689		0.744693	
1.1	39	35	39	35	38	35	37	34	37	34	36	34
	0.68885		0.703775		0.718196		0.730973		0.743522		0.755986	
1.2	40	35	39	35	38	34	38	34	37	34	37	34
	0.694616		0.70985		0.724622		0.739288		0.752122		0.764671	

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

Table 7: Effect of c_{h_1} and c_{r_2} on the optimal values

c_{h1} / c_{r2}	0.006		0.007		0.008		0.009		0.01		0.011	
3	35	37	34	37	33	36	32	35	32	35	31	34
	0.562557		0.577559		0.591813		0.605683		0.619037		0.632065	
3.5	35	38	34	37	33	36	32	35	32	35	31	35
	0.564629		0.579275		0.59347		0.607279		0.620633		0.632951	
4	35	38	34	37	33	36	32	35	32	36	31	35
	0.566403		0.58099		0.595126		0.608875		0.621751		0.633654	
4.5	35	38	34	37	33	36	32	36	32	36	31	35
		0.568176	0.582706		0.596783		0.610192		0.622483		0.634356	
5	35	38	34	37	33	37	32	36	32	36	31	35
	0.569949		0.584421		0.598219		0.610924		0.623215		0.635059	
5.5	35	38	34	38	33	37	32	36	32	36	31	35
	0.571723		0.585905		0.598981		0.611656		0.623946		0.635761	

Table 8: Effect of c_{h_2} and c_s on the optimal values

c_{h2} / c_{s}	0.01		0.011		0.012		0.013		0.014		0.015	
6	34	30	33	29	33	29	32	28	32	28	31	27
	0.559854		0.579678		0.598989		0.618152		0.636849		0.655391	
7	36	32	35	31	34	30	34	30	33	29	33	29
	0.5878		0.60832		0.6287		0.648599		0.668356		0.687667	
8	37	33	36	32	36	32	35	31	35	31	34	30
	0.613436		0.634927		0.655946		0.676762		0.697229		0.717445	
9	37	34	37	34	36	33	37	33	36	32	36	32
	0.63714		0.659405		0.681352		0.702948		0.724093		0.745113	
10	39	36	38	35	37	34	37	34	36	33	36	33
	0.658962		0.681807		0.704464		0.726728		0.7488		0.770561	
11	40	37	39	36	39	36	38	35	37	34	37	34
	0.679541		0.703039		0.72631		0.749101		0.771788		0.794052	

Table 9: Effect of c_{h_2} and c_{r_1} on the optimal values

c_{h2} / c_{r1}	0.012		0.013		0.014		0.015		0.016		0.017	
0.75	35	33	34	32	33	31	33	31	33	31	32	30
	0.655662		0.675952		0.696178		0.715965		0.735751		0.755042	
0.8	35	33	34	32	33	31	33	31	33	31	32	30
	0.661314		0.681624		0.701871		0.721658		0.741445		0.760759	
0.85	35	32	35	32	34	31	34	31	33	30	33	30
	0.666932		0.68717		0.707301		0.727034		0.746706		0.765932	
0.9	35	32	35	32	34	31	34	31	33	30	33	30
	0.671257		0.691494		0.711583		0.731316		0.750941		0.770167	
0.95	35	32	35	32	34	31	34	31	34	30	34	30
	0.675581		0.695819		0.715865		0.735598		0.755173		0.774125	
1	36	32	36	32	35	31	35	31	34	30	34	30
	0.679587		0.699618		0.719319		0.738814		0.757917		0.776868	

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

Table 10: Effect of c_{h_2} and c_{r_2} on the optimal values

c_{h2} / c_{r2}	0.012		0.013		0.014		0.015		0.016		0.017	
2.5	36	32	36	32	35	31	35	31	34	30	34	30
	0.679931		0.699962		0.71973		0.739225		0.758413		0.777365	
3	35	32	35	32	35	31	35	31	34	30	34	30
	0.680019		0.700256		0.720141		0.739636		0.75891		0.777861	
3.5	35	32	35	32	34	31	35	31	34	30	34	30
	0.680075		0.700313		0.720338		0.740046		0.759406		0.778358	
4	35	32	35	32	34	31	34	31	33	30	34	30
	0.680132		0.70037		0.720402		0.740135		0.7597		0.778854	
4.5	35	32	35	32	34	31	34	31	33	30	33	30
	0.680189		0.700427		0.720466		0.740199		0.759772		0.778999	
5	35	32	35	32	34	31	34	31	33	30	33	30
	0.680246		0.700484		0.72053		0.740263		0.759845		0.779071	

Table 11: Effect of c_s and c_{r_1} on the optimal values

c_s / c_{r1}	6		7		8		9		10		11	
0.5	32	31	33	33	34	35	35	36	35	37	36	38
	0.520918		0.545697		0.568678		0.589853		0.609871		0.628745	
0.55	32	31	34	33	35	35	36	36	36	37	37	38
	0.528152		0.553657		0.577177		0.598884		0.61914		0.638388	
0.6	33	31	35	33	35	34	36	36	37	37	38	38
	0.534011		0.560708		0.584562		0.60691		0.627499		0.6472	
0.65	34	31	35	33	36	34	37	36	38	37	38	38
	0.539671		0.566374		0.591075		0.613847		0.635036		0.655015	
0.7	34	31	35	32	36	34	37	35	38	37	39	38
	0.543823		0.571555		0.596722		0.620002		0.641799		0.661972	
0.75	34	31	35	32	37	34	38	35	39	37	40	38
	0.547975		0.575761		0.601546		0.625613		0.647485		0.668296	

Table 12: Effect of c_s and c_{r_2} on the optimal values

c_s / c_{r2}	9		10		11		12		13		14	
2.5	34	29	35	30	36	31	37	32	38	33	38	34
	0.587972		0.608885		0.628377		0.646635		0.663832		0.679977	
3	34	29	35	30	36	31	37	32	38	33	38	34
	0.588523		0.609261		0.628637		0.646818		0.663964		0.680007	
3.5	34	29	35	30	36	31	37	32	37	33	38	34
	0.589075		0.609637		0.628898		0.647002		0.664033		0.680036	
4	34	29	35	30	36	31	37	32	37	33	38	34
	0.589626		0.610012		0.629158		0.647186		0.664072		0.680065	
4.5	34	29	35	30	36	31	36	32	37	33	38	34
	0.590178		0.610388		0.629418		0.647304		0.664111		0.680095	
5	33	29	34	30	35	31	36	32	37	33	38	34
	0.590556		0.610735		0.629606		0.647357		0.66415		0.680124	

[16] Neuts, M.F., (1981), *Matrix-geometric solutions in stochastic models: An algorithmic approach*, Johns Hopkins University Press, Baltimore.
[17] Ohno, K. and Ishigaki, T., (2001), A multi-item continuous review inventory system with compound Poisson demands, *Mathematical Methods of Operations Research,* **53,** 147–165.
[18] Silver, E. A., (1974), A control system for coordinated inventory replenishment, *International Journal of Production Research,* **12,** 647–671.
[19] Sivakumar, B., Anbazhagan, N. and Arivarignan, G., (2005), A two-commodity perishable inventory system, *ORiON,* **21(2)**, 157 - 172.
[20] Sivakumar, B., Anbazhagan, N. and Arivarignan, G., (2005), Two-commodity continuous review perishable inventory system, *International Journal of Information and Management Sciences,* **17(3)**, 47 - 64.
[21] Sivakumar, B., (2008), Two-commodity inventory system with retrial demands, *European Journal of Operational Research*, **187**, 70 - 83.
[22] Srinivasan, S. K. and Ravichandran, N., (1994), Multi-item (S, s) inventory model with Poisson demand, general lead time and adjustable reorder size, in: G.V. Krishna Reddy et al. (Eds.), (Reviewed edited), Proceedings of the Conference of Stochastic Models, *Optimization and Computer Application*, *Wiley Eastern Limited*, 226–236.
[23] Takagi, H. (1993), Queueing analysis: A foundation of performance evaluation, Discrete-Time Systems, *J.Math.Phy.Sci.*, **3**, North-Holland, Amsterdam.
[24] Thomstone, T. M. and Silver, E. A., (1975), A coordinated inventory control system, *International Journal of Production Research*, 581–602.
[25] Woodward, M. E. (1994), Communication and computer networks: Modelling with Discrete-time queues, *IEEE Computer Society Press*, Los Alamitos, CA.
[26] Yadavalli, V. S. S., Anbazhagan, N. and Arivarignan, G., (2004), A two-commodity continuous review inventory system with lost sales, *Stochastic Analysis and Applications,* **22,** 479–497.
[27] Yadavalli, V. S. S., Van Schoor, C. De W. S. and Udayabaskaran., (2006), A substitutable two-product inventory system with joint-ordering policy and common demand, *Applied Mathematics and Computation,* **172 (2),** 1257–1271.