International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 3 Issue 9, September- 2014

A Fixed Point Theorem on Semi-Metric Space using Occasionally Weakly Compatible Mappings

B. Vijayabasker Reddy
Department of Mathematics, Sreenidhi Institute of Science and Technology,Ghatkesar, Hyderabad, India-501 301

V. Srinivas
Department of Mathematics,University college of Science,Saifabad,Osmania UniversityHyderabad, India.

***Abstract*- The aim of this paper is to prove common fixed point theorem for four self mappings in semi -metric space using the concept of occasionally weakly compatible. This theorem generalizes the result of Bijendra Singh and M.S Chauhan[1].**

***Keywords*- Semi- metric space ,coincidence point, weakly compatible, occasionally weakly compatible, Fixedpoint.**

I. INTRODUCTION

The concept of semi-metric space is introduced by Menger, which is a generalization of metric space.Cicchese introduced the notion of a contractive mappings in semi-metric space and proved the fixed point theorem.In 2006 Jungck and Rhoades introduced the concept of Occasionally weakly compatible mappings which generalizes weakly compatible mappings.

II PRELIMINARIES

Definition 1 : (X ,d) is said to be Semi- metric space if and only if it satisfies the following conditions:

M_1: d(x ,y)=0 if and only of x=y.

M_2:d(x ,y)=d(y ,x) if and only if x=y

for any x,y∈X.

Definition 2 **:** Let A and B be two self mappings of a semi metric space (X, d) then A and B are said to be weakly compatible mappings if they commute at their coincidence points.

Definition 3 **:** Let A and B be two self maps of a semi metric space (X ,d) then A and B are said to be occasionally weakly compatible mappings if there is a coincidence point x∈X of A and B at which A and B are commute.

Remark 1: Weakly compatible mappings are occasionally compatible mappings but converse is not true.

Example 1: Let (X, d) be semi-metric space with X=[1/2,5] and d(x ,y)=$(x-y)^2$.
Define two self mappings A and B as follows

$$A(x)=\begin{cases} x^2 & if\ \frac{1}{2}\le x<1 \\ 2x-1 & if\ \ x\ge 1\end{cases} \quad and$$

$$B(x)=\begin{cases} 2x & if\ \frac{1}{2}\le x<1 \\ x^2 & if\ \ x\ge 1\end{cases}$$

Clearly, X=1/2 and x=1 are two coincidence points. If x=1 then A(1)=1=B(1) which gives AB(1)=1=BA(1).If x=1/2 then A(1/2)=B(1/2)=1/4 but AB(1/2)≠BA(1/2).Therefore A and B are occasionally weakly compatible but not weakly compatible

Lemma 1: Let (X ,d) be a semi-metric space, A,B are occasionally weakly compatible mappings of X. If the self mappings A and B on X have a unique point of coincidence w=Ax=Bx. Then w is unique common fixed point of A and B.

Proof: Since A and B are occasionally weakly compatible mappings, there exists a point x∈X such that Ax=Bx=w and ABx=BAx.Thus AAx=ABx=BAx Which gives Ax is also point of coincidence of A and B. since the point of coincidence w=Ax is unique then, BAx=AAx=Ax ,and w=Ax is a common fixed point of A and B.If z is any common fixed point of and A and B then z=Az=Bz=w by the uniqueness of the point of coincidence**.**

IJERT

(This work is licensed under a Creative Commons Attribution 4.0 International License.)

III MAIN RESULT

Theorem 1: Let A,B,S,T,P and Q be self maps on a semi metric space (X ,d) If

(i) (AP,S) and (BQ,T) are occasionally weakly compatible mappings.

(ii)

$$[d(APx,BQy)]^2 \le k_1 \begin{bmatrix} d(APx,Sx)\,d(BQy,Ty)+ \\ d(BQy,Sx)d(APx,Ty) \end{bmatrix} + k_2 \begin{bmatrix} d(APx,Sx)d(APx,Ty)+ \\ d(BQy,Ty)d(BQy,Sx) \end{bmatrix}$$

Where x,y∈X and $k_1+2k_2 \le 1, k_1,k_2 \ge 0$

then AP,BQ,S and T have a Common fixed point. Further if AP=PA,BQ=QB Then A,B,P,Q,S and T have a common fixed point ,

Proof: (AP,S) and (BQ,T) are occasionally weakly compatible, then there exists some x,y∈X such that

APx=Sx and BQy=Ty. Using (ii) we claim APx=BQy.

$$[d(APx,BQy)]^2 \le k_1 \begin{bmatrix} d(APx,APx)\,d(BQy,BQy)+ \\ d(BQy,APx)d(APx,BQy) \end{bmatrix} + k_2 \begin{bmatrix} d(APx,APx)d(APx,BQy)+ \\ d(BQy,BQy)d(BQy,APx) \end{bmatrix}$$

$$[d(APx,BQy)]^2 \le k_1 [d(BQy,APx)d(APx,BQy)]$$

$$[d(APx,BQy)]^2 (1-k_1) \le 0$$

This is contradiction. So APx=BQy.

Therefore APx=BQy=Sx=Ty.

if there is another point of coincident say,w such that APz=Sz=w then APz=Sz=BQy=Ty. Which gives APz=APx implies z=x.

Hence w=APx=Sx for w∈X is the unique point of coincidence of AP and S.By lemma (1.1) w is a fixed point of AP and S Hence APw=Sw=w.Similarly there exists a common fixed point u∈X such that u=BQu=Tu.

Suppose u≠w

Put x=w and y=u in (ii)

$$[d(w,u)]^2 = [d(APw,BQu)]^2 \le k_1 \begin{bmatrix} d(APw,Sw)\,d(BQu,Tu)+ \\ d(BQu,Sw)d(APw,Tu) \end{bmatrix} + k_2 \begin{bmatrix} d(APw,Sw)d(APw,Tu)+ \\ d(BQu,Tu)d(BQu,Sw) \end{bmatrix}$$

$$[d(w,u)]^2 \le k_1 [d(u,w)d(w,u)]$$

$$[d(w,u)]^2 (1-k_1) \le 0$$

This is contradiction. There fore u=w.Hence w is unique common fixed point of AP,BQ,S and T.

If AP=PA and BQ=QB then Aw=A(APw)=A(Paw)=AP(Aw).

Put x=w and y=Aw in (ii)

$$[d(APw,BQ(Aw))]^2 \le k_1 \begin{bmatrix} d(APw,Sw)\,d(BQ(Aw),T(Aw))+ \\ d(BQ(Aw),Sw)d(APw,T(Aw)) \end{bmatrix} + k_2 \begin{bmatrix} d(APw,Sw)d(APw,T(Aw))+ \\ d(BQ(Aw),T(Aw))d(BQ(Aw),Sw) \end{bmatrix}$$

IJERT

$$[d(w,Aw]^2 \le k_1 [d(Aw,w)d(w,Aw)]$$

$$[d(w,Aw]^2 \le k_1 [d(w,Aw]^2$$

$$[d(w,Aw]^2 (1-k_1) \le 0$$

Which gives $w = Aw$

Pw= A(Pw)=P(Aw)=w.
Similarly we have Bw=Qw=w.
Hence A,B,S,T,P and Q have unique fixed point.

Example 2 **:**

Let (X,d) be the semi-metric Space with $X = [0,1/2]$ and $d = (x-y)^2$.

Define Self mappings A,B,T,S,P *and* Q as

$$A(x) = \frac{2x+1}{4}, B(x) = \frac{4x+1}{6}, T(x) = \frac{4x+3}{10},$$

$$S(x) = \frac{6x+1}{8}, P(x) = \frac{2+6x}{10} \ \ and \ \ Q(x) = \frac{2x+3}{8}.$$

.

(This work is licensed under a Creative Commons Attribution 4.0 International License.)

Also the mappings satisfy all the conditions of theorem 1. Here the common fixed point is 1/2

REFERENCES:

[1] Bijendra Singh and S.Chauhan," On common fixed poins of four mapping"s, Bull.Cal.Math.Soc.,88,301-308,1998

[2] k.jha,M .Imdad and U.rajopadhyaya,"Fixed point theorems for Occasionally Weakly compatible Mappings in semi metric space",Annals of pure and Applied Mathematics Vol 5,No.2,2014.

[3] K.Menger Untersuchungenuberallgemeine,Math Annalen 100,75-163, 1928

[4] M. Aamri and D.EI.Moutawakil," Common fixed points under contractive conditions in symmetric Space" ,Applied Mathematics E-Notes,3156-162, 2003

[5] G.jungck and B.E Rhoades ,"Fixed point theorems for occasionally weakly compatible mappings",Fixed Point Theory,7,280-296, 2006.

[6] B.D Pant and S Chauhan,"Common fixed point theorem for occasionally weakly compatible mappings in menger space",Srveys in Maths 1-7, Appl 2001.

[7] W.A.Wilson",OnSemi-metric space",Amer.J.Math,53,361-373,1931.

[8] M.A.A1-Thagafi and Naseer Shahzad,"A note on Occasionally weakly compatible Maps",Int.journal of Math.Analysis,Vol.3, ,no 2,55-58,2009.

IJERT

(This work is licensed under a Creative Commons Attribution 4.0 International License.)