A Fuzzy Supply Chain Model for Defective Items

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ABSTRACT

In this paper, we consider a supply-chain (SC) production-inventory control system consisting of a supplier, producer (having a warehouse and a production center) and retailer. The whole system is considered for a finite time period with fuzzy demand for finished products and fuzzy inventory costs. Here shortages are fully backlogged. There are fuzzy chance constraints on the transportation costs for both producer, retailer and also a space constraint for producer is considered. Then for the integrated case the model is formulated as fuzzy chance constraint programming problem where constraint is satisfied with some predefined degree of necessity. As optimization of fuzzy objective is not well defined a necessity based return of the objective is optimized under the constraint. Then the model is transferred to a crisp one using fuzzy extension principle and solved using LINGO software. For non-integrated case the model is solved applying an appropriate interactive fuzzy decision making (IFDM) method for multi-objective. The fuzzy model provides the decision maker with alternative decision plans for different degrees of satisfaction. This proposal is tested by using data from a real supply chain. Results indicate the efficiency of proposed approach in performance measurement.

KEYWORDS

Supply chain model, Necessity, Interactive Fuzzy Decision Making Method.

1. Introduction

A supply chain model (SCM) is a network of supplier, producer, distributor and customer which synchronizes a series of inter-related business processes in order to plan for: (*i*) optimal procurement of raw materials from open market, (*ii*) transportation of raw materials into warehouse, (*iii*) production of the goods in the production centre and (*iv*) distribution of these finished goods to retailer for sale to the customers. With a recent paradigm shift to the supply chain (SC), the ultimate success of a firm may depend on its ability to link supply chain members seamlessly.

One of the earliest efforts to create an integrated supply chain model dates back to Bookbinder *et al.*[3], Cachon and Zipkin [4], Cohen and Lee [9], Newhart *et al.* [24] etc. They developed a production, distribution and inventory (PDI) planning system that integrated three supply chain segments comprised of supply, storage / location and customer demand planning. The core of the PDI system was a network model and diagram that increased the decision makers insights into supply chain connectivity. The model, however was confined to a single-period and single-objective problem. Viswanathan and Piplani [31] concerned an integrated inventory model through common replenishment in the SC. All the above SCMs are considered with constant, known demand and production rates.

Gradually the time varying demand over a finite planning horizon has attracted the attention of researchers (Donaldson [10], Chang and Dye [5] and others. This type of demand is observed in the case of fashionable goods, seasonable products, etc. Moreover, there are a lot of item which deteriorate continuously. Articles (Zhou *et al.* [36] and others) on inventory model of deteriorating items are available in the literature. Rau *et al.* [29] developed an integrated SCM of a deteriorating item with shortages. Ben-Daya and Al-Nassar [3] developed an integrated inventory production system in a three-layer supply chain. Jaber and Goyal [18] coordinated a three-level supply chain. Yan *et al.*[39] developed an integrated production-distribution model for a deteriorating inventory item. Sajadieh *et al.* [33] developed an integrated vendor buyer model with stock-dependent demand. Recently, Sana [34] developed a production-inventory model of imperfect quality products in a three-layer supply chain and Ben-Daya *et al.* [4] also developed an integrated production inventory model with raw material replenishment considerations in a three layer supply chain. All the above SCMs are developed in crisp environment.

After the development of fuzzy set theory by Zadeh [34], it has been extensively used in different field of science and technology to model complex decision making problems. It has been applied to model real life inventory control problems during last two decades [1, 2, 13, 14] etc. Since Zimmermann [37, 38] first introduced fuzzy set theory into the ordinary linear programming (LP) and multi-objective linear programming (MOLP) problems, several fuzzy mathematical programming techniques have developed by researchers to solve fuzzy production and/or distribution planing problems. Santoso *et al.* [30] developed a stochastic programming approach for supply chain network design under uncertainty. Leung *et al.* [19] developed stochastic programming approach for multi-site aggregate production planning. Inuiguchi and Ramik [15] reviewed several existing techniques and newly developed ideas in fuzzy mathematical programming. Petrovic *et al.* [26] described fuzzy modelling and simulation of a supply chain in an uncertain environment to determine the stocks levels and order quantities for each inventory for obtaining an acceptance delivery performance of a reasonable total cost for the whole supply chain, in which two sources of uncertain customer demand and external supply of raw materials were identified and modelled by fuzzy sets. Moreover, Petrovic *et al.* [27] developed a heuristic based on fuzzy set theory to determine the order quantities for each inventory in a supply chain in the presence of uncertainties associated with customer demand, deliveries along the supply chain and external or market supply to provide an acceptance service level of the supply chain at reasonable total costs. Additionally, Petrovic [28] developed a special purpose simulation tool, SCSIM, for analyzing supply chain behavior and performance in an uncertain environment. The SCSIM were comprised of two types of models: supply fuzzy analytical models to determine the optimal order-up-to levels for all inventories and simulation model to evaluate supply chain performance achieved over time by applying the order-up-to levels recommended by the fuzzy models. Nair and Closs [23] examined of the impact of coordinating supply chain policies and price markdowns on short life cycle product retail performance. Lee and Kim [17] developed a production distribution planning in supply chain considering capacity constraints. Lee *et al.* [18] developed a production-distribution planning in supply chain using a hybrid approach.

Chen and Lee [6] designed a supply chain scheduling model as a multi-products, multi-stages and multi-periods mixed integer nonlinear programming problem with uncertain market demand, to satisfy conflict objectives theorin the compromised preference levels on product prices from the sellers and buyers point of view were simultaneously taken into account. Wang and Shu [32] presented a fuzzy supply chain model by combining possibility theory and genetic algorithm approach to provide an alternative framework to handle supply chain uncertainties and to determine inventory strategies. Chen and Huang [7] proposed a fuzzy model by combining fuzzy set theory with programm evaluation and review technique(PERT) to calculate the total cycle time of a supply chain system. That fuzzy model adopted triangular fuzzy numbers to describe these uncertain variables and the promise delivery possibility index is defined to indicate the order fulfilment degree of a supply chain system based on the fuzzy completion time and fuzzy due date. Xie *et al.* [33] designed a two-level hierarchical method to inventory management and control in serial supply chains, in which the supply chain operated under imprecise customer demand and was modelled by fuzzy sets. More recently, Liang [20] presented a fuzzy programming approach for solving the manufacturing/ distribution planning decisions (MDPD) problems with fuzzy goals and certain constraints in a supply chain under uncertain environment. That approach adopts the piecewise linear membership function to represent the fuzzy goals of the DM for the integrated MDPD problems and achieves more flexible doctrines through an interactive decision-making process. Related investigations on solving the fuzzy MDPD problems included Kumar *et al.* [16], Chen *et al.* [8].

In this paper, we consider a supply-chain (SC) production-inventory control system consisting of a single supplier, single producer (having a warehouse and a production center) and retailer. The imprecise demands of the goods are made to the retailer by the customers. These goods are produced (along with a defectiveness) from a raw material in the producers production center with controllable production rate. Producer store these raw materials in a warehouse purchasing these from a supplier and the supplier collects these raw materials from open market / nature at a constant collection rate. The SC starts with the collection of raw materials, then storage and production and ends with the distribution of finished goods to the retailer and sale of those units by the retailer to the customers. The whole system is considered for a finite time period with fuzzy demand for finished products and fuzzy inventory costs. Here shortages are fully backlogged. There are fuzzy chance constraints on the transportation costs for both producer and retailer and also a space constraint is considered. Then for the integrated case the model is formulated as fuzzy chance constraint programming problem where constraint is satisfied with some predefined degree of necessity. A necessity based return of the objective is optimized under the constraint. Then the model is transferred to a crisp one using fuzzy extension principle and solved using LINGO software. For non-integrated case the model is solved applying an appropriate interactive fuzzy decision making method (IFDM) for multi-objective is applied to solve the model. The fuzzy model provides the decision maker with alternative decision plans for different degrees of satisfaction. This proposal is tested by using data from a real supply chain. Results indicate the efficiency of proposed approach in performance measurement. A numerical example has been considered to illustrate the model. The outline of this paper is as follows. Section-2 contains discussion on the basics of fuzzy set theory connecting to this work. Section-3 contains relevant assumptions and notations connected to the model. Section-4 and Section-5 present the mathematical formulation and model formulation respectively of the proposed supply chain model. In Section-6 and Section-7, the illustration with a numerical example and practical implementation are presented respectively and the final section contains the concluding remark and future researches.

2. Prerequisite Mathematics

Any fuzzy subset $\tilde{A}$ of $\Re$(where $\Re$ represents the set of real numbers) with membership function $\mu_{\tilde{A}} : \Re \longrightarrow [0,1]$ is called a fuzzy number. Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers with membership functions $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ respectively. Then taking degree of uncertainty as the semantics of fuzzy number, according to Zadeh [35], Dubois and Prade [11], Liu and Iwamura [21]:

$$\text{Pos } (\tilde{A} \star \tilde{B}) = \sup\{\min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)), x, y \in \Re, x \star y\} \tag{1}$$

where the abbreviation Pos represent possibility and $\star$ is any one of the relations $>, <, =, \leq, \geq$. Analogously if $\tilde{B}$ is a crisp number, say b, then

$$\text{Pos } (\tilde{A} \star b) = \sup\{\mu_{\tilde{A}}(x), x \in \Re, x \star b\} \tag{2}$$

On the other hand necessity measure of an event $\tilde{A} \star \tilde{B}$ is a dual of possibility measure. The grade of necessity of an event is the grade of impossibility of the opposite event and is defined as:

$$\text{Nes } (\tilde{A} \star \tilde{B}) = 1 - \text{Pos } (\overline{\tilde{A} \star \tilde{B}}) \tag{3}$$

where the abbreviation Nes represents necessity measure and $\overline{\tilde{A} \star \tilde{B}}$ represents complement of the event $\tilde{A} \star \tilde{B}$.

If $\tilde{A}, \tilde{B} \in \Re$ and $\tilde{C} = f(\tilde{A}, \tilde{B})$ where $f : \Re \times \Re \rightarrow \Re$ be a binary operation then membership function $\mu_{\tilde{C}}$ of $\tilde{C}$ can be obtained using fuzzy extension principle [11, 34] as

$$\mu_{\tilde{C}}(z) = \sup \{\min (\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)), x, y \in \Re, \text{ and } z = f(x, y), \forall z \in \Re\} \tag{4}$$

According to this principle if $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers (TFNs) with positive components then $\tilde{A}+\tilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3)$ is a TFN. Furthermore if $a_2 - a_1$, $a_3 - a_2$, $b_2 - b_1$, $b_3 - b_2$ are small then $\tilde{A}.\tilde{B} = (a_1.b_1, a_2.b_2, a_3.b_3)$ is approximately a TFN [11].

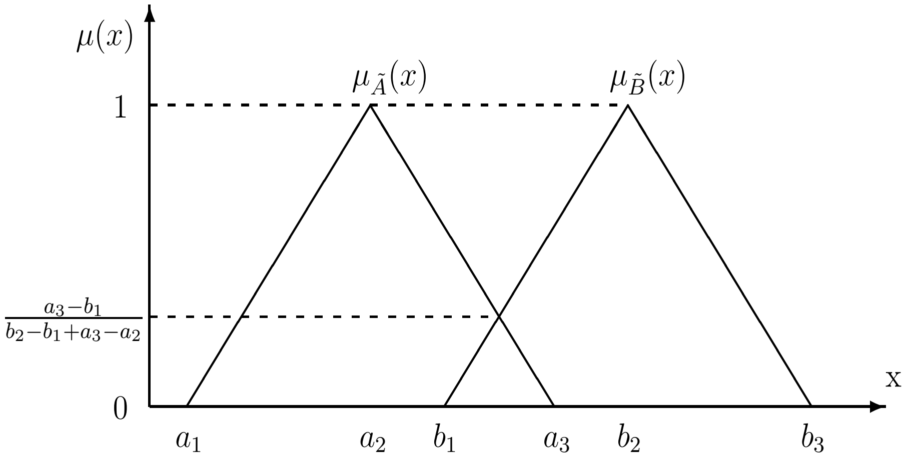

Fig-1: Comparison of two TFNs $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$

Lemma-1: If $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be TFNs with $0 < a_1$ and $0 < b_1$ then $\text{Nes}(\tilde{b} > \tilde{a}) \geq \alpha$ iff $\dfrac{a_3 - b_1}{b_2 - b_1 + a_3 - a_2} \leq 1 - \alpha$.

Proof: We have $\text{Nes}(\tilde{b} > \tilde{a} \geq \alpha \Rightarrow \{1 - Pos(\tilde{b} \leq \tilde{a})\} \geq \alpha \Rightarrow Pos(\tilde{b} \leq \tilde{a}) \leq 1 - \alpha$
So from Fig-1 it is clear that

$$Pos(\tilde{b} \leq \tilde{a}) = \begin{cases} 1 & \text{for } a_2 \geq b_2 \\ \dfrac{a_3 - b_1}{b_2 - b_1 + a_3 - a_2} & \text{for } a_2 \leq b_2 \text{ and } a_3 \geq b_1 \\ 0 & \text{otherwise} \end{cases}$$

Since $0 \leq \alpha \leq 1$, $Pos(\tilde{b} \leq \tilde{a}) \leq 1 - \alpha$, iff $\dfrac{a_3 - b_1}{b_2 - b_1 + a_3 - a_2} \leq 1 - \alpha$, and hence the result follows.

Lemma-2: If $\tilde{a} = (a_1, a_2, a_3)$ be a TFN with $0 < a_1$ and b be a crisp number then $\text{Nes}(b > \tilde{a}) \geq \alpha$ iff $\dfrac{a_3 - b}{a_3 - a_2} \leq 1 - \alpha$.

Proof: Proof follows from Lemma-1(Put $b_1 = b_2 = b_3 = b$ in Lemma-1).

2.1. Interactive Fuzzy Decision Making Method

Considering the imprecise nature of DM's judgment, DM may have different fuzzy or imprecise goals for each of the objective functions and hence interactive approach is used for the man-machine interaction.

Pay-off matrix

Here DM first derive the membership functions for the objective functions f_j, (j=1,2,...,k) respectively from DM's viewpoint with the help of individual minimum and individual maximum by non-linear optimization method(GRG).

Membership function

With the help of individual minimum and maximum, the DM can formulate and select any one from among the following three types of membership functions. (i) Linear membership functions. (ii) Quadratic membership functions. (iii) Exponential membership functions. The membership functions for the corresponding objective functions f_j, (j=1,2,...,k) may be written as

Type-1: Linear membership function

For each objective function, the corresponding linear membership functions are as follows:

$$\mu_{f_j}(x) = \begin{cases} 0 & \text{for } f_j^0 > f_j(x) \\ 1 - \dfrac{f_j^1 - f_j(x)}{f_j^1 - f_j^0} & \text{for } f_j^0 \leq f_j(x) \leq f_j^1 \\ 1 & \text{for } f_j(x) > f_j^1 \end{cases} \tag{5}$$

Type-2: Quadratic membership function

For each objective function, the corresponding quadratic membership functions are as follows:

$$\mu_{f_j}(x) = \begin{cases} 0 & \text{for } f_j^0 > f_j(x) \\ 1 - \left(\dfrac{f_j^1 - f_j(x)}{f_j^1 - f_j^0}\right)^2 & \text{for } f_j^0 \leq f_j(x) \leq f_j^1 \\ 1 & \text{for } f_j(x) > f_j^1 \end{cases} \tag{6}$$

Type-3: Exponential membership function

For each objective function, the corresponding exponential membership functions are as follows:

$$\mu_{f_j}(x) = \begin{cases} 0 & \text{for } f_j^0 > f_j(x) \\ \alpha_r \left[1 - e^{-\beta_K \left(\frac{(f_j^1 - f_j(x))}{(f_j^1 - f_j^0)}\right)}\right] & \text{for } f_j^0 \leq f_j(x) \leq f_j^1 \\ 1 & \text{for } f_j(x) > f_j^1 \end{cases} \tag{7}$$

The constants can be determined by asking the DM to specify the three points such that and is the tolerance of j-th objective function f_j .

Parametric values

The DM gives the goal parametric values for the membership function which are determined following Sakawa. et. al (2004) as:

$$\begin{aligned} f_j^1 &= f_j(x_i^{max}) \\ f_j^0 &= \underset{l \neq j}{\text{Min}} \{f_l(x_i^{max})\} \end{aligned}$$

Level of significant

After determining the different linear/non-linear membership functions (MF) for each of the objective functions to generate a candidate for the saticficing solution following Bellman and Zadeh (1970) and Zimmermann (1976), the DM is asked to specify his / her reference level of achievement for the membership values. Let $\overline{\mu}_{f_j}$ is the reference membership level of the objective function. The better reference membership levels are attainable for the better requirement that can be formulated as

$$\underset{x_i \epsilon X}{\text{Min}} \ \underset{1 \le l \le j}{\text{Max}} (\overline{\mu}_{f_l} - \mu_{f_l}) \tag{8}$$

which is equivalent to

$$\begin{aligned} \text{Max} \quad & \lambda \\ \text{where} \quad & \lambda \le (\overline{\mu}_{f_j} - \mu_{f_j}) \ , \end{aligned}$$

Preferential Objective

Suppose that objective f_T is more important than f_S (S, T=1,2,...k. and $S \neq T$) which is expressed as $f_S \prec f_T$. It is reasonable for us to hope that objective with higher priorities will also have higher level of satisfaction, this means if is the solution obtained from finding the maximum level of significant, then conditions of priority can be described as:

$$\mu_{f_S}(x_i^*) \le \mu_{f_T}(x_i^*)$$

Now after obtaining λ^* , if the DM selects Z_T as the most important objective function from among the all objective functions f_j (j=1,2,...k). Then the problem becomes (for $\lambda = \lambda^*$)

$$\begin{aligned} \text{Max} \quad & f_T(x_i) \\ \text{subject to} \quad & \lambda^* \le (\overline{\mu}_{f_j} - \mu_{f_j}) \\ \text{where} \quad & 0 \le \lambda \le 1 \end{aligned}$$

3. Assumptions and Notations

The following assumptions and notations are used in developing the proposed SCM.

3.1. Assumptions

The following assumptions are used for the proposed SCM.

(i) The model is developed for a finite time horizon.
(ii) A single supplier, producer and retailer are considered.
(iii) Only one type of raw material and finished products are considered.
(iv) Collection rate of raw material, production rate of the produced goods are constant.
(v) demand rate of finished goods met by the retailer is imprecise in nature.
(vi) holding cost, set-up cost, purchasing cost by retailer, total warehouse space, total transportation cost to transport raw materials from suppliers to production warehouse, total transportation cost to transport the produced goods from producer to retailer are taken as fuzzy in nature.
(vii) Producer possesses two systems- a warehouse and a production center.
($viii$) Shortages of goods are allowed and fully backlogged.
(ix) Multiple lot-size deliveries per order are considered instead of a single delivery per order.
(x) Lot size is the same for each delivery.
(xi) Space constraints to the producer is allowed.

(xii) There is limited transportation cost.

3.2. Notations

The following notations are used for the proposed SCM.

For supplier

(i) $q_s(t)$ = inventory level at time t.
(ii) C = collection rate of the supplier(a decision variable).
(iii) Q_s =maximum inventory at each interval.
(iv) $\widetilde{h}_s$ = fuzzy holding cost per unit quantity per unit time.
(v) $\widetilde{H}_s$ = total holding cost which is fuzzy in nature.
(vi) p_s = per unit purchasing cost of goods(constant).
(vii) $\widetilde{A}_s$ = fuzzy ordering cost per unit quantity.
$(viii)$ $\widetilde{TC}_s$ = supplier's raw material cost which is fuzzy in nature.

For producer's warehouse

(i) $q_{PW}(t)$ = inventory level at time t.
(ii) U = production rate of the finished goods (a decision variable).
(iii) Q_{PW} =maximum inventory at each interval.
(iv) $\widetilde{h}_{PW}$ = fuzzy holding cost per unit quantity per unit time.
(v) $\widetilde{H}_{PW}$ = total holding cost which is fuzzy in nature.
(vi) p_{PW} = per unit production cost of goods(constant).
(vii) $\widetilde{A}_{PW}$ = fuzzy ordering cost per unit quantity.
$(viii)$ $\widetilde{TC}_{PW}$ = fuzzy total cost of raw materials for producer's warehouse.

For the producer

(i) $q_P(t)$ = inventory level at time t.
(ii) λ = defective rate of production.
(iii) Q_P =maximum inventory of produced goods at each interval.
(iv) $\widetilde{h}_P$ = fuzzy holding cost per unit quantity per unit time.
(v) $\widetilde{H}_P$ = total holding cost which is fuzzy in nature.
(vi) p_P = per unit production cost of goods(constant).
(vii) $\widetilde{A}_P$ = fuzzy ordering cost per unit quantity.
$(viii)$ $\widetilde{TC}_P$ = fuzzy total cost for producer's finished goods.

For the retailer

(i) $q_R(t)$ = inventory level at time t.
(ii) $\widetilde{D}$ = demand rate of the produced goods which is fuzzy in nature.
(iii) Q_R =maximum inventory at each interval.
(iv) $\widetilde{h}_R$ = fuzzy holding cost per unit quantity per unit time.
(v) $\widetilde{H}_R$ = total holding cost which is fuzzy in nature.
(vi) $\widetilde{p}_R$ = per unit purchasing cost of goods which is fuzzy in nature.
(vii) $\widetilde{A}_R$ = fuzzy ordering cost per unit quantity.

(viii) $\widetilde{C}_3$ = per unit shortage cost which is fuzzy in nature.
(ix) $\widetilde{S}_R$ = total amount of shortage.
(x) $\widetilde{TC}_R$ = fuzzy total cost for the retailer.

Common notations

(i) T = order cycle.
(ii) n = number of deliveries per order cycle (a decision variable).
(iii) t = delivery cycle.
(iv) T_1 = length of time of each of the n equal sub intervals of order cycle(a decision variable).
(ii) T_R = total shortage period.
(iii) $\widetilde{W}$ = fuzzy total space to keep the raw materials in the warehouse to keep the finished goods.
(iv) $\widetilde{T}_{11}$ = fuzzy total transportation cost to transport the raw materials from supplier's to production warehouse.
(v) $\widetilde{T}_{21}$ = fuzzy total transportation cost to transport the produced goods from producer to retailer.

4. Mathematical Formulation

This paper develops a supply-chain system which consists with a supplier, a producer and a retailer. The supplier is to collect the raw material at a constant collection rate, this raw material is purchased by producer and then transported and stored in his / her warehouse, from which raw material is used for production and finished goods are produced at a production rate which is taken as control variable. Then the goods are purchased by a retailer, who sells these goods in a market with imprecise demand. The system is considered over a finite time horizon and hence several cycles of procurement, production, etc are repeated within the said time period. There are some resource constraints for the producer and retailer on purchasing the raw materials and finished goods respectively. For the retailer, the model is developed with shortages which are fully-backlogged. The purpose of this study is to find the optimal collection rate, optimal production rate, the number of cycles to each partner and length of time of each of the n equal sub intervals of order cycle so that total or individual costs are minimum.

4.1. Inventory model of supplier's raw material

In this model supplier collect raw material from nature and satisfies the producers warehouse. Therefore supplier's raw material inventory quantity $q_s(t)$ at any time t can be expressed as

$$\frac{dq_s}{dt} = C, \qquad iT_1 \leq t \leq (i+1)T_1, \quad i = 0, 1, 2, ..., n-2. \tag{9}$$

Now from the help of boundary condition $q_s(iT_1) = 0$ the inventory at any time t, $q_s(t)$, is given by:

$$q_s(t) = C(t - iT_1). \tag{10}$$

(11)

and using the boundary condition $q_s((i+1)T_1) = Q_s$ we get

$$Q_s = CT_1. \tag{12}$$

(13)

Holding cost of raw material is

$$= \widetilde{h}_s \int_{iT_1}^{(i+1)T_1} q_s(t)dt \tag{14}$$

$$= \frac{\widetilde{h}_s CT_1^2}{2} \tag{15}$$

So total holding cost of raw material is

$$\begin{aligned} H_s &= \sum_{i=0}^{n-2} \frac{\widetilde{h}_s CT_1^2}{2} \\ &= (n-2)\frac{\widetilde{h}_s CT_1^2}{2} \end{aligned} \tag{16}$$

Total collection cost of raw material is

$$\begin{aligned} C_s &= \sum_{i=0}^{n-2} p_s CT_1 \\ &= (n-2)p_s CT_1 \end{aligned} \tag{17}$$

The total raw material cost for the supplier is the sum of the set up cost, collection cost and holding cost as follows:

$$\widetilde{TC_s} = \widetilde{A_s} + (n-2)p_s CT_1 + (n-2)\frac{\widetilde{h}_s CT_1^2}{2} \tag{18}$$

4.2. Inventory model of raw material in producer's warehouse

The inventory level of raw material at the producer's warehouse at time t, q_{PW} determine by the linear differential equation

$$\frac{dq_{PW}}{dt} = -U, \qquad (i+1)T_1 \le t \le (i+2)T_1, \quad i = 0,1,2,...,n-2 \tag{19}$$

As shown in the figure-1 the inventory conditions for the model are:

$$q_{PW}((i+1)T_1) = Q_{PW} \text{ and } q_{PW}((i+2)T_1) = 0 \text{ for } i = 0,1,2,....,n-2$$

Therefore using the condition $q_{PW}((i+2)T_1) = 0$ the inventory at any time t is given by:

$$q_{PW}(t) = U\{(i+2)T_1 - t\} \tag{20}$$

Using the condition $q_{PW}((i+1)T_1) = Q_{PW}$ we get

$$Q_{PW} = UT_1 \tag{21}$$

Holding cost of raw material is

$$= \widetilde{h}_{PW} \int_{(i+1)T_1}^{(i+2)T_1} q_{PW}(t)dt \tag{22}$$

$$= \frac{\widetilde{h}_{PW} U T_1^2}{2} \tag{23}$$

So total holding cost of raw material is

$$\begin{aligned} \widetilde{H}_{PW} &= \sum_{i=0}^{n-2} \frac{\widetilde{h}_{PW} U T_1^2}{2} \\ &= (n-2)\frac{\widetilde{h}_{PW} U T_1^2}{2} \end{aligned} \tag{24}$$

Purchasing cost of raw materials$=p_{PW}Q_{PW}$
Total purchasing cost of raw materials

$$\begin{aligned} \widetilde{P}_{PW} &= \sum_{i=0}^{n-2} p_{PW}Q_{PW} \\ &= (n-2)p_{PW}Q_{PW} \\ &= (n-2)p_{PW}UT_1 \end{aligned} \tag{25}$$

The total raw material cost for the producer's warehouse is the sum of the set up cost, purchasing cost of raw material and holding cost as follows:

$$\widetilde{TC}_{PW} = \widetilde{A}_{PW} + (n-2)p_{PW}UT_1 + (n-2)\frac{\widetilde{h}_{PW} U T_1^2}{2} \tag{26}$$

4.3. Inventory model of producer's finished goods

The finished goods inventory level for the producer with unknown production rate U is described by the following differential equation:

$$\frac{dq_P}{dt} = (1-\lambda)U, \quad (i+1)T_1 \le t \le (i+2)T_1, \quad i = 0,1,2,...,n-2 \tag{27}$$

The boundary conditions are $q_P\{(i+1)T_1\} = 0$ and $q_P\{(i+2)T_1\} = Q_P$
Therefore using the condition $q_P\{(i+1)T_1\} = 0$ the inventory at any time t is given by:

$$q_P = (1-\lambda)U\{t-(i+1)T_1\} \tag{28}$$

Using the condition $q_P\{(i+2)T_1\} = Q_P$, we get,

$$Q_P = (1-\lambda)UT_1 \tag{29}$$

In this case holding cost is

$$= \widetilde{h}_P \int_{(i+1)T_1}^{(i+2)T_1} q_P(t)dt \tag{30}$$

$$= \frac{\widetilde{h}_P(1-\lambda)UT_1^2}{2} \tag{31}$$

So total holding cost of raw material is

$$\begin{aligned}\widetilde{H}_{PW} &= \sum_{i=0}^{n-2} \frac{\widetilde{h}_P(1-\lambda)UT_1^2}{2} \\ &= (n-2)\frac{\widetilde{h}_P(1-\lambda)UT_1^2}{2} \qquad (32)\end{aligned}$$

Production cost=$p_P Q_P$
Total production cost

$$\begin{aligned}\widetilde{P}_P &= \sum_{i=0}^{n-2} p_P Q_P \\ &= (n-2)(1-\lambda)p_P Q_P \\ &= (n-2)(1-\lambda)p_P UT_1 \qquad (33)\end{aligned}$$

The total cost for the producer due to finished goods can be expressed as the sum of the setup cost, production cost and holding cost as follows:

$$\widetilde{TC}_P = \widetilde{A}_P + (1-\lambda)(n-2)p_P UT_1 + (1-\lambda)(n-2)\frac{\widetilde{h}_P UT_1^2}{2} \qquad (34)$$

4.4. Inventory model for the retailer for finished goods

If $q_R(t)$ be the inventory of the finished goods at any time t for the retailer with imprecise demand $\widetilde{D}$ and if t_{b_i} be the time of shortage for the retailer in the i-th cycle, the governing differential equations are:

$$\frac{dq_R}{dt} = -\widetilde{D}, \qquad (i+2)T_1 \le t \le (i+3)T_1, \quad i = 0,1,2,....,n-2 \qquad (35)$$

As shown in the figure-1 the inventory conditions for the model are:

$$q_R((i+2)T_1) = Q_R \text{ and } q_R((i+2)T_1 + T_R) = 0$$
$$\text{and } q_R((i+3)T_1) = S_R \text{ for } i = 0,1,2,....,n-2$$

Therefore using the condition $q_R\{(i+1)T_1\} = 0$ the inventory at any time t is given by:

$$q_R = \widetilde{D}\{(i+2)T_1 - t\} + Q_R, \qquad (i+2)T_1 \le t \le (i+2)T_1 + T_R, \quad i = 0,1,2,....,n-2 \qquad (36)$$

Using the condition $q_R\{(i+2)T_1 + T_R\} = 0$, we get,

$$Q_R = \widetilde{D}T_R \qquad (37)$$

Using the condition $q_R((i+3)T_1) = S_R$, we get,

$$q_R = \widetilde{D}\{(i+3)T_1 - t\} + S_R, \qquad \{(i+2)T_1 + T_R\} \leq t \leq (i+3)T_1, \quad i = 0, 1, 2, ..., n-2 \tag{38}$$

Now using $q_R\{(i+2)T_1 + T_R\} = 0$, we get,

$$S_R = \widetilde{D}(T_1 - T_R) \tag{39}$$

In this case holding cost is

$$= \widetilde{h}_R \int_{(i+2)T_1}^{\{(i+2)T_1+T_R\}} q_R(t)dt \tag{40}$$

$$= \widetilde{h}_R(Q_R T_R - \frac{\widetilde{D}T_R^2}{2}) \tag{41}$$

So total holding cost of finished goods is

$$\widetilde{H}_R = \sum_{i=0}^{n-2} \widetilde{h}_R(Q_R T_R - \frac{\widetilde{D}T_R^2}{2})$$

$$= (n-2)\widetilde{h}_R(Q_R T_R - \frac{\widetilde{D}T_R^2}{2}) \tag{42}$$

Shortage cost

$$= C_3 \int_{\{(i+2)T_1+T_R\}}^{(i+3)T_1} [D\{(i+3)T_1 - t\} + S_R]dt \tag{43}$$

$$= C_3(S_R T_R + \frac{\widetilde{D}T_R^2}{2}) \tag{44}$$

Total shortage cost$(\widetilde{TS_R})$

$$= \sum_{i=0}^{n-2} C_3(S_R T_R + \frac{\widetilde{D}T_R^2}{2}) \tag{45}$$

$$= (n-2)C_3(S_R T_R + \frac{\widetilde{D}T_R^2}{2}) \tag{46}$$

Purchasing cost=$\widetilde{p}_R Q_R$
Total purchasing cost$(\widetilde{TP_R})$

$$= \sum_{i=0}^{n-2} \widetilde{p}_R Q_R \tag{47}$$

$$= (n-2)\widetilde{p}_R \widetilde{D} T_R \tag{48}$$

The total cost for the retailer due to finished goods can be expressed as the sum of the setup cost, production cost, holding cost and shortage cost as follows:

$$\begin{aligned}\widetilde{TC}_R &= \widetilde{A}_R + (n-2)p_R\widetilde{D}T_R + (n-2)\widetilde{h}_R(Q_RT_R - \frac{\widetilde{D}T_R^2}{2}) \\ &+(n-2)C_3(S_RT_R + \frac{\widetilde{D}T_R^2}{2}) \end{aligned} \tag{49}$$

4.5. Model-1 (Integrated Formulation)

Assuming the whole system is owned and managed by a single concern / management the problem reduces to a single objective minimization problem as:

$$\min \widetilde{TC}_I \approx \min \sum_{i=1}^{n} \left\{ \widetilde{TC}_s + \widetilde{TC}_{PW} + \widetilde{TC}_P + \widetilde{TC}_R \right\} \tag{50}$$

Subject to
$$Q_P + Q_{PW} \leq \widetilde{W} \ldots\ldots\ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots(C-1)$$
$$t_0^{'} + t_1^{'} Q_S \leq \widetilde{T}_1 \ldots\ldots\ldots \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots(C-2)$$
$$t_0^{''} + t_1{''} Q_R \leq \widetilde{T}_2 \ldots\ldots\ldots \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots(C-3)$$

4.6. Model-2 (Non-integrated Formulation)

In this formulation, members of the chain are assumed to be different from each other but they operate / work together in collective / collaborative manner. Hence, the problem is to find the no of cycles n, $(t_{ci})_k$, $(t_{bi})_k$, $(t_{p_i})_k$, $(t_{pwi})_k$ and the production rate P_0, P_1 and corresponding the order quantities which minimizes the total cost in the finite time horizon of each member simultaneously with the said Chance-Constraints.i.e.,

obj-1:	$\min\ \widetilde{TC}_S$	(51)
obj-2:	$\min\ \{\widetilde{TC}_{PW} + \widetilde{TC}_P\}$	(52)
obj-3:	$\min\ \widetilde{TC}_R$	(53)

Subject to (C-1),(C-2) and (C-3).

5. Procedure for Defuzzification

5.1 For Model-1 (Integrated Formulation)

Since $\widetilde{TC}_I$ is fuzzy in nature minimize $\widetilde{TC}_I$ is not well defined. So instead of minimize $\widetilde{TC}_I$ one can minimize F such that necessity of the event $\widetilde{TC}_I < F$ exceeds some predefined level $\alpha(0 < \alpha < 1)$ according to companies requirement. Similarly as fuzzy constraints are also not well defined, necessity of the constraints (C-1,C-2,C-3) must exceed some predefined level $\alpha_i(0 < \alpha_i < 1)(i = 1, 2, 3)$ as proposed by Maiti and Maiti [22]. Then the problem reduces to

$$\left.\begin{array}{l}\text{Minimize} \quad F \\ \text{Subject to Nes}(\tilde{TC}(s,Q,r) < F) > \alpha \\ \qquad\qquad \text{Nes}(Q_P + Q_{PW} \leq \widetilde{W}) > \alpha_1 \\ \qquad\qquad \text{Nes}(t_0^{'} + t_1^{'} Q_S \leq \widetilde{T}_1) > \alpha_2 \\ \qquad\qquad \text{Nes}(t_0^{''} + t_1{''} Q_R \leq \widetilde{T}_2) > \alpha_3 \end{array}\right\} \tag{54}$$

Now, let us consider $\tilde{h}_s = (h_{s1}, h_{s2}, h_{s3})$, $\tilde{A}_s = (A_{s1}, A_{s2}, A_{s3})$, $\tilde{h}_{PW} = (h_{PW1}, h_{PW2}, h_{PW3})$, $\tilde{A}_{PW} = (A_{PW1}, A_{PW2}, A_{PW3})$, $\tilde{h}_P = (h_{P1}, h_{P2}, h_{P3})$, $\tilde{A}_P = (A_{P1}, A_{P2}, A_{P3})$, $\tilde{D} = (D_1, D_2, D_3)$, $\tilde{C}_3 = (C_{31}, C_{32}, C_{33})$, $\tilde{h}_R = (h_{R1}, h_{R2}, h_{R3})$, $\tilde{A}_R = (A_{R1}, A_{R2}, A_{R3})$, $\tilde{p}_R = (p_{R1}, p_{R2}, p_{R3})$, $\tilde{W} = (W_1, W_2, W_3)$, $\tilde{T}_{11} = (T_{111}, T_{112}, T_{113})$, $\tilde{T}_{21} = (T_{211}, T_{212}, T_{213})$, as TFNs then $\widetilde{TC}_I$ becomes a TFN $(TC_{I1}, TC_{I2}, TC_{I3})$. Then using Lemma-1 and Lemma-2 the above problem reduces to:

$$\left.\begin{array}{l}\text{Minimize} \quad F \\ \text{Subject to } \alpha TC_{I3} + (1-\alpha) TC_{I2} \leq F \\ \qquad\qquad \frac{W_2 - (Q_P + Q_{PW})}{W_2 - W_1} \geq \alpha_1 \\ \qquad\qquad \frac{T_{112} - (t_0^{'} + t_1^{'} Q_s)}{T_{112} - T_{111}} \geq \alpha_2 \\ \qquad\qquad \frac{T_{212} - (t_0^{''} + t_1^{''} Q_s)}{T_{212} - T_{211}} \geq \alpha_3 \end{array}\right\} \tag{55}$$

which is equivalent to

$$\left.\begin{array}{l}\text{Minimize} \quad F = \alpha TC_{I3} + (1-\alpha) TC_{I2} \\ \text{Subject to } \frac{W_2 - (Q_P + Q_{PW})}{W_2 - W_1} \geq \alpha_1 \\ \qquad\qquad \frac{T_{112} - (t_0^{'} + t_1^{'} Q_s)}{T_{112} - T_{111}} \geq \alpha_2 \\ \qquad\qquad \frac{T_{212} - (t_0^{''} + t_1^{''} Q_s)}{T_{212} - T_{211}} \geq \alpha_3 \end{array}\right\} \tag{56}$$

For some assumed parametric values we get the optimal value of the problem by using LINGO software.

5.2 For Model-2 (Non-integrated Formulation)

As this is a multi-objective problem, to optimize the problem, we have used Interactive Fuzzy Decision Making (IFDM) Technique as follows: Considering the imprecise nature of the decision maker's (DM's) judgements, it is natural to assume that the DM may have fuzzy or imprecise goals for each of the objective functions

$$min(TCS_{sl}(x), TCS_{su}(x), TCPW_{sl}(x), TCPW_{su}(x), TCP_{sl}(x), TCP_{su}(x),$$
$$TCR_{sl}(x), TCR_{su}(x))$$

Let a goal assigned by the DM to an objective is stated as "somewhat larger than A". This type of statement can be quantified by eliciting a corresponding membership function. To derive the membership function $\mu_{TC_r(x)}$ for each of the objective functions 'r'

$TC_r(x), (r = 1, 2, 3, ..., m)$ we first calculate individual minimum $TC_r^{min}(x)$ and maximum $TC_r^{max}(x)$ under the given constraints. With the help of individual minimum and maximum, the DM can select his membership functions from different types of membership functions (i.e., linear, quadratic, exponential etc.). The membership function for each of the objective functions $TC_r(x), (r = 1, 2, 3, ..., m)$ may be written as

$$\mu_{TC_r}(x) = \begin{cases} 0 & \text{for } L_r > TC_r(x) \\ d_r(TC_r(x)) & \text{for } L_r \leq TC_r(x) \leq U_r \\ 1 & \text{for } TC_r(x) > U_r \end{cases}$$

where L_r and U_r are chosen such that $TC_r^{min}(x) \leq L_r \leq U_r \leq TC_r^{max}(x)$. $d_r(TC_r(x))$ is a strictly monotone increasing continuous function of $TC_r(x)$ which may be linear or non-linear. *Type-1 : Linear membership function*

For each objective function, the corresponding linear membership functions are as follows:

$$\mu_{TC_r}(x) = \begin{cases} 0 & \text{for } L_r > f_r(x) \\ 1 - \dfrac{U_r - TC_r(x)}{U_r - L_r} & \text{for } L_r \leq TC_r(x) \leq U_r \\ 1 & \text{for } TC_r(x) > U_r \end{cases} \tag{57}$$

Type-2 : Quadratic membership function

For each objective function, the corresponding quadratic membership functions are as follows:

$$\mu_{TC_r}(x) = \begin{cases} 0 & \text{for } L_r > TC_r(x) \\ 1 - \left(\dfrac{U_r - TC_r(x)}{U_r - L_r}\right)^2 & \text{for } L_r \leq TC_r(x) \leq U_r \\ 1 & \text{for } TC_r(x) > U_r \end{cases} \tag{58}$$

Type-3 : Exponential membership function

For each objective function, the corresponding exponential membership functions are as follows:

$$\mu_{TC_r}(x) = \begin{cases} 0 & \text{for } L_r > TC_r(x) \\ \alpha_r\left[1 - e^{-\beta_r\left(\frac{(U_r - TC_r(x))}{(U_r - L_r)}\right)}\right] & \text{for } L_r \leq TC_r(x) \leq U_r \\ 1 & \text{for } TC_r(x) > U_r \end{cases} \tag{59}$$

The constants $\alpha_r > 1$ and $\beta_r > 0$ can be determined by asking the DM to specify the three

points $L_r, TC_r^{0.5}(x)$ and U_r such that $TC_r^{min}(x) \leq L_r \leq TC_r^{0.5}(x) \leq U_r \leq TC_r^{max}(x)$ where $TC_r^{0.5}(x)$ represents the value of $TC_r(x)$ such that the degree of membership function $\mu_{TC_r}(x)$ is 0.5. After determining the different linear / non-linear membership functions for each of the objective functions proposed and following Zimmermann [1976,1978] the given problem can be formulated as:

$$\text{MIN } \lambda \tag{60}$$

$$\text{subject to} \quad \lambda \geq \mu_{TC_r}(x), \;\; x \in S, \, , \;\; 0 < \lambda \leq 1, (r = 1, 2, 3, ..., m)$$

Now the DM will select the membership functions for the corresponding objective functions. With the help of three different types of membership functions given by (29)-(31), above problem can be restated as

$$\begin{aligned} & \text{Min } \lambda && (61) \\ \text{subject to} \quad & \lambda \geq \mu_{TC_r}(x), \quad (\text{if } r\text{-th objective} \in \text{Type-1}) \\ \text{or} \quad & \lambda \geq \mu_{TC_r}(x), \quad (\text{if } r\text{-th objective} \in \text{Type-2}) \\ \text{or} \quad & \lambda \geq \mu_{TC_r}(x), \quad (\text{if } r\text{-th objective} \in \text{Type-3}) \\ & x \in S, \quad 0 < \lambda \leq 1 \end{aligned}$$

6. Numerical Experiment

To illustrate the proposed supply-chain model, the following input data are consider.

Table-4.1: Input data:

SCM	Ordering cost	Purchasing/Prod cost	Holding cost	defective rate	Demand rate	Transportation cost
Supplier	(45, 50, 55)	(18, 20, 22)	(3.5, 4, 4.5)	–	–	(5, 0.5)
Prod. warehouse	(75, 80, 85)	(35, 40, 45)	(4.5, 5, 5.5)	–	–	–
Prod. centre	(95, 100, 105)	(4.5, 5, 5.5)	(5.5, 6, 6.5)	0.02	–	
Retailer	(85, 90, 95)	(45, 50, 55)	(4.5, 5, 5.5)	–	(110, 120, 130)	(5, 0.5)

The corresponding results of Models-1a and 1b and Model-2 are presented respectively in Table-4.2 and Table-4.3.

Table-4.2: Optimum solutions for Integrated Model:

Time Period	No.of cycle	C	U	QS	QR	Supplier Cost	Producer Cost	Retailer Cost	Total Cost
0.1028	10	121.21	121.21	1.24	1.23	30.65	30.08+93.85	33.85	188.43

Table-4.2: Solutions of Non-Integrated Model by IFDM:

λ	Time Period	No.of cycle	C	U	QS	QR Cost	Supplier Cost	Producer Cost	Retailer
0.49	0.146	3	133.07	133.07	0.194	0.175	35.09	40.0+95.09	50.00

7. Practical Implementation

In developing countries like India, Bangladesh, Nepal, etc., products like cloths, etc., are produced under small scale industry sector. The cottons are supplied by some suppliers who collect these from villagers or village markets. The product center purchases and stores these cottons. Cloths products (which become defect at the time of production) are produced and sold to retailers who later sell these in the market. In this process, there may be some resource constraints like limited capacity for space, limitation on transportation cost, etc. Similar process is also followed in rice mills. Here some middle men collect paddy from villages and supply to a rice-mill owner. The rice-mill owner makes a temporary stock of paddy and produces rice out of it. This rice is sold to retailers for sale. Here both in the time of production from paddy to rice few rice loses.

8. Conclusion

This paper addresses optimal order placement and delivery rate policies for a three stage SCM. Here, demand of goods is fuzzy. There are fuzzy chance constraints on the transportation costs for both producer and retailer and also a space constraint is considered.

An appropriate IFDM for multi-objective are applied to solve the models.
Till now, most of the supply-chain models are formulated as a single-objective problem which is far from the realistic situation. This is because normally, the partners of SCM are different, not under a single management. Hence, though single management system fetches minimum cost for the system, it is impractical. Again, if the interest of only one partner is looked into, his / her cost goes down maximum at the cost of the other partners' interests (their costs shoot up). This is also not acceptable. Hence, only possible way is to satisfy the interests of all partners of the system simultaneously in a maximum possible way i.e., a compromise solution for all members. In this paper, we have shown that the total cost is minimum for integrated model, as expected.

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