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A Low Computational Complexity Algorithm for PTS based PAPR Reduction Scheme in OFDM Systems

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Abstract

Peak-to-Average Power Ratio (PAPR) is the major drawback in Orthogonal Frequency Division Multiplexing (OFDM) systems, which may result in signal distortion, loss of orthogonality in OFDM signals etc. Hence it is the most concerned problem in OFDM systems, which has to be reduced. Of all the available PAPR reduction techniques, Partial Transmit Sequence (PTS) scheme is the most efficient and attractive one. But, the original PTS scheme requires large computations in order to transmit the optimum signal with low PAPR from all available combinations of candidate signals. This paper proposes a new efficient PTS scheme which utilizes the correlation property among all available candidate signals to reduce the computational complexity.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a parallel multicarrier transmission scheme, where a high-rate serial data stream is split up into a set of low-rate sub streams, each of which is modulated on a separate subcarrier. The basic principle of OFDM is to split a high-rate data stream into a number of lower rate streams that are transmitted simultaneously over a number of subcarriers. Because the symbol duration increases for lower rate parallel subcarriers, the relative amount of dispersion in time caused by multipath delay spread is decreased. Intersymbol interference is eliminated almost completely by introducing a guard time in every OFDM symbol. In the guard time, the symbol is cyclically extended to avoid intercarrier interference.

OFDM is a multicarrier technique which is used commonly all over the world. It is widely adopted & standardized in the world. A number of applications [4], [5] and standards which use OFDM include Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), WiFi (IEEE 802.11a/g/j/n), World Wide Interoperability for Microwave Access (WiMAX-IEEE 802.16), Ultra Wide Band Wireless Personal Area Network (UWB Wireless PAN-IEEE 802.15.3a) and Mobile Broadband Wireless Access (MBWA-IEEE802.20). One of the main reason to use OFDM is to increase the Robustness against frequency-selective fading or narrowband interference. Inverse Fast Fourier Transform (IFFT) and FFT are used in OFDM to multiplex the signals together at the transmitter, and demultiplex the signals in the receiver, respectively.

OFDM has many advantages like, it is an efficient way to deal with multipath fading, it is robust against narrowband interference and etc. Beside all these advantages, the major disadvantage and exclusive drawback of the OFDM is the PAPR which increases the complexity of A/D & D/A converters [7] and also reduces the efficiency of the RF power amplifiers. Hence this problem of high PAPR has become a major concerned problem in the OFDM systems. Thus several PAPR reduction techniques [13] have been implemented which are categorized as the Signal Distortion techniques (like Clipping), Coding techniques and Scrambling Sequence techniques (like PTS scheme). Of all the PAPR techniques, PTS scheme has been found to an efficient and attractive method that has several advantages over others. In PTS, an input data sequence is divided into a number of disjoint subblocks, which are then weighted by a set of phase factors to create a set of candidate signals. Finally, the candidate with the lowest PAPR is chosen for transmission. This PTS scheme has a major drawback that it requires a very large number of computations to identify and select the optimum candidate signal that has a low PAPR from all the available combinations of candidate signals. This computational complexity has been reduced by implementing several modified techniques like iterative flipping, but all these techniques are implemented by reducing & eliminating the some of the candidate signals which has caused in the information loss.

This paper proposes a new PTS scheme to reduce the PAPR in OFDM systems with low & much reduced computational complexity. In this paper, we reduce the Computational Complexity in PTS scheme, by using the correlation property between the available candidate signals, instead of reducing the number of candidate signals and thus no information is being lost. This scheme achieves the PAPR reduction same as the conventional PTS scheme and reduces the computational complexity to a large extent. In section II, we introduce the concept of OFDM signal and overview of the PAPR problem. The conventional PTS scheme has been explained in the section III. The new PTS scheme to reduce the computational complexity has been implemented in section IV. Section V gives the computational analysis and focuses on the simulation results. The last section i.e. section VI concludes our paper.

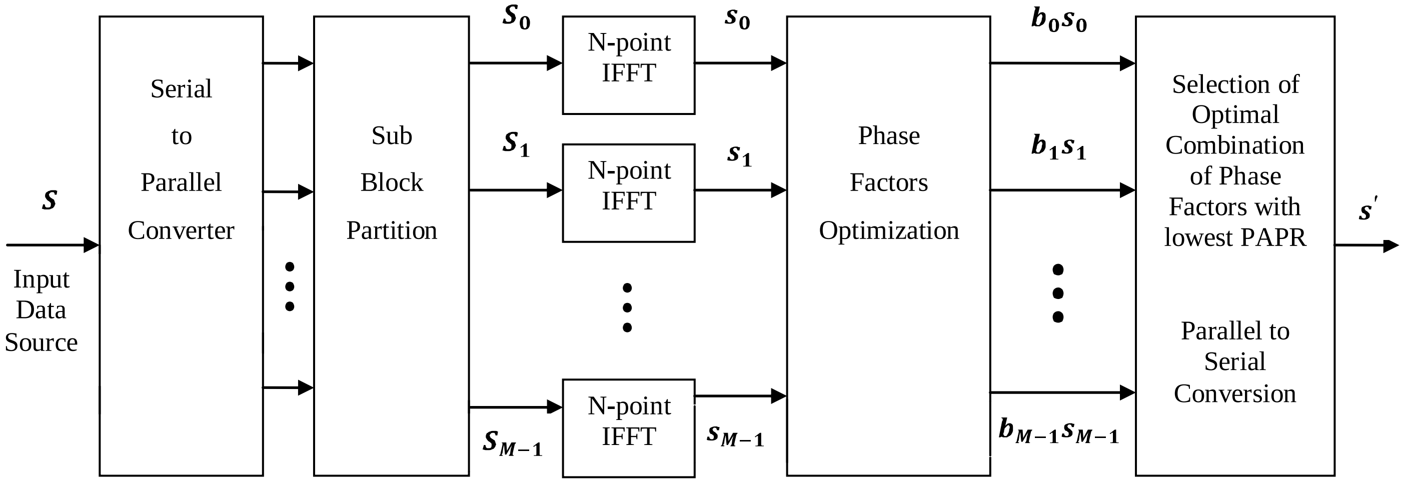

Figure 1. Partial Transmit Sequence Scheme Block Diagram

2. Background

2.1. OFDM Signal

Let there be N different subcarriers used to transmit the parallel information and let $S_{n,k}$ denote the kth complex modulated symbol, then the nth OFDM block [6] is given by

$$s_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_{n,k}\; g_k(t-nT) \qquad (1)$$

where the kth subcarrier signal is given by

$$g_k(t) = \begin{cases} e^{j2\pi k\Delta f t} & \forall\, t \in [0,T_s] \\ 0 & otherwise \end{cases} \qquad (2)$$

where $\Delta f = \frac{1}{T_s}$.

Thus, the outputs s_n [12], [14] of the N-point IDFT of S_k are given by

$$s_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left(\frac{j2\pi kn}{N}\right). \qquad (3)$$

where $j^2 = -1$.

2.2. PAPR

The Peak-to-Average Power Ratio (PAPR) of the OFDM signal is defined as the ratio of the Maximum power delivered by the Average power of the OFDM signal. It is expressed as given below

$$PAPR = 10 \cdot \log_{10} \frac{Max\,\{|s_n|^2\}}{E\,[|s_n|^2]}\ (dB) \qquad (4)$$

In other words,

$$PAPR = 10 \cdot \log_{10} \frac{\max\limits_{0 \le t < NT} |s_n|^2}{\frac{1}{NT}\int_0^{NT} |s_n|^2\, dt} \qquad (5)$$

where $|s_n|$ denotes the magnitude of s_n and $E[\cdot]$ denotes the expectation operation. Here, the peak power occurs when the N modulated symbols are added with the same phase.

3. Conventional PTS Scheme

Partial Transmit Sequence (PTS) scheme [11] is the PAPR reduction technique in which, only part of the data of varying sub-carrier with low PAPR is transmitted which covers all the information to be sent in the signal as a whole. The block diagram of the PTS scheme is as shown in the Figure 1, above.

As shown in Figure 1, in the Partial Transmit Sequence technique, input data block S is partitioned into M disjoint sub-blocks such that each sub-block is given by

$$S_m = \left[S_{m,0}, S_{m,1}, \cdots, S_{m,N-1}\right]^T \quad m = 1,2,\cdots,M \qquad (6)$$

such that $\sum_{m=1}^{M} S_m = S$. Now, these sub-blocks are combined to minimize the PAPR in the time domain. The L times oversampled time domain signal of $S_m, m = 1,2,\cdots,M$, is obtained by taking the inverse discrete fourier transform of length NL on S_m concatenated with (L-1)N zeroes. These are called the Partial Transmit Sequences.

Here, the complex phase factors given by $b_m = e^{j\theta_m}$ $m = 1,2,\cdots,M$ are introduced to combine the partial transmit sequences. The set of complex phase factors is denoted by $b = [b_1, b_2, \cdots, b_M]^T$.

Now, the time domain signal after combining all the partial transmit sequences by the use of the complex phase factors is given by

$$s'(b) = \sum_{m=1}^{M} b_m \cdot s_m \tag{7}$$

where,

$$s'(b) = [s'_0(b), s'_1(b), \cdots, s'_{NL-1}(b)]^T.$$

In other words, the time domain partial transmit sequence by using IDFT can be expressed as

$$s' = IDFT(S') = \sum_{m=1}^{M} b_m \, s_m \tag{8}$$

Now, the objective is to find the set of phase factors which minimizes the PAPR. This implicates that we need to find the set of optimum phase factors, which when used to combine the M disjoint blocks will yield a candidate signal with low PAPR. Now, if we assume that there are W phase factors, then it would require for the PTS to search W^M combinations to get an optimal set of phase factors which yields low PAPR signal. This indicates that the number of search increases [8] exponentially with number of blocks M. Thus, this conventional PTS scheme requires a large computations to get an optimal candidate signal with low PAPR.

4. A New Reduced Complexity PTS Scheme

Here, we propose a new PTS scheme which reduces the computational complexity without reducing the number of candidate signals. In all available combinations there may be certain pairs which may have the same relations. Thus, we use the correlation property among these phase factors in each subset, such that the computational complexity is reduced.

To start with our proposed method, let us first define the phase factors in a phase set. Let us denote the number of phase factors as W. If we take W=2, then it indicates that there are two phase factors, one is in-phase & other is out-of-phase factor with phase factor set as {1,-1}. In same way, if we take W=4, then the phase factors set consists of {1,-1,j,-j} which indicates real & imaginary, in-phase & out-of-phase factors.

Now, knowing these phase factors, we can create a fundamental combination known as Prototype Vectors. We derive all the other vectors from this prototype vector. For example, if W=2 & M=2, then the prototype vectors are {1,1} & {1, j). Because of using the correlation property, all the vectors derived from same prototype vector differ each other by a sign change only.

Based on these phase factor, number of subblocks of the PTS, by knowing all the vectors, we can write the candidate signals. Hence by using the correlation property and knowing all the vectors, we can write the first candidate signal derived from first prototype vector as

$$s_{1,1} = s_1 + s_2 + \cdots + s_M \tag{9}$$

Now, we can derive the second candidate signal from first candidate signal by using the sign change property as

$$s_{1,2} = s_{1,1} - sign(b_{1,1,m}) \cdot 2s_m \tag{10}$$

Where $b_{i,k,m}$ represents the kth phase weighting vector based on the ith prototype vector and is applied to the mth subblock of the PTS OFDM transmitted signal and $sign(A)$ indicates the sign of A.

Similarly, we can write the i+1th candidate signal derived from this first prototype vector as

$$s_{1,i+1} = s_{1,i} - sign(b_{1,i,m}) \cdot 2s_m \tag{11}$$

Now, we can derive the first candidate signal that of the 2nd prototype vector denoted by $s_{2,1}$ from $s_{1,prev}$ as given by

$$s_{2,1} = s_{1,prev} + b_{1,prev,m} \left(A_{2,m} - 1\right)s_m \tag{12}$$

Where $prev = 2^{M-1}$ indicates the previous prototype vector, and $A_{i,j}$ is the value which denote the change of the real & imaginary phase factors in the various prototype vectors.

So in general, we can write the i+1th candidate signal derived from the second prototype vector as

$$s_{2,i+1} = s_{2,i} - sign(b_{2,i,m}) \cdot 2s_m \tag{13}$$

Combining all the above equations from 9 to 13, we can summarize the general equations to get the candidate signals as given below

$$s_{i+1,1} = s_{i,prev} + b_{i,prev,m} \left(A_{i+1,m} - 1\right)s_m \tag{14}$$

And

$$s_{k+1,i+1} = s_{k+1,i} - b_{k+1,i,m} \cdot 2s_m \tag{15}$$

With all the above equations, we get the all possible candidate signals with reduced computational complexity. From these candidate signals, we choose the one with minimum PAPR for transmission.

Now, let us see the above reduced computational complexity PTS algorithm considering an example. Let us do the partial transmission of sequences by taking 3 subblocks i.e. M=3 and let us consider 4 phase factors set i.e. W=4.

Rewriting the equations 14 & 15 using M=3 & W=4 to get the candidate signals, we can write

$$s_{1,i+1} = s_{1,i} - sign(b_{1,i,m}) \cdot 2s_m \tag{16}$$

$$s_{2,1} = s_{1,4} + sign(b_{1,4,3}) \left(A_{2,3} - 1\right)s_3 \tag{17}$$

$$s_{2,i+1} = s_{2,i} - sign(b_{2,i,m}) \cdot 2s_m \qquad (18)$$

$$s_{3,1} = s_{2,4} + sign(b_{2,4,2})\,(A_{3,2} - 1)s_2 \qquad (19)$$

and similarly we can write the equations so on till we get the last candidate signal i.e. $s_{4,4}$. So, by using the equations from 16 to 19, we can write the candidate signals as follows

$$s_{1,1} = s_1 + s_2 + s_3 \qquad (20.1)$$

$$s_{1,2} = s_1 - s_2 + s_3 \qquad (20.2)$$

$$s_{1,3} = s_1 - s_2 - s_3 \qquad (20.3)$$

$$s_{1,4} = s_1 + s_2 - s_3 \qquad (20.4)$$

$$s_{2,1} = s_1 + s_2 - js_3 \qquad (20.5)$$

$$s_{2,2} = s_1 - s_2 - js_3 \qquad (20.6)$$

$$s_{2,3} = s_1 - s_2 + js_3 \qquad (20.7)$$

$$s_{2,4} = s_1 + s_2 + js_3 \qquad (20.8)$$

$$s_{3,1} = s_1 + js_2 + js_3 \qquad (20.9)$$

$$s_{3,2} = s_1 - js_2 + js_3 \qquad (20.10)$$

$$s_{3,3} = s_1 - js_2 - js_3 \qquad (20.11)$$

$$s_{3,4} = s_1 + js_2 - js_3 \qquad (20.12)$$

$$s_{4,1} = s_1 + js_2 - s_3 \qquad (20.13)$$

$$s_{4,2} = s_1 - js_2 - s_3 \qquad (20.14)$$

$$s_{4,3} = s_1 - js_2 + s_3 \qquad (20.15)$$

$$s_{4,4} = s_1 + js_2 + s_3 \qquad (20.16)$$

Here, note that, due to the correlation property the computational complexity has been reduced to a large extant. Now, from these available candidate signals, we transmit the one with minimum PAPR.

5. Analytical & Simulation Results

5.1. Computational Complexity Analysis

In the previous section, we have seen how we had reduced the computational complexity of the proposed system. Here in this section, we analyse the computational complexity reduction by defining the term Computational Complexity Reduction Ratio (CCRR) of the proposed PTS scheme comparing with that of the conventional PTS scheme. The CCRR is given by

$$CCRR = \left(1 - \frac{Proposed\ PTS\ complexity}{Conventional\ PTS\ complexity}\right) \times 100\ \% \qquad (21)$$

From equation 14 it is clear that the number of multiplications required by the proposed PTS is given by

$$\alpha_{mul} = N \cdot \left[(\frac{W}{2})^{M-1} - 1\right] \qquad (22)$$

And the number of multiplications required by the conventional PTS is given by

$$\beta_{mul} = N \cdot \left[C_{M-1}^{1}\left(\frac{W}{2} - 1\right) + 2C_{M-1}^{2}\left(\frac{W}{2} - 1\right)^{2} + \cdots + (M-1)C_{M-1}^{M-1}\left(\frac{W}{2} - 1\right)^{M-1}\right] \cdot 2^{M-1} \qquad (23)$$

Similarly, From equation 15, we can say that the number of additions required by the proposed PTS scheme has reduced to N because it is only necessary to calculate the s_m. Hence, the ratio of the addition complexity of proposed scheme to that of conventional PTS scheme is $\frac{1}{M-1}$.

Now, if we take, M=6, then we get Addition CCRR as 80% and Multiplication CCRR as 98% with the proposed PTS scheme compared to the conventional PTS scheme. With this, we can say that the proposed PTS scheme can reduce the computational complexity to a very extant when compared to the conventional PTS scheme.

5.2. Simulation Results

In this proposed PTS scheme, we have not reduced the number of candidate signals as in other cases, but we have used the correlation property in order to reduce the computational complexity. In this regard we can prove this by observing the Complementary Cumulative Distribution Function CCDF [10] which is given by

$$P(PAPR > z) = 1 - P(PAPR \le z)$$

$$= 1 - (1 - e^{-z})^{N}$$

Here, taking z as $PAPR_0$, the Complementary Cumulative Distribution Function CCDF is given as the

$$CCDF\ (\Pr[PAPR > PAPR_0])$$

Here, as we have used the correlation property instead of reducing the number of candidate signals, we achieve the PAPR reduction as same as in the case of conventional PTS scheme. This is illustrated in the Fig. 2 in which we can see the coincidence of the theoretical evaluation and computer simulation results. The results have been simulated in the MATLAB.

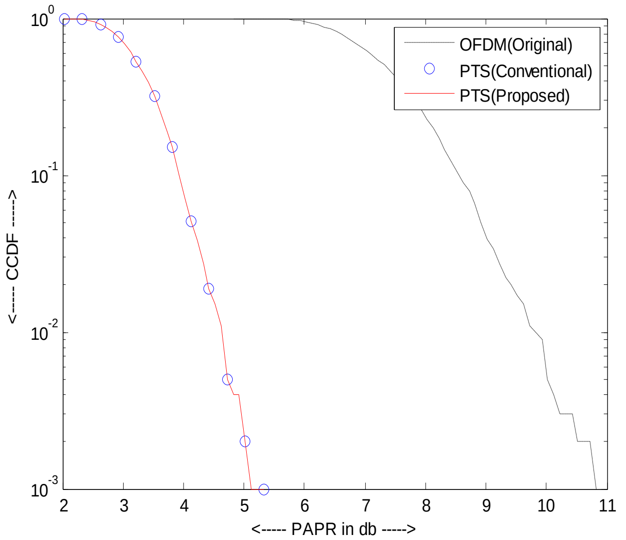

Figure 2. CCDF

6. Conclusion

Conventional PTS scheme employs large computations to reduce the PAPR which is a major drawback in the OFDM systems. As analysed in section V, our new proposed PTS scheme reduces the computational complexity of the conventional PTS scheme to a very large extant. Also, the simulation results show that the proposed system achieves same PAPR reduction as the conventional system. Hence our proposed PTS scheme not only reduces the computational complexity to a great extant but also achieves the same PAPR reduction as that of the conventional system.

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