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A Mathematical Approach in the Formulation of a Direct Evaporative Cooling Device

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***Abstract*—The paper is concerned with the development of a mathematical approach for a direct contact heat exchanger. An air washer has been considered for the analysis and a one dimensional model using mass and energy balance equation has been developed. The new solution presented for the model shows an excellent agreement with the Literature available in the air conditioning. The effect on air temperature inside the air washer by the different parameters such as feeding water temperature, air flow rate and air humidity ratio have been investigated. It has been proved mathematically that as the cooling water temperature at the inlet of air washer and the flow rate of air is decreased the effect of air washer will be more prominent. The model can also be applied to the cooling tower with some modification**

Keywords—Air cooling rate, heat exchanger, direct evaporative cooling.

NOMENCLATURE

a - Surface area per unit of chamber volume [m^2/m^3]

A_{cs} - Cross-sectional area of the air washer [m^2]

C_L - specific heat of the water [J/ kg K]

C_{pm} - specific heat of the moist air at constant pressure [J/kgK]

G - flow rate per unit cross-sectional area of the air washer [kg/s-m^2]

h - heat transfer coefficient [W/m^2K]

H - Specific enthalpy [J/ kg]

H_i - Specific enthalpy value in the air-water interface [J/kg]

h_a - air heat transfer coefficient [W/m^2K]

h_L - water heat transfer coefficient [W/m^2K]

H_{fg} - enthalpy of vaporization [J/ kg K]

K_M- mass transfer coefficient [kg/s-m^2]

x - Axial position [m]

L - Length of the air washer [m]

$h_a/K_M C_{pm}$ - Lewis relation

T - Temperature [^{0}C]

W - Air humidity ratio [kgwv/ kgda]

W_i - air humidity ratio in the air-water interface [kgwv/ kgda]

Subscripts

a - relative to the dry air

H - Relative to the heat transfer

M - Relative to the mass transfer

i - relative to the air water interface

L - relative to the water

0 - property evaluated at the 0^0C.

I. INTRODUCTION

High energy cost and problems associated with environmental concern, call for efficiency improvement of conventional air conditioners and all systems related to cooling the air. Use of evaporative cooling has grown up in recent years because of their high efficiency. It deals with evaporation of water induced by the passage of an air flow, thus decreases the air temperature. When water evaporates into the air to be cooled, simultaneously humidifying, it that is called as direct evaporative cooling(DEC).The heat and mass transfer process ceases when the air reaches its saturation temperature. The main advantage of evaporative cooling is that the efficacy of the process increases when the climate is extremely hot and dry.

Moreover, most of the air conditioning systems used in the application of thermal comfort operate with VCRS(Vapour compression refrigeration systems).These systems utilises non-renewable energy source and are responsible for increasing the demand of electric power in the summer season. The refrigerant of VCRS is also responsible for ozone layer depletion. As a result, a lot of effort has been devoted for developing air conditioning systems that employ renewable energy sources and working fluids that causes no damage to the environment.

Different approaches have been developed concerning the modelling of heat and mass transfer processes in evaporative cooler. Stoecker and Jones (1982) developed a numerical method for obtaining the number of transfer units in cooling towers. Jaber and Webb (1989) applied effectiveness-NTU approach used by conventional heat exchangers to evaporative systems. Ismail and Mahmoud (1994) developed a mathematical model for the analysis of air washers. Jin et al. (2007) presented a literature review of the mathematical models employed for cooling tower. Niksiar and Rahimi (2009) developed a numerical study concerning the energy and exergy efficiencies in an evaporative cooler. A mathematical model of the problem was developed through the application of the mass, energy and exergy balance equations to the cooler. The results showed that the system presents low exergy efficiency, although it had demonstrated high energy efficiency. An analysis of the influence of some operational and design parameters in the exergy destruction was also discussed. There are several articles in the literature that investigated air-conditioning systems in conjunction with evaporative panels. However, only few have been published on evaporative cooling using airwashers. As mentioned above, Jaber and Webb (1989) developed an analytical solution based on the Effectiveness- NTU method. However, this solution cannot be used to predict the temperature and humidity fields inside each point of the system. In the available literature very few work have been done over the analytical method applied for air washers.

The main contributions of the present work are the modelling of the heat and mass transfer processes that occurs in these devices, and obtaining an analytical solution by using MATLAB 7.8.0. The work will help in the developing of accurate numerical models to the desiccant air-conditioning systems employing air washers. Heat transfer resistance in liquid film and variation of both humid air and water properties into the air washer has been taken into account in the presented modelling.

II. MATHEMATICAL MODELLING OF THE SYSTEM

The fig-1 shows a schematic representation of an air washer in which the related physical and geometric parameters are represented, As the air passes through the air washer (length L) the water evaporates. Let *dx* be a small elementary length in the air washer and the variation in the different properties over this length *dx* is represented. It has been assumed that air and water travel parallel to each other in same direction.

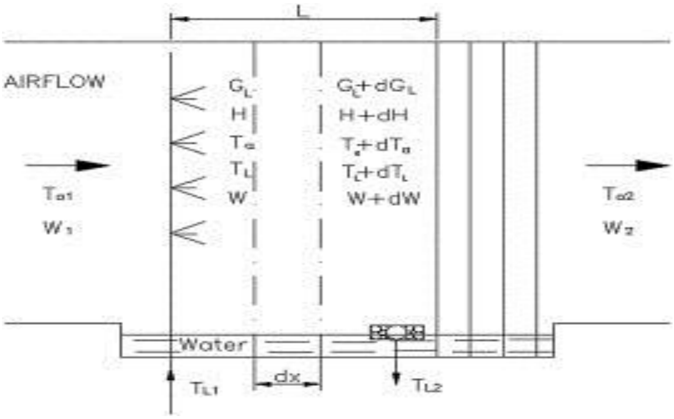

Fig.1. Air Washer

The equations that describe the physical processes are obtained applying the mass and energy balances in the differential element *dx* as described in the following sections.

A. Mass transfer to air

As the air passes through the length *dx* there will be reduction in water flow rate due to the evaporation and the mass transfer is given by

$$dG_L = G_a dW = K_m a_m (W_i - W) dx \quad (1)$$

W - Specific humidity of air

W_i - Air humidity ratio at the air water interface

B. Heat Transfer to air

The air washer can be used for heating or cooling of the air. As the air passes through the air washer the air temperature will increase due to sensible heat transfer between the air and the air water interface. The resulting equation will be

$$G_a C_{pm} dT_a = h_a a_H (T_i - T_a) dx \quad (2)$$

C. Total heat Transfer

The variation in the air enthalpy is the result of sensible heat transfer (due to the difference in the temperature of air and water) and Latent heat Transfer (due to the mass transfer).The resulting equation will be

$$G_a\left(C_{pm} dT_a + h_{fg} dW\right) = \left[h_a a_m (T_i - T_a) + K_m a_m \ (W_i - W) h_{fg}\right] dx \quad (3)$$

In direct contact devices, the super facial areas of heat and mass transfer may be regarded as identical (a_m=a_h). Neglecting the variations in the water vaporization latent heat with the temperature and considering the Lewis relation ($h_a/K_m C_{pm}$) is equal to unity Equation (3) can be expressed as

$$G_a dH = K_m a_m \left[\left(C_{pm} T_i + W_i h_{fg}\right) - \left(C_{pm} T_a + W h_{fg}\right)\right] dx \quad (4)$$

$$G_a dH = K_m a_m [H_i - H] dx \quad (5)$$

Where $[H_i - H]$ is called as enthalpy potential

D. Energy balance

As the air gets cooled in the air washer, it results in increase of water energy, applying energy conservation principle we get,

$$G_a dH = -G_L C_L dT_L \tag{6}$$

$$-G_L C_L dT_L = h_L a_H (T_L - T_i) dx \tag{7}$$

The above equations corresponds to ordinary differential equations and the solutions of these equation will give the relation for different variables: $W(x)$, $T_a(x)$, $H(x)$, $T_L(x)$ and $T_i(x)$.

The following are the boundary conditions prescribed at the inlet of the air washer:

$W(0)=W_1$; $T_a(0)=T_{a1}$; $H(0)=H_1$; $T_L(0)=T_{L1}$; $T_i(0)=T_{i1}$

III. Solution Process

In the literature the above equation are solved by graphical procedure which can be time consuming. Very few papers have been written on analytical solution of simultaneous heat and mass transfer problem related to air washer.

A linear adjustment of the air saturation curve in an enthalpy- temperature chart has been considered. Therefore,

$$H_i(T_i) = a_1 T_i + a_0 \tag{8}$$

Where a_1 and a_0 are the adjusted coefficient, the coefficients were determined for the temperature range between 150°C and 300°C which represent well the working condition imposed in the air washer. In this paper the values of a_0 and a_1 are obtained as given below:

$a_1 = 3.833$; $a_0 = -18.026$

From equation (5), (6) and (7) and considering $a_h = a_m$ we obtain

$$\frac{H_i - H}{T_L - T_i} = \frac{h_L}{K_m} \tag{9}$$

$$\int_{H1}^{H} dH = \int_{T_{L1}}^{T_L} \frac{G_L C_L dT_L}{Ga} \tag{10}$$

Using Boundary conditions laid before we get, G_L, G_a, C_L in the equation are assumed to be constant

$$H - H_1 = \frac{G_L C_L}{Ga}(T_L - T_{L1}) \tag{11}$$

Substituting equation (8) and (11) into equation (9) we obtain a linear relationship between T_L and T_i given as

$$T_L = b_1 T_i + b_0 \tag{12}$$

$$b_0 = \frac{-H_L - \frac{G_L C_L T_{L1}}{Ga} + a_0}{\frac{h_L}{K_m} - \frac{G_L C_L}{G_a}} \tag{13}$$

$$b_1 = \frac{a1 - \frac{h_L}{K_m}}{\frac{h_L}{K_m} - \frac{G_L C_L}{G_a}} \tag{14}$$

Differentiating equation (12) we obtain

$$dT_L = b_L dT_i$$

Substituting equation (12) and (14) into equation (7) we obtain the following ordinary differential equation

$$\frac{dT_i}{dx} = \frac{h_l a_H b_0}{G_L C_L b_L} - T_i \left[\frac{h_L a_H (b_1 - 1)}{G_L C_L b_1}\right] \tag{15}$$

$$T_i(x) = C_1 e^{\frac{-h_L a_H (b_1-1)x}{G_L C_L b_1}} - \frac{b_0}{b_1 - 1} \tag{16}$$

Further,

$$C_1 = \frac{T_{L1} - b_0}{b_1} + \frac{b_0}{b_1 - 1} \tag{17}$$

Substituting Equation (16) in Equation (2) we obtain the following differential equation to determine the air temperature T_a

$$\frac{dT_a}{dx} + P(x) T_a = Q(x) \tag{18}$$

Where P(x) and Q(x) are given by

$$P(x) = \frac{h_a a_H}{G_a C_{pm}} \tag{19}$$

$$Q(x) = P(x) T_i(x) \tag{20}$$

Equation (18) is a first order linear differential equation and the general solution has been described in the available literature (Boyce and DiPrima, 2006).

$$T_a(x) = e^{-\int p(x)dx} \int Q(x) e^{\int P(x)dx} \, dx + C\, e^{-\int P(x)dx} \tag{21}$$

Where C represent the integration constant

Solving equation (18) and applying the initial condition given before, we will obtain the following expression for the air temperature T_a:

$$T_a(x) = \frac{C_2 e^{\frac{h_L a_H (b_1-1)x}{G_L C_L b_1}}}{\frac{h_a a_H}{G_a C_{pm}} - \frac{h_L a_H (b_1-1)}{G_L C_L b_1}} - \frac{b_0}{b_1 - 1} + C_3 e^{\frac{h_a a_H x}{G_a C_{pm}}} \tag{22}$$

Where 'x' denotes the axial position in the air washer and the constants of integration are given as below:

$$C_2 = C_1 \frac{h_a a_H}{G_a C_{pm}} \tag{23}$$

$$C_3 = T_{a1} - \left[\frac{C_2}{\frac{h_a a_H}{G_a C_{pm}} - \frac{h_L a_H (b_1-1)}{G_L C_L b_1}} - \frac{b_0}{b_1 - 1}\right] \tag{24}$$

Using a linear approach for the air saturation curve in a temperature – humidity chart, we obtain the following relation:

$$W_i(T_i) = d_1 T_i + d_0 \tag{25}$$

$d_1 = 0.0011$; $d_0 = -0.0069$ where d_1 and d_0 are adjusted coefficients.

Substituting Equation (25) into Equation (1), the mass balance for the humid air can be rewritten in the following form:

$$\frac{dW}{dx} + P(x)\, W = Q(x) \tag{26}$$

$$P(x) = \frac{K_m a_m}{G_a} \tag{27}$$

$$Q(x) = \frac{K_m a_m}{G_a}[d_0 + d_1 T_i(x)] \tag{28}$$

The general solution of equation (26) is given by

$W(x) =$

$$d_0 - \frac{d_1 b_0}{b_1-1} + \frac{K_m a_m d_1 C_1 e^{\frac{h_L a_H (b_1-1)x}{G_L C_L b_1}}}{K_m a_m - \frac{G_a h_L a_H (b_1-1)}{G_L C_L b_1}} + C_4 e^{\frac{K_m a_m x}{G_a}} \quad (29)$$

Where C4 is the constant of integration and can be obtained from the initial conditions as given below.

$$C_4 = W_1 - d_0 + \frac{K_m a_m d_1 C_1}{K_m a_m - \frac{G_a h_L a_H (b_1-1)}{G_L C_L b_1}} + \frac{d_1 b_0}{b_1-1} \quad (30)$$

IV. RESULTS AND DISCUSSION

A. Comparision with published results

The analytical solutions obtained from solving the equations were validated by using a case study from chapter 5 of the ASHRAE handbook Fundamentals (1997). In this chapter for the case study, a graphical procedure is utilised described by Kusuda (1957). The data and properties of air washer used in this case study are described in Table 1.

B. List of Properties

Data Set and Properties of the air washer used for the case study 1.

Water temperature at inlet T_{L1}= 35°C

Air temperature at inlet T_{a1} = 18.3°C

Air mass flow rate per unit area G_a = 1.628 kg/(s-m^2)

Spray ratio G_L/G_a = 0.70

Air heat transfer coefficient per cubic metre of chamber volume

$h_a\, a_H$ = 1.34 kW/(m^3·K)

Liquid heat transfer coefficient per cubic metre of chamber volume

$h_L\, a_H$ = 16.77 kW/(m^3·K)

Air volumetric flow rate Q = 3.07 m^3/s

The above air washer has been modelled according to the present method and solved analytically by using MATLAB 7.8.0. The graphs have been plotted as shown in Fig.2, Figure shows the temperature distribution of water, air and air-water interface throughout the length of air washer.

There is a good match between the graphical solution and the analytical solution (Fig.2), except in some areas. These minor differences take place as a result of the linearization procedure used in the present method for the air saturation curve. The errors can also come into picture due to selection of correlation of h_L and it tends to decrease with the reducing of temperature range. The solution can be employed under different air inlet conditions.

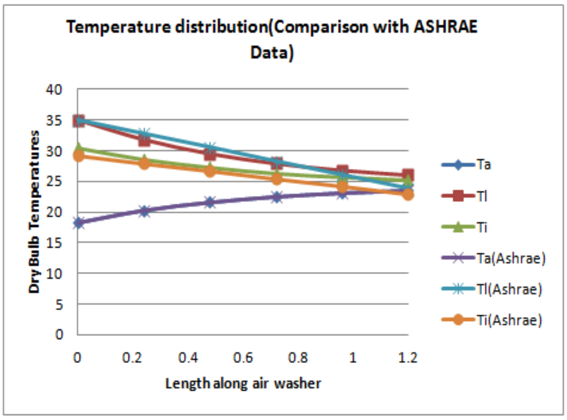

Fig.2. Temperature Distribution throughout the air-washer (comparison)

The analytical solution was tested for heating and humidifying for the case study from ASHRAE handbook. The solution was also tested to study the performance of air washer for cooling and humidifying process. The analytical solution was solved by using MATLAB 7.8 and the results are depicted in form of graphs. In the present analysis it has been assumed that a linear relationship exist between the w_i and T_i whereas in the ASHRAE chart the actual relationship have been considered.

C. Effect of the air dry bulb temperature

In order to assess the effect of dry bulb temperature in air cooling rate, three distinct values have been considered: 25, 30 and 35^{0}C. In the present case study, the inlet air humidity ratio was considered equal to 0.006 kgwv/ kgda while the inlet water temperature was assumed equal to 20^{0}C. Fig. 3 shows the air temperature profiles along the axial position in the air washer. The investigations showed that for a certain humidity ratio value, when increasing the air dry bulb temperature at the inlet, greater temperature drop is obtained. This happens because higher dry bulb temperatures with constant humidity ratio mean smaller relative humidity. When the air relative humidity is small, there emerges a high difference in the partial pressure of water vapour, favouring in this way, the mass transfer between the water and the air. Consequently, more sensible heat is extracted from the air for water evaporation, resulting in larger temperature drop in the air. In the present case study, when inlet air temperature was set to 35^{0}C, the temperature drop was about 9^{0}C. However, when the inlet temperature was 25^{0}C, there occurred a temperature drop of only 4^{0}C.

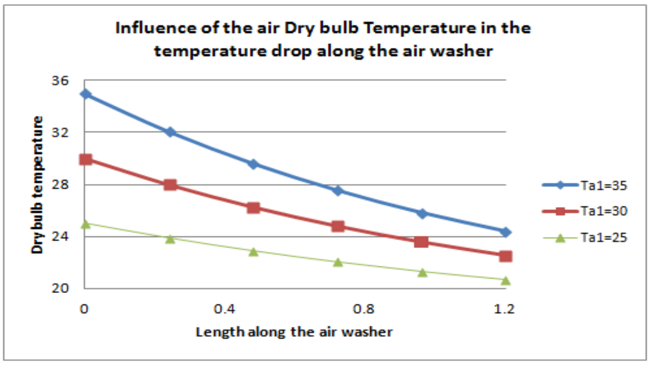

Fig.3. Influence of air dry bulb temperature in the temperature drop along the air washer

D. Effect of inlet water temperature

For this investigation the air dry bulb temperature at inlet was set to 35^{0}C; while the inlet humidity ratio is maintained at 0.006 kgwv/kgda. Figure 4 shows the results for three different values for the inlet water temperature: 15, 20 and 25^{0}C. Fig. 4, shows that the reduction in the feed water temperature decreases the air temperature at the exit of the air washer. The heat exchange between the air and water increases due to reduction in cooling water temperature. Therefore, a reduction in the feed water temperature increases the air cooling rate.

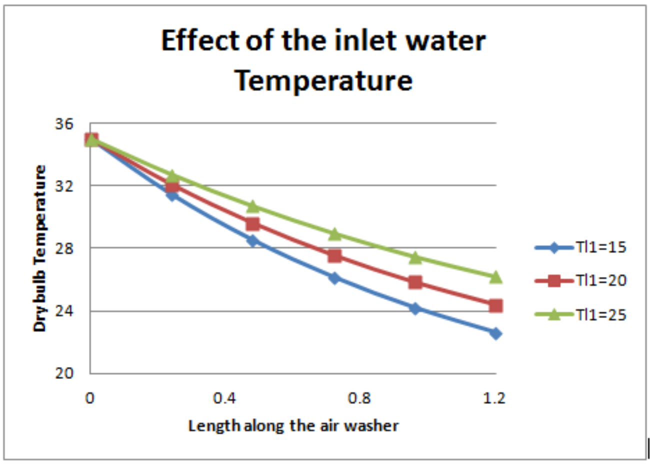

Fig.4. Effect of inlet water temperature in the air cooling rate

E. Influence of applied Flow rate

Fig.5 shows the effect of the applied air flow rate on the air temperature profiles along the washer. The air dry bulb temperature at the inlet was set to 35^{0}C, while the inlet humidity ratio was maintained in 0.010 kgwv/kgda. Three different values for the mass flow rate per unit cross-sectional area of the washer are considered: 1.628, 3.256, and 4.884 kg/sm^2. The inlet water temperature was maintained at 15^{0}C. From Fig. 5, it is possible to observe that an increase in the applied flow rate reduces the air cooling rate. This occurs as a result of the reduction in the contact time between the water and the air as the applied flow rate increases, and as a consequence the temperature of air leaving the air washer increases.

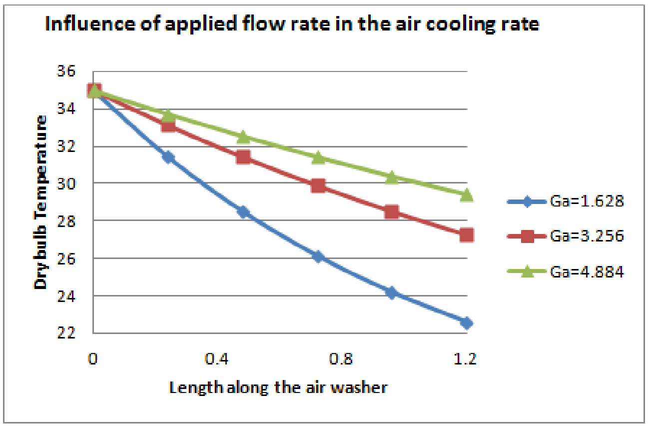

Fig.5. Effect of the applied air flow rate in the air cooling rate

F. Behaviour of water and air in the air washer

Fig.6 shows the water, the liquid gas interface, and the air temperature profiles along the washer. The air dry bulb temperature at the inlet of the washer was maintained at 35^{0}C, while the inlet humidity ratio was maintained at 0.010 kg/ kgwv. The inlet water temperature was set to 15^{0}C. From Fig. 6, it can be observed that the warm and dry air transfers sensible heat for the water causing its heating. It has been observed, however, that the temperatures profiles approach each other as the air flows along the air washer. In fact, if the contact area between the air and the water were infinite these temperature profiles would be the same at the exit of the air washer. The same is obtained by the analytical solution as shown in Fig.6.The profiles of the interface humidity ratio and the air humidity ratio along the washer is shown in Fig 8. It can be observed that the air humidity ratio increases as a consequence of the mass transfer between the air and the water. The air humidifying process happens because the humidity ratio in the interface liquid gas is higher than the air humidity ratio.

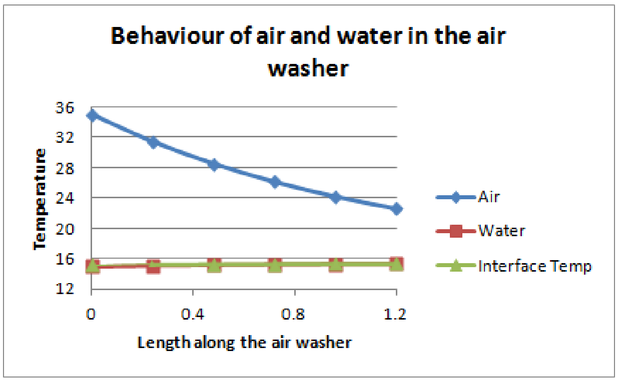

Fig.6. Water and interface temperature profile along the air washer

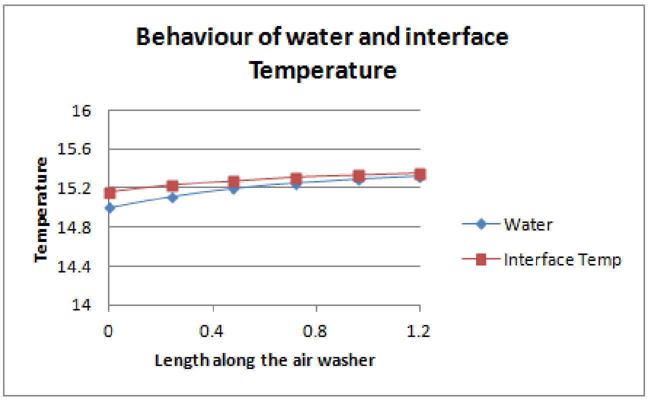

Fig.7. Effect on Water and interface Temperature along the air washer

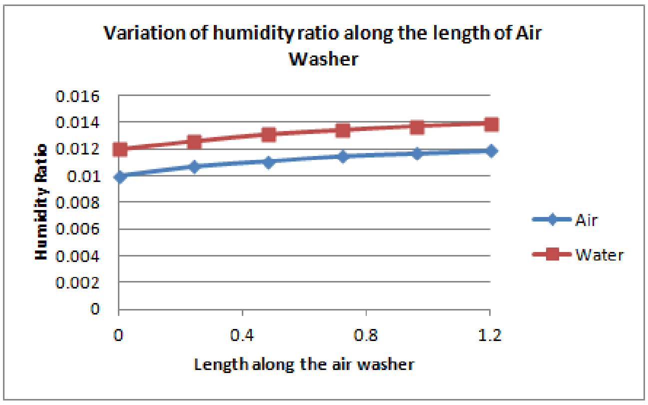

Fig.8. Air Humidity ratio profile along the air

V. DISCUSSION

The main aim of the paper is to develop a theoretical approach for simultaneous heat and mass transfer for the air washer and the same concept can be applied to the other evaporative cooling devices. The equation developed were solved by MATLAB 7.8.0 and shown in form of graphs. The obtained results have been compared with ASHRAE data book and found to match with the literature available. The error found is very small and it is due to assumption of linear approach for the air saturation curve.

It has been obtained that the air washers are more efficient in hot and dry climate. It produces lowest exit as compared to other. As the applied flow rate increases the contact time between air and water reduces and the cooling rate decreases. The approach can also be utilised for the cooling tower with slight modifications.

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