A Mathematical Model for Nonlinear Optimization Which Attempts Membership Functions to Address the Uncertainties

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Abstract:

The problem of optimizing an objective function that exists within the constraints of equality and inequality is addressed by nonlinear programming (NLP). A linear program exists if all of the functions are linear; otherwise, the problem is referred to as a nonlinear program. The development of highly efficient and robust linear programming (LP) algorithms and software, the advent of high-speed computers, and practitioners' wider understanding and portability of mathematical modeling and analysis have all contributed to LP's importance in solving problems in a variety of fields. However, due to the nature of the nonlinearity of the objective functions and any of the constraints, several practical situations cannot be completely explained or predicted as a linear program. Efforts to overcome such nonlinear problems quickly and efficiently have made rapid progress in recent decades. The past century has seen rapid progress in the field of nonlinear modeling of real-world problems. Because of the uncertainty that exists in all aspects of nature and human life, these models must be viewed through a system known as a fuzzy system. In this article, a new fuzzy model is proposed to address the vagueness presented in the nonlinear programming problems (NLPPs). The proposed model is described; its mathematical formulation and detailed computational procedure are shown with numerical illustrations by employing trapezoidal fuzzy membership functions (TFMFs).Here, the computational procedure has an important role in acquiring the optimum result by utilizing the necessary and sufficient conditions of the Lagrangian multipliers method in terms of fuzziness. Additionally, the proposed model is based on the previous research in the literature, and the obtained optimal result is justified with TFMFs. A model performance evaluation was completed with different set of inputs, followed by a comparison analysis, results and discussion. Lastly, the performance evaluation states that the efficiency level of the proposed model is of high impact. The code to solve the model is implemented in LINGO, and it comes with a collection of built-in solvers for various problems.

Keywords: nonlinear optimization; fuzzy nonlinear programming problem; Lagrangian multiplier method in terms of fuzziness; fuzzy numbers; trapezoidal membership functions; ranking index.

Introduction

NLP typically describes rather more significant challenges than LP. The situation is perhaps always difficult if all of the constraints are linear and the objective function is nonlinear. For example, the feasible set may or may not be convex, and the optimum result could be placed within the feasible set, on its boundary, or at its vertex. For the most part, the scientific programming issue manages the ideal use or distribution of constrained assets to meet the ideal goal. The fuzzy NLP issue is valuable in taking care of issues due to the uncertain, emotional nature of the problematic definition, or due to its precise arrangement. In this case, an objective function must improve while working within certain constraints. Ref [1] introduced the theory of fuzzy and fuzzy decision-making, and the right decision used in decision problems to attain the optimum result [2]. Finally, the evaluation of the optimal results for the mentioned two cases reveals the newness and cost effectiveness so far fuzzy model, addressing the ambiguity and providing significantly more optimum values.

Literature Survey

This section highlights certain identified research work collections of existing fuzzy NLP, as shown below: Fuzzy programming techniques are likely to have a broader range of applications for nonlinear optimization and also stochastic optimization, specifically for allocation problems in supply chain management. A genetic algorithm technique has been used to illustrate the nonlinear transportation problem as an improved version of their previous findings for linear transportation problems, which obtained feasibility due to chromosome structures and genetic operators [19]. An innovative application for nonlinear network flow problems has been presented, which is strong enough to handle mixed-integer nonlinear optimization problems that in corporate an online a transportation problem with the best solution[20].

The outcomes are compared to a proposed approach for the design of the lowest cost canal sections in Newton's method, which is applied to KKT conditions for the constrained into unconstrained NL optimization problems with standard algorithms [28].The fuzzy-based Lagrangian method can be described as the digital information mechanism to support vector machines for readily accessible biomedical data interpretation [29].

Preliminaries

In this section, some essential primary concepts and backgrounds are outlined in fuzzy mathematics[5,6].Nowitseemstoaddressafewdefinitionswhicharemostrequired:

Definition1

Let$T$=[t_1,t_2,t_3,t_4]*beatrapezoidalfuzzynumberwiththefollowingMF,*

$$\mu_T(x) = \begin{Bmatrix} \frac{x-t1}{t2-t1}, t1 \le x \le t2 \\ 1, t2 \le x \le t3 \\ \frac{x-t3}{t3-t4,}, t3 \le x \le t4 \end{Bmatrix}$$

TheMF$\mu_T(x)$*isillustratedintheFigure1below*

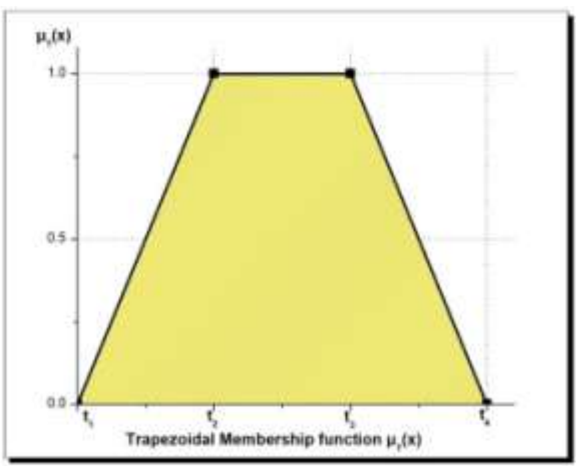


Figure1. TrapezoidalMembershipfunction$\mu T(x)$.

Definition2

LetafuzzysetTinXandanyrealnumberαin[0,1],thentheα-cutof T, denoted by $^{\alpha}T$ *is the crisp set* $^{\alpha}T = \{x \in X : \mu_T(x) \ge \alpha\}$. *For illustration, let T be a fuzzy set whosemembershipfunctionisgivenasabove*$\mu_T(x)$.

Tofindtheα-cutofT,whereα$\in$[0,1],*letus set the reference functions of T to each left and right.*

Expressing xtoα, where$x^{(1)} = (t_2 - t_1)\alpha + t_1$*and*$x^{(2)} = t_4 + (t_3 - t_4)\alpha$ *which provides the α−cut of T is*

$$^{\alpha}T = [x^{(1)}, x^{(2)}] = [(t_2 - t_1)\alpha + t_1, t_4 + (t_3 - t_4)\alpha].$$

An Optimization Model for Fuzzy Nonlinear Programming

Research emphasison fuzzy optimization issues in the area of NLP is mainly limited. However, little attention has focused on NLP, such as within quadratic programming, separable programming and search methods, and many others. However, apart from that, there are several numerous forms of fuzzy NLP addressed extensively in various significant issues, mostly in complex industrial systems. Research emphasis on problems of fuzzy optimization in the field of NLP is generally limited. Furthermore, there is little interest in NLP to address the vagueness soft the issues. Besides this, in many real issues, many kinds of fuzzy NLPs occur, mainly in complex manufacturing systems. This cannot be signified and enlightened by traditional models. Meanwhile, scientific studies on modeling techniques and enhancing approaches for NLP in fuzzy situations are important not only from the frame work of fuzzy optimization but also in the application of the challenges.

Numerical Illustration

This section outlines two illustrative examples that canbe used to optimize the models for addressing the problem of fuzzy NLP using TFMF and its mathematical calculations[5–7,32]. In Case(i),the fuzzy model explains the procedure using the MF approach,and inCase(ii), the same problem was investigate during the robus tranking approach.

The NLP in the manner of fuzziness is as follows, and the fuzzified form of the considered NLPP can be stated as below:

Minimize

$$[-1,0,2,3]{x^{(k)}}^2 + [-1,0,2,3]{x^{(k)}}^2 + [-1,0,2,3]{x^{(k)}}^2, \text{forall } k = 1,2,3,4.$$

$x^{(k)}, x^{(k)}, x^{(k)} \geq 0$, forall$k$=1,2,3,4 1 2

$[-2x_2+2x_2\lambda, 0, 4x_2-4x_2\lambda, 6x_2-6x_2\lambda]=0$

$[-2x_3, -\lambda, 4x_3-3\lambda, 6x_3-4\lambda]=0$

$$2x_1-x_2^2-12, 3x_1+x_3-13, 5x_1+2x_2^2+3x_3-15, 6x_1+3x_2^2+4x_3-16=0$$

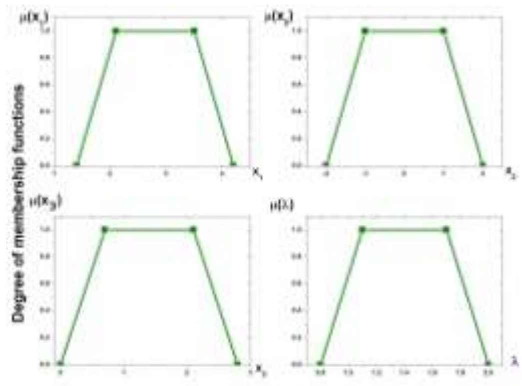
Solving the above the equations results in the extreme points,they are;

Extremepoint1:$(x_\circ,\lambda_\circ)$=[(1,1.5,2.5,3),(−2,0,4,6),(0,0.5, 1.5,2),(−1,0,2,3)]
Extremepoint2:$(x_\circ,\lambda_\circ)$=[(1,1.5,2.5,3),(−2,0,4,6),(0,0.5, 1.5,2),(−1,0,2,3)]

Extremepoint3:$(x_\circ,\lambda_\circ)$=[(1.4,2.1,3.5,4.2),(−2,−1,1,2),(0, 0.7,2.1,2.8),(0.8,1.1,1.7,2)]

By employing the sufficiency conditions to evaluate whether the extreme point sare maximum or minimum. Hence, the sufficient conditions for the LMM for minimizing the above NLPP as H=

[−2,−1,1,2]	[2,3,5,6]	[0,1,3,4]x_2	,1,3,4]
[2,3,5,6]	[0,1,3,4]	[−2,−1,1,2]	[−2,−1,1,2]
[0,1,3,4]x_2	[−2,−1,1, 2]	[0,1,3,4]−[0,1,3, 4]λ	[−2,−1,1,2]
[0,1,3,4]	[−2,−1,1, 2]	[−2,−1,1,2]	[0,1,3,4] 1 2



The fuzzy vectors (x1), (x2), (x3) and the Lagrangian multiplier (λ)

$\lambda^{(k)}$=[0.8,1.1,1.7,2]&$Z^{(k)}$=[−9.8,0,19.6,29.4],*for all* k=1,2,3,4.

Case(ii):The Robust Ranking Approach for NLP with Fuzzy MFs

The NLP in the manner off fuzziness i s as follows, and the fuzzified form of the considered NLPP can be stated as below:

Minimize

$[-1,0,2,3]x^{(k)^2}+[-1,0,2,3]x^{(k)^2}+[-1,0,2,3]x^{(k)^2}$,*for all* k=1,2,3,4.

Subject to the constraints,

$[2,3,5,6]x^{(k)}+[-1,0,2,3]x^{(k)^2}+[0,1,3,4]x^{(k)}=[12,13,15,16]$, *for all* k=1,2,3,4.

$x^{(k)}, x^{(k)}, x^{(k)} \geq 0$, *for all* k=1,2,3,4

The confidence interval for each degree α & the trapezoidal structures will be characterized by the functions of α.

Therefore,

$[x^{(1)}, x^{(2)}]=T^L T^U=[(t_2-t_1)\alpha+t_1, t_4+(t_3-t_4)\alpha]$

$=[\alpha-1, \alpha-3]$

$R(T)=R[-1,0,2,3]=(0.5)*T^L T^U d\alpha=(0.5)(2)d\alpha=1$

Using the proposed approach in the previous section, the fuzzy NLPP can be modified to the conventional crisp problem; the crisp problem is

$MinimizeZ = x_1^2 + x_2^2 + x_3^2$

Subject to the constraints,

$$4x_1 + x_2^2 + 2x_1 = 14; x_1, x_2, x_3 \geq 0.$$

Now apply the existing conventional approach to the NLPP by using necessary and sufficientconditionsoftheLMMandobtainedtheoptimumsolutionfortheaboveis

$x_1 = 2.8, x_2 = 0, x_3 = 1.4$ $\lambda = 1.4$ & Minimum $Z = 9.8$.

Models Performance Evaluation with Different Sets of Inputs

This section is encapsulated to determine the efficiency of the fuzzy model and its solutions. For this efficiency test,we have considered four different sets of inputs in fuzzy format and then, using the ranking function provided in the earlier section, wehave defuzzified all these inputs to obtain the equivalent crisp number. The fuzzy inputs are available in Table 1.With the defuzzified value, we have solved the model for each set using LINGO software and we have obtained the optimal solution for the NLPP.The results are given in Table1 and here it canbe easily observed that for any arbitrary set of trapezoidal fuzzy inputs,the model is solvable and gives the optimal solution. The code to solve the model is implemented in LINGO, and it comes with a collection of built-in solvers for various problems. The modeling environment is strictly aligned to the LINGO solver and because of this interconnectivity, it transmits problems directly to memory which results in the minimization of compatibility issues between the solver and modeling components. It uses multiple CPU cores for model simulation, thus giving faster results

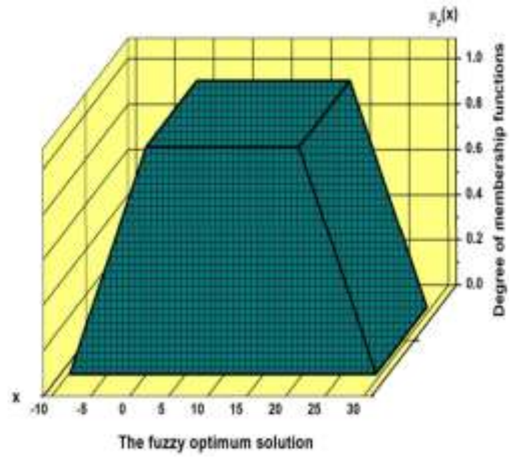
Results and Discussion

Employing the suggested model numerical illustrations demonstrate that the optimum value of the FNLPP is [−9.8, 0, 19.6, 29.4], which might be a fresh attempt to clear the vagueness.The optimum solution for the fuzzified NLPPs will be continuously greater than −9.8 and lessthan 29.4, and the most likely outcome will be some where in the range of 0 and 19.6.The varieties in cost with significance probability can be seen in Figure 3. Additionally, obtained fuzzy optimum solutions x_{ij} might be empirically comprehended.

The decision maker perception,the entire value of the fuzzy NLPP,will be higherth−9.8and lessthan 29.4.

The decision-maker for the entire fuzzy NLPP estimations are going to be bigger than or sufficient to 0 and less than or equivalent to 19.6. The extent of the favors of the decision-maker for the rest of the estimations of the entire fuzzy NLPP value has frequently been attained as below: Here x describes the significance of the entire NLPP,and also the perception of decision- makers for $\mu_{min}(X)$.

$$\mu_{min}(X) = \begin{cases} \dfrac{x+9.8}{9.8} \; for -9.8 \leq x \leq 0 \\ 1 for \; 0 \leq x \leq 9.6 \\ \dfrac{x-29.4}{9.8} for \; 19.6 \leq x \leq 29.4 \\ 0, \quad otherwise \end{cases}$$



Conclusions

Finally, an effort has been made to create a model that solves the problem of NLP in a fuzzy environment. The fuzzy version of the problem has been addressed using the necessary and sufficient conditions of Lagrangian multipliers in terms of fuzziness with the aid of an umerical illustration. This approach clarifies by solving two numerical illustrations; one is using MFs, and the other, the approach of robust rankings.MFs provide a significant role in the creation of a model in a fuzzy context.Most of there search techniques have been discussed in establishing only the MFs for the fuzzy objectives or constraints.However, this approach solved the mutually contradictory complexity ofthe objectives as well as constraints using MFs.This model offers an efficient approach to dealing with the problems of NLP.

Therefore, the optimal solution has been signified through fuzziness with in the result and discussion.Additionally, the solution is explained by the manner of TFMs which have models of performance evaluation with different sets of inputs. This shows that the efficiency level of the model is of high impact. The code to solve the model is implemented in LINGO, and comes with a collection of built-insolvers for various problems. Furthermore, the comparison analysis could be a newly-designed effort to solve NLPs under fuzziness. The model focuses on addressing the decision-makers uncertainties and subjective experiences, and can help to solve decision-making issues. The model's future scope suggests that the model be used in other types of NLPPs or suitable nonlinear optimization models in upcoming models, preferably optimization models, under numerous fuzzy situations.

References

1. Zadeh,L.A. Information and Control.In Fuzzy Sets; World Scientific: Singapore,1965;Volume8,pp.338–353.
2. Zimmermann, H.-J. Fuzzy Sets, Decision Making, and Expert Systems; Springer Science & Business Media:Berlin/Heidelberg, Germany, 2012; Volume 10.
3. Vasant, P.; Nagarajan, R.; Yaacob, S. Fuzzy linear programming:A modern tool for decision making.In Computationa lIntelligence for Modelling and Prediction;Springer:Berlin/Heidelberg,Ger many,2005;pp.383–401.
4. Kheirfam,B.;Hasani,F.Sensitivity analysis for fuzzy linear programming problems with fuzzy variables.Adv.Model.Optim. 2010,12,257–272.
5. Palanivel, K. Contributions to the Study on Some Optimization Techniques in Fuzzy Membership Functions; Bharathidasan University:Trichy, Tamil Nadu, India, 2013.
6. Palanivel,K Fuzzy commercial traveler problem of trapezoidal membership functions with in the sort of α optimum solution using ranking technique. Afr. Mat. 2016,27,263–277. [CrossRef]
7. Saranya,R.;Palanivel,K.Fuzzy nonlinear programming problem for inequality constraints with alpha optimal solution in terms of trapezoidal membership functions.Int. J. Pure Appl. Math. 2018, 119, 53–63.
8. Tang,J.;Wang,D.Anonsym metric model for fuzzy nonlinear programming problems with penalty coefficients. Comput.Oper. Res.1997, 24, 717–725. [CrossRef].
9. Tang,J.;Wang,D.;Ip,A.; Fung,R.Ahybridgenetic algorithm for a type of nonlinear programming problem.Comput.Math.Appl. 1998,36,11–22.[CrossRef]
10. Fung, R.Y.; Tang, J.; Wang, D. Extension of a hybrid genetic algorithm for nonlinear programming problems with equality and inequalityconstraints.Comput.Oper.Res.2002,29,261–274. [CrossRef]
11. Sarimveis,H.;Nikolakopoulos,A.Alineupevolutionary algorithm for solving nonlinear constrained optimization problems.Comput.Oper.Res.2005,32,1499–1514.[CrossRef]
12. Syau,Y.-R.;StanleyLee,E.Fuzzy convexity and multi objective convex optimization problems.Comput.Math.Appl.2006,52,351–362.[CrossRef]
13. Chen, S.-P. Solving fuzzy queueing decision problems via a parametric mixed integer nonlinear programming method. Eur. J. Oper. Res. 2007,177,445–457. [CrossRef]
14. Qin,X.S.;Huang,G.H.;Zeng,G.M.;Chakma,A.;Huang,Y.F.Aninterval-parameter fuzzy nonlinear optimization mode lfor stream water quality management under uncertainty. Eur. J. Oper. Res. 2007, 180, 1331–1357. [CrossRef]
15. Kassem, M.A.E.-H. Stability achievement scalarization function for multiobjective nonlinear programming problems.Appl. Math.Model.2008, 32, 1044–1055. [CrossRef]
16. Shankar, N.R.; Rao, G.A.; Latha, J.M.; Sireesha, V. Solving a fuzzy nonlinear optimization problem by genetic algorithm.Int. J. Contemp. Math. Sci. 2010,5,791–803.
17. Abd-El-Wahed, W.F.; Mousa, A.A.; El-Shorbagy, M.A. Integrating particle swarm

optimization with genetic algorithms for solvingnonlinearoptimizationproblems.J.Comput.Appl.Math.2011,235,1446–1453.[CrossRef]

18. Jameel,A.F.; Sadeghi, A. Solving nonlinear programming problem in fuzzy environment. Int.J.Contemp.Math.Sci.2012, 7, 159–170.

19. Michalewicz,Z.;Vignaux,G.A.;Hobbs,M.A nonstandard genetic algorithm for the nonlinear transportation problem. ORSAJ. Comput.1991, 3, 307–316. [CrossRef]

20. Ilich,N.;Simonovic,S.P.Anevolutionprogramfor non-linear transportation problems.J.Heuristics2001,7,145–168.[CrossRef]

21. Hedar, A.-R.; Allam, A.A.; Deabes, W. Memory-Based Evolutionary Algorithms for Nonlinear and Stochastic Programming Problems. Mathematics2019, 7, 1126. [CrossRef]

22. Klanšek, U. A Comparison between Milp and Minlp Approaches to Optimal Solution of Nonlinear Discrete Transportation Problem. Transport 2014, 30,135–144. [CrossRef]

23. Das, A.; Bera, U.K.; Maiti, M. A profit maximizing solid transportation model under a rough interval approach. IEEE Trans. Fuzzy Syst.2016, 25, 485–498. [CrossRef]

24. Das, A.; Lee, G.M. A Multi-Objective Stochastic Solid Transportation Problem with the Supply, Demand, and Conveyance Capacity Following the Weibull Distribution. Mathematics2021, 9, 1757. [CrossRef]

25. Ahmadini,A.A.H.;Varshney,R.;Ali,I.Onmultivariate-multi objective stratified sampling design under probabilistic environment:A fuzzy programming technique. J.KingSaud. Univ.-Sci. 2021,33,101448. [CrossRef]

26. Khan, M.F.; Modibbo, U.M.; Ahmad, N.; Ali, I. Nonlinear optimization in bi-level selective maintenance allocation problem. J. KingSaud.Univ.-Sci.2022,34,101933. [CrossRef]

27. Ali,S.S.;Barman,H.;Kaur,R.;Tomaskova,H.; Roy,S.K.Multi-Product Multi Echelon Measurements of Perishable SupplyChain: Fuzzy Non-Linear Programming Approach. Mathematics 2021, 9, 2093. [CrossRef]

28. El-Sobky, B.; Abo-Elnaga, Y.; Mousa, A.A.A.; El-Shorbagy, M.A. Trust-Region Based Penalty Barrier Algorithm for Constrained Nonlinear Programming Problems: An Application of Design of Minimum Cost Canal Sections.Mathematics 2021,9, 1551. [CrossRef]

29. Gupta, D.; Borah, P.; Sharma, U.M.; Prasad, M. Data-driven mechanism based on fuzzy Lagrangian twin parametric-margin support vector machine for biomedical data analysis. NeuralComput. Appl. 2021,1–11. [CrossRef]

30. Lin,L.;Lee,H.-M.Fuzzy nonlinear programming for production inventory based on statistical data.J.Adv.Comput.Intell.Intell. Inform. 2016, 20, 5–12. [CrossRef]