A Method to Solve Multi Criteria Decision Making Problems based on Fuzzy Numbers

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Abstract:- **Multi-Criteria decision making (MCDM) method is a technique where alternatives or options are assessed based on a set of criteria. Most important decisions in organizations are finalized by group of experts. Human judgments including preferences are often vague and cannot be estimated in exact numerical values. This paper proposes a fuzzy approach under the linguistic frame work to obtain optimal solution for Multi Criteria Decision Making problems. To accomplish this, an aggregate-deviation method based on triangular fuzzy numbers is proposed. A fuzzy decision matrix plays an important role in our research problem.**

Keywords:- ***Multi Criteria Decision Making, Triangular fuzzy numbers, Linguistic Variables, Aggregation Operators, Fuzzy Decision Matrix.***

I. INTRODUCTION

Decision Making

Decision making can be defined as a process of specifying a problem, identifying and evaluating criteria or alternatives and selecting a preferred alternative among possible ones (Chen, 2005).

Multi-criteria Decision Making (MCDM)

Multi-criteria decision making (MCDM) method is a technique where alternatives or options are assessed based on a set of criteria and it is one of the most widely used methods in decision making (Hwang & Yoon, 1981). MCDM methods have been employed in many areas such as engineering, agricultural, banking, energy, forestry, health services and education. General form of MCDM problem with *m* alternatives and *n* criteria can be illustrated in matrix format as follows:

$Alternatives/Criteria$	$C1$	$\ldots\ldots Cn$
$A1$	$-$	$-$
.	.	.
.	.	.
Am	$-$	$-$

Fuzzy MCDM

In real life, decision makers often make evaluation based on a set of criteria which are normally vague and imprecise. Due to this, fuzzy set was introduced particularly in representing the vague information or criteria. Fuzzy set theory was first utilized in solving decision making problem by Bellman and Zadeh in 1970. The key concept of fuzzy set theory is that its elements have a varying grade of membership, ranging from 0 to 1. The boundaries of these fuzzy sets are not sharp or imprecise. The individual membership in a fuzzy set is represented by the *degree of compatibility* (Klir et.al, 1997) and fuzzy sets are used to describe linguistic values for example "very good," "good," "fair," "poor," and "very poor". Instead of using exact numbers as input values, fuzzy numbers were utilized in representing these linguistic terms.

The introduction of fuzzy set theory also motivates many researchers in integrating the theory with some of the classical MCDM methods. Pioneer work in incorporating fuzzy element into decision making was done by introducing an algorithm for rating and ranking multiple aspects of alternatives using fuzzy sets. Decision makers' opinions can be expressed in terms of linguistic variables.

II. PRELIMINARIES

Fuzzy Set :

A fuzzy set is a pair (A, f) where A is a set and f:A→[0,1]. For each x in A the value f(x) is called the grade of membership of x in (A, f).

Triangular fuzzy number:

Let l, m, u in **R** , l$< m < u$. The fuzzy number t:R→[0,1] denoted by

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$$t = \begin{cases} 0 & if\ x < l \\ \frac{x-l}{m-l} & if\ l \le x \le m \\ \frac{u-x}{u-m} & if\ m \le x \le u \\ 0 & if\ x > u \end{cases}$$

is called a triangular fuzzy number. *Example*: [1, 2, 3]

Linguistic Variable[5]:

A linguistic variable is a variable whose values are words or sentences in a natural or artificial language. These linguistic variables can be expressed in positive triangular fuzzy numbers.

III. PROPOSED METHOD

- Collect the evaluation of alternatives by expert decision makers with respect to all criteria in terms of linguistic variables and we can form a decision matrix.
- Replace each linguistic variable by corresponding fuzzy number.
- Aggregate the fuzzy numbers in column wise based on criteria C_1, C_2 ,……..
- Aggregate the fuzzy numbers column wise based on decision makers P_1, P_2 ,…..
- Find the deviation of each triangular fuzzy number.
- The fuzzy number with minimum deviation comes first in ranking order [ascending].

IV COMPUTATIONAL ASPECTS

Suppose group of expert decision makers want to select a most suitable candidate from several alternatives based on some criteria.

STEP 1

Evaluation of alternatives by expert decision makers with respect to all criteria in terms of linguistic variables.

Linguistic frame work [5]

Very Poor	VP	(0,0,1)
Poor	P	(0,1,3)
Medium Poor	MP	(1,3,5)
Fair	F	(3,5,7)
Medium Good	MG	(5,7,9)
Good	G	(7,9,10)
Very Good	VG	(9,10,10)

Evaluation table

		C_1			C_2			……			…C_m		
P_1	A_1	L_1	L_2	L_3	…	..	…	..	..	..	..	..	..
	. .												
	A_n												
P_2	A_1											..	..
. . P_k	. .												
	A_n												

Here C_1, C_2,…. are the criteria. A_1, A_2, A_3…. are the alternatives. P_1,P_2,…. are the decision makers.L_1, L_2,… are the linguistic variables.

STEP 2

To construct a fuzzy decision matrix replace each linguistic variable by corresponding fuzzy number.

STEP 3

Aggregate the fuzzy numbers in column wise based on criteria C_1, C_2,…….. by using the formula [7],

$$L_{ag} = M_{ag} - \frac{1}{\prod_1^n \frac{1}{[M_{ag}-ui]}^{\frac{1}{n}}}, \quad M_{ag} = [\prod_1^n m_i]^{\frac{1}{n}} \quad \&$$

$$U_{ag} = M_{ag} + \frac{1}{\prod_1^n \frac{1}{[li - M_{ag}]}^{\frac{1}{n}}}$$ for all fuzzy numbers (l_i,m_i,u_i).

Now we have the set of fuzzy numbers(L_{ag} , M_{ag} , U_{ag}).
and we can form the table,

	P_1	P_2………..	…..P_k
A_1	$F_{ag\ 1}$	$F_{ag\ 2}$………	
. . .	. . .		
A_m	……..		………

Where $F_{ag\ 1}$, $F_{ag\ 2}$……… are fuzzy numbers.

STEP 4

Aggregate fuzzy numbers column wise using the same formula based on decision makers P_1, P_2,…..
We get

A_1	$F_{ag\ 1}$
A_2	$F_{ag\ 2}$
.	.
A_m	$F_{ag\ m}$

STEP 5

Find the deviation in triangular fuzzy number by using the formula $D_f = (u - l) + \frac{m-l}{3+l}$ where (l,m,u) is triangular fuzzy number.

STEP 6 [CONCLUSION]

The fuzzy number with minimum deviation (D_f) comes first in ranking order [ascending].

V NUMERICAL EXAMPLE

Suppose 3 expert decision makers want to select a most suitable computer programmer from 3 alternatives based on 5 criteria which are attitude, communication skills, hardworking, general knowledge, and programming knowledge.

STEP 1

Collect the evaluation of alternatives by expert decision makers with respect to all criteria in terms of linguistic variables and we can form a decision matrix[6].

	C1			**C2**			**C3**			**C4**			**C5**		
	A1	**A2**	**A3**	**A1**	**A2**	**A3**	**A1**	**A2**	**A3**	**A1**	**A2**	**A3**	**A1**	**A2**	**A3**
P1	MG	G	VG	G	VG	MG	F	VG	G	VG	VG	VG	F	VG	G
P2	G	G	G	MG	VG	G	G	VG	MG	G	VG	VG	F	MG	G
P3	MG	MG	F	F	VG	VG	G	G	VG	VG	VG	MG	F	G	MG

STEP 2

Replace each linguistic variable by corresponding fuzzy number.

FUZZY DECISION MATRIX						
		C1	**C1**	**C3**	**C4**	**C5**
P1	**A1**	(5,7,9)	(7,9,10)	(3,5,7)	(9,10,10)	(3,5,7)
	A2	(7,9,10)	(9,10,10)	(9,10,10)	(9,10,10)	(9,10,10)
	A3	9,10,10	(5,7,9)	(7,9,10)	(7,9,10)	(7,9,10)
P2	**A1**	(7,9,10)	(5,7,9)	(7,9,10)	(7,9,10)	(3,5,7)
	A2	(7,9,10)	(9,10,10)	(9,10,10)	(9,10,10)	(5,7,9)
	A3	(7,9,10)	(7,9,10)	(5,7,9)	(9,10,10)	(7,9,10)
P3	**A1**	(5,7,9)	(3,5,7)	(7,9,10)	(9,10,10)	(3,5,7)
	A2	(5,7,9)	(9,10,10)	(7,9,10)	(9,10,10)	(7,9,10)
	A3	(3,5,7)	(9,10,10)	(9,10,10)	(5,7,9)	(5,7,9)

STEP 3

Aggregate the fuzzy numbers in column wise based on criteria C_1, C_2 ,……..

		C1	**C2**	**C3**	**C4**	**C5**	**Aggregation**
P1	**A1**	(5,7,9)	(7,9,10)	(3,5,7)	(9,10,10)	(3,5,7)	(6.57,6.91,10.12)
	A2	(7,9,10)	(9,10,10)	(9,10,10)	(9,10,10)	(9,10,10)	(8.82, 9.79, 10)
	A3	(9,10,10)	(5,7,9)	(7,9,10)	(7,9,10)	(7,9,10)	(6.71, 8.74, 9.66)
P2	**A1**	(7,9,10)	(5,7,9)	(7,9,10)	(7,9,10)	(3,5,7)	(6.39, 7.6, 9.24)
	A2	(7,9,10)	(9,10,10)	(9,10,10)	(9,10,10)	(5,7,9)	8.69, 9.12,9.71)
	A3	(7,9,10)	(7,9,10)	(5,7,9)	(9,10,10)	(7,9,10)	(6.71,8.74,9.66)
P3	**A1**	(5,7,9)	(3,5,7)	(7,9,10)	(9,10,10)	(3,5,7)	(3.97,6.91,8.05)
	A2	(5,7,9)	(9,10,10)	(7,9,10)	(9,10,10)	(7,9,10)	(6.71, 8.93, 9.55)
	A3	(3,5,7)	(9,10,10)	(9,10,10)	(5,7,9)	(5,7,9)	(5.27, 7.55, 9.02**)**

We get

	P1	**P2**	**P3**
A1	(6.57,6.91,10.12)	(6.39, 7.6, 9.24)	(3.97,6.91,8.05)
A2	(8.82, 9.79, 10)	8.69, 9.12,9.71)	(6.71, 8.93, 9.55)
A3	(6.71, 8.74, 9.66)	(6.71,8.74,9.66)	(5.27, 7.55, 9.02**)**

STEP 4

Aggregate the fuzzy numbers column wise based on decision makers P_1, P_2 ,…..

	Total evaluation
A1	(6.07, 7.02, 8.93)
A2	(8.46, 9.07, 9.73)
A3	(6.73, 8.15, 9.40)

STEP 5

Find the deviation in triangular fuzzy number by using

$D_f = (u - l) + \frac{m-l}{3+l}$.

For A_1 (l=6.07,m=7.02,u=8.93) **D_f=2.96**

For A_2 (l=8.46,m=9.07,u=9.73) **D_f=1.32**

For A_3 (l=6.73, m=8.15, u=9.40) **D_f=2.82**

STEP 6

The fuzzy number A_2 with minimum deviation **D_f=1.32** comes first in ranking order [ascending]. The final ranking order is A_2,A_3,A_1.

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