

# A Neural Network based Individual Control Chart

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**Abstract**—Control chart techniques have been used in industries to monitor a process in quality improvement. In many manufacturing industries Shewhart X chart and Tukey's charts are used to monitor single observation data. The uses of Artificial Neural Network (ANN) model have recently been recommended as Statistical Quality control (SQC) tool. In this paper, neural network scheme is developed to for monitoring process mean. The performance of X chart, Tukey's chart and ANN model is evaluated by Average Run Length (ARL) using the simulation under normal and non-normal distributions. The ARL comparison between traditional X chart, Tukey's chart and ANN model shows that ANN is effective.

**Keywords**— Average run length, Inter-quartile range, Control chart, Artificial Neural Networks.

## I. INTRODUCTION

Now days, it had been seen that there is a technical modernization in manufacturing industry to achieve high quality of production and to fulfill the customer requirement. This technical modernization includes the use of information and communication technology, computerized machineries, robotic inspection, automation etc, which make the manufacturing process more advanced, but complicated (Masood and Hassan 2010). Hence it is necessary to develop the some advanced tools monitor and to control the overall this modern manufacturing process. The Statistical Process Control (SPC) has been used to maintain the superior quality of production with lowest production cost by minimizing the defective product production. In manufacturing process control charts are key point of SPC used to monitor the quality of production. The Precision of control chart results is completely based on its assumption of independence of random variable and symmetry in its probability distribution (Montgomery 2009). It has observed that assumption of these traditional control charts are violated in modern manufacturing process, this makes the traditional control charts less sensitive to process variation and affects on efficiency of control chart results (Junsub, Victor, Praybutok and Howart 2001). Hence it is time to develop some alternative methods to traditional control chats, which is not based on any theoretical assumptions and can have ability to work under the non-normal condition also.

Recently the concepts of Artificial Neural Network (ANN) have been attempted as competitive alternative to existing SPC techniques. Smith (1993) proposed a single ANN for Shewhart X-bar and R control charts for large shifts in mean or variance detection simultaneously. Junsub, Victor, Praybutok and Howart (2001) made the ARL comparison between the neural network and X-bar control chart under

non-normal distribution and showed that the neural network can be a better alternative for X-bar to detect the sudden change in process mean. Barghash (2011) compared the diverse neural network, X-bar and CUSUM control charts in terms of ARL for small mean shift detection. Alhammadi and Adams (2013) developed the ANN for monitoring the non-conformities in a passion process; they also compared the performance of ANN under the varying numbers of nodes in hidden layers. The ANN can be applied in industry both to production process modeling and to production control. ANNs are used instead of or along with SPC techniques. These are potential tools for recognizing shifts in process parameters as data independency is not an assumption in ANN theory. One of the most important approaches to this is compare statistical control charts to ANN.

This paper aims to compare the efficiency of traditional Shewhart individual X chart, Tukey's control chart with ANN to detecting process mean shift in terms of ARL under the normal and non-normal distributions. To study the consequence of skewness and kurtosis changes in probability distribution on the process monitoring of control charts, gamma distribution is used as skewed distribution where as t distribution as a symmetric distribution.

## II. INDIVIDUAL CONTROL CHARTS

For individual control charts the individual observations ( $n = 1$ ) are plotted one by one at a time. This type of control chart has been found useful more particularly in process control when only one observation is obtained per lot or batch of material. This situation often arises when:

- There is no basis for rational sub-grouping.
- Data comes available relatively slowly, and it is inconvenient to allow sample sizes of  $n > 1$  to accumulate before analysis.
- Repeat measurements on the process differ only because of laboratory or analysis error, as in many chemical processes.

Shewhart individual control chart and Tukey's control chart both are well known individual control chart used to monitoring the mean of single observation value.

### A. Shewhart Individual X Chart

There are many situations in which the sample size used for process monitoring consists of an individual measurement. In such situations, the Shewhart Individual X control chart is useful. Shewhart Individual X control chart is used when we collect samples one at a time and there is no logical grouping of data. Let  $X_1, X_2, \dots, X_n$  be a random sample of size  $n$  from

normal distribution with mean  $\mu$  and variance  $\sigma^2$ . Without loss of generality we assume  $\mu = 0$  and  $\sigma^2 = 1$ . The  $3\sigma$  control limits of individual control chart are given as follow.

$$\begin{aligned} \text{UCL} &= +3 \\ \text{LCL} &= -3 \end{aligned} \quad (1)$$

The charts operates by plotting  $X$  values on the chart with above control limits. If the plotted points lay outside the limits the process is considered to be out-of-control.

### B. Tukey's Control Chart

The Tukey's control chart is an individual control chart like Shewhart individual control chart, in which box plot technique is applied to calculate the control limits. Let  $X_1, X_2, \dots, X_n$  be a random sample of size  $n$  from normal distribution with mean  $\mu$  and variance  $\sigma^2$ . Without loss of generality we assume  $\mu = 0$  and  $\sigma^2 = 1$ . The  $3\sigma$  control limits of individual control chart are given as follow.

$$\begin{aligned} \text{UCL} &= Q_3 + k * \text{IQR} \\ \text{LCL} &= Q_1 - k * \text{IQR} \end{aligned} \quad (1)$$

The charts operates by plotting  $X$  values on the chart with above control limits. Where IQR is stands for Inter Quartile Range is given by  $\text{IQR} = Q_3 - Q_1$ . For the standard Tukey's control chart parameter  $k$  is usually set as 1.5. Under a normal distribution assumption, the IQR will cover 50% of the population and the interval between  $Q_3 + k * \text{IQR}$  and  $Q_1 - k * \text{IQR}$  will cover 99.3% of the population.

### III. ARTIFICIAL NEURAL NETWORK

The word neural network is referred to a network of biological neurons in nervous system that process and transmit the information. ANN is an artificial representation of human brain that tries to simulate its learning process. More specifically, ANN is mathematical model that tries to simulate the structure and/or functional aspects of biological neural network or system. Generally ANNs consist of three types of layers: input layers, output layers and hidden layers. Input layers receive inputs from sources external to the system under study while the output layers send signals out of the system. Hidden layers are those whose inputs and outputs are within the system and are necessary for the network to learn interdependencies in the model. The layers in a neural network are connected by links and each link has a numeric weight associated with it. ANN has great learning ability by using set of input values with respective known output values. This learning process in which weights are adjusted with respect to achieve desired output values is called training of ANN. In this study, we used the feed forward pattern recognition network because it had been proven that a feed forward neural network can learn any input-output relationship given enough neurons in the hidden layer and to be an effective system for patterns classification problems.

To construct ANN Control Chart (ANNCC), we implemented feed forward pattern recognition neural network consist of one neurons in input layer, ten neurons in hidden layer and one neurons in output layer with scaled conjugate gradient backpropagation as network training function. ANN trained for classify manufacturing process as in-control state and out-of-control state. In this study, to train the ANN 10,000 simulated samples of in-control and out-of-control data were used. The simulated data was organized as input matrix and

target matrix in such a way that the input matrix consist of in-control and out-of-control data together, while the target matrix had 0 value for all corresponding in-control observations in input matrix and 1 value for all corresponding out-of-control observations in input matrix. The input and target matrix defined above was taken as corresponding input and target values for feed forward pattern recognition neural network. To train ANN, we divided whole data sets into training set, validation set and test sets. The training set is used to coach the network for desire classification. Training continues as long as the network continues improving on the validation set. The performance of network was measured in terms of mean squared error, which was rapidly decreased as the network was trained. The trained neural network was tested with the testing samples. Which give us a sense of how well the network will perform when applied to real data. One measure of how well the neural network has fit the data is the confusion matrix. The confusion matrix shows the percentages of correct and incorrect classifications of proposed network. For trained network the misclassifications percentages should be very small. If this is not the case then further training, or training a network with more hidden neurons, would be advisable. In this study the network with confusion matrix the percentages incorrect classifications less than or equal to 5% was accepted as trained network. Once the network is trained sufficiently, the appropriate quantile was taken to as cutoff value; again the same trained ANN was implemented to the out-of-control data with shift in desirable mean size  $\delta_\mu = 0.5, 1, 1.5, 2, 3, 4$ .

In order to determine the UCL of ANN outputs, we first generated pure in-control state data. Then applied the trained neural network to this data, network generate the output within interval  $[0, 1]$ . As per the training, ANN gives the output values close to 0 for in-control state, where output values close to 1 for out-of-control state. A suitable quantile values for the in-control ANN output is used to get an UCL value, which is taken as cutoff point to decide the in-control and out-of-control state of the process (Alhammadi and Adams (2013)). An intensive search was conducted to find the appropriate quantile value which gives nominal ARL of 370. That is value above the given quantile represents out-of-control point and below which represents in-control points. In this work we observed that, the nominal ARL was detected for greater than 95% quantile values for various normal and non-normal distributions.

In SPC, ARL is mostly used to monitoring the capability of control chart. It is defined as the average number of points that must be plotted before a point indicates an out-of-control condition (Montgomery 2009). For given process the ARL was based on 10,000 simulated replications and calculated as,

$$\text{ARL} = \frac{\text{Total points}}{\text{Total number of out of control points}}$$

For the purpose of comparison of ANN with other Control charts, data was first generated from in-control distribution. To find the nominal 370 in-control ARL for ANN the appropriate UCL value must be detected by extensive searching method. The exhaustive quantile range  $[95, 99.8]$  was used as UCL of ANNCC for various distributions used in this work so that in-control ARL must be 370 approximately.

IV. DATA GENERATION AND RESULTS

In order to compare the performance of ANN scheme with traditional individual X charts, data was simulated from various symmetric distributions and the asymmetric distributions. In this study to evaluate the process monitoring capability in terms of ARL, control chart are tested under distributions like standard normal distribution, gamma distribution and t distribution with various parameters.

If the process quality characteristic x has a gamma distribution with parameters a and b, then its mean and variance will be ab and ab<sup>2</sup> respectively. The corresponding probability density function is given as follow

$$f(x) = \frac{x^{a-1} e^{-\frac{x}{b}}}{b^a \Gamma(a)} \quad x > 0, a, b > 0 \quad (3.8)$$

Similarly, if the process quality characteristic x has t distribution with parameter v, mean and variance will be 0 and  $\frac{v}{v-2}$  respectively. The corresponding probability density function is given as follow

$$f(x) = \frac{\Gamma[(v+1)/2]}{\Gamma(v/2)} \frac{1}{\sqrt{v\pi}} \left(1 + \frac{x^2}{v}\right)^{-\left(\frac{v+1}{2}\right)}, -\infty < x < +\infty \quad (3.9)$$

It is observed that in case of gamma distribution as the value of parameter a increases the gamma distribution tends to a normal distribution, while in case of t distribution as parameter v increases the kurtosis of t distribution will get closer to the normal distribution (Torng and Lee (2008)). Therefore, in order to compare the effect of skewness change and kurtosis change on the performances of control charts, we kept parameter b = 1 fixed and choose parameter values as a = 4, 2, 1 for gamma distribution and for t distribution value of parameter v is selected as v = 30, 10, 4.

For the purpose of performance comparison of proposed control chart, in-control ARL values for underlying process distributions are adjusted to 370 by selecting an appropriate parameter k with corresponding  $\alpha = 0.0027$ . Then we can compare their performance at various shifts of size. All data in this present study was simulated by MATLAB software.

TABLE I. ARL PERFORMANCE OF TUKEY'S CHART, X CHART AND ANN SCHEME

Distribution	Control Chart	$\delta_\mu$						
		0.0	0.5	1.0	1.5	2.0	3.0	4.0
N(0,1)	Tukey's chart (k=1.7227)	370.37	155.19	47.62	15.24	6.23	1.98	1.19
	X chart (k=3.00)	370.37	149.25	44.44	15.38	6.58	2.01	1.19
	ANN chart	370.37	90.09	25.77	9.93	4.53	1.68	1.12
G(4,1)	Tukey's chart (k=2.645)	370.37	212.77	90.91	42.55	21.01	5.97	2.06
	X chart (k=3.886)	370.37	212.77	90.91	42.37	20.92	5.96	2.06
	ANN chart	370.37	212.77	90.09	42.37	20.92	5.96	2.06
G(2,1)	Tukey's chart (k=3.192)	370.37	238.10	117.65	61.35	32.68	10.44	3.41
	X chart (k=4.251)	370.37	238.10	116.28	61.35	32.57	10.42	3.40
	ANN chart	370.37	238.10	119.05	62.11	33.22	10.64	3.45
G(1,1)	Tukey's chart (k=3.95)	370.37	208.33	116.28	70.42	40.98	14.75	5.12
	X chart (k=4.6)	370.37	208.33	116.28	70.92	40.98	14.77	5.14
	ANN chart	370.37	208.33	116.28	70.42	40.98	14.75	5.12
t(30)	Tukey's chart (k=1.93)	370.37	181.82	58.82	20.04	8.21	2.22	1.23
	X chart (k=3.079)	370.37	181.90	58.85	20.09	8.27	2.29	1.23
	ANN chart	370.37	95.24	33.11	12.41	5.42	1.80	1.16
t(10)	Tukey's chart (k=2.327)	370.37	243.90	117.65	45.66	17.39	3.50	1.43
	X chart (k=3.52)	370.37	243.90	119.05	46.73	17.73	3.53	1.43
	ANN chart	370.37	161.29	68.03	25.51	9.85	2.41	1.24
t(4)	Tukey's chart (k=4.15)	370.37	322.58	232.56	153.85	105.26	31.25	5.88
	X chart (k=4.755)	370.37	333.33	232.56	156.25	106.38	31.85	5.02
	ANN chart	370.37	178.57	128.21	68.97	32.47	5.53	1.73

From Table 1 it can be seen that for normal distribution the proposed ANN chart has very small ARL values for various mean shifts. That is the proposed ANN chart is more sensitive for detecting the shift in process mean as compared to the traditional individual X chart and Tukey's chart. It also can be seen that in case of skewed distributions the ANN chart is performing approximately the same as traditional individual X chart and Tukey's chart. For skewed distribution ANN chart has the same ARL values for various parameter values of Gamma distribution. For t distribution with parameter value v=30, 10, 4, we can infer that for small mean shift of size  $\delta_\mu = 0.5, 1, 1.5$  and 2 the ANN chart exhibits more superiority over the individual X chart and Tukey's chart. Comparing the ARL values presented in Table 1 it is clear

that performance of ANN chart has vast superiority over the traditional control charts in case of normal distribution and t distribution, while in case of gamma distribution the ANN chart is performing equally.

V. CONCLUSION

Overall it was seen that ANN charts can be used as the best alternative for monitoring single measurement. The advantages of ANN chart over traditional control charts is that, ANN chart is free from statistical assumptions. In this study, it has been found that in case of normal distribution the proposed ANN chart is very sensitive for mean shift detecting and has the high process monitoring capability. It can be inferred that the proposed ANN chart has the same process monitoring capability as that of individual X

chart and Tukey's chart for skewed distributions, which is ANN charts has good ability to detect the out-of-control point even there is a skewness change. We can also conclude that proposed control chart is not affected by change in kurtosis and performing superior to detect the process mean shift than any other situations. The ANN charts can be utilized as the good alternative process control tool in manufacturing industries wherein the assumption on normality get violated.

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