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A New Hypothesis Of Projective Tensor Product And It's Consequences

Mr. Ajay Kumar[1], Prof. Seema Sharma[2] Dr. Sushil Kumar Jamariar[3]
Department of Mathematics
1. Dr. C.V. RAMAN UNIVERSITY, Bhagwanpur, Vaishali , Bihar,
2. Dr. C.V. RAMAN UNIVERSITY, Khandwa, (M.P)
3. Dr. C.V. RAMAN UNIVERSITY, Bhagwanpur, Vaishali , Bihar,

ABSTRACT

This Paper Presents the Study of the Projective Tensor Product and its consequences by defining the projective topology π on Locally convex spaces E and F; U & V be the closed absolutely convex neighborhoods of Q in E and F respectively , forming the set τ(U⊗V) = absolutely convex hull of U⊗V in E⊗F, it is proved in this paper that the projective topology π is the finest locally convex topology on E⊗F for which the Canonical mapping Ψ : E X F → E⊗F is continuous. Tensor products are used to describe systems consisting of multiple subsystems. Each systems are described by a vector in a Hilbert Space.

KEYWORDS : Projective Topology, Locally convex spaces, Convex hull, Canonical mapping, Automorphism, Canonical Isomorphism.

INTRODUCTION:

The strongest locally convex topological vector space (TVS) topology on X⊗Y the tensor product of two locally Convex TVSs making the Cononical map $\otimes : X \times Y \rightarrow X \otimes Y$ defined by sending (x,y) ϵ $X \times Y$ to X⊗Y Continous is called projective topology or π Topology . When X⊗Y is equipiped with this topology then it is denoted by $X\otimes_{\pi}Y$ and called the projective tensor product of X and Y. Halub (1) and Kothe (2,3) are the pioneer worker of the present area. In fact the present work is the extension of work done by Tomiyama (6), Studied analytically about projective Tensor Product.

Here, we use the following definitions and fundamental ideas:

Definition - I

If V and W be vectors space of finite dimension then $V \otimes W$ is fnite diminsional and its dimension is product of the dimensions of V and W. This result from the fact that $V \otimes W$ is formed by taking all Tansor Products of basis element of V and basis element of W.

Definition – II

Let E and F be locally convex spaces , and let U & V be the closed absolutely convex neighborhood's of O in E and F respectively, forming the set τ (U⊗V) = absolutely convex, hull of U⊗V in E⊗F , (E⊗F is denoted as tensorial product of E & F).

Definition – II

If {U} and {V} are neighborhood bases in E and F respectively with U,V closed absolutely then the family { τ (U⊗V)} is a neighbourhood basis of a locally convex topology on E⊗F

This topology is called the projective topology on E⊗F and is denoted as E.

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Properties

1. Associativity as a Vector Space operation
   $(U\otimes V)\otimes W \cong U\otimes(V\otimes W)$

Where U,V,W be canonical isomorphism that maps

$(U\otimes V)\otimes W$ to $U\otimes(V\otimes W)$

This follows omitting parenthesis in the tensor product of more than two vector space or vectors.

2. Commutativity as a vector space operation
   $V\otimes W = W\otimes V$

Where V & W is commutative in the sense that is a canonical isomorphism.

That maps $V\otimes W = W\otimes V$

In fact V= W the tensor product of vectors is not commutative that is $V\otimes W \neq W\otimes V$

In general $X\otimes Y \rightarrow Y\otimes X$ from $V\otimes V$ to itself induces a linear auto morphism that is called braiding map.

Proposition 1 : Let p(x) and q(y) be the semi-norms defined by U and V respectively. The set $\tau\,(U\otimes V)$ is absorbing and thus defines a semi-norm . The semi-norm of $\tau\,(U\otimes V)$ is given by

$$p\otimes q\,(-Z) = \text{in } f_1 \textstyle\sum . p(x_i) q(y_i) \qquad \text{-------------} \quad (1)$$

Where the infimum is taken over all representations

$$-Z = \textstyle\sum . x_i \otimes y_i \quad \text{in } E\otimes F.$$

Proof : First we show $\tau\,(U\otimes V)$ is absorbing . Let

$-Z = \sum . x_i \otimes y_i$ be an element of $E\otimes F$. we observe that

U if $p(X_i) \neq 0$

And $\frac{yi}{q(yi)}$ V if $q(Y_i) \neq 0$

Also $p(X_i) = 0$ iff $p_i x_i$ U all $p > 0$ and

$q(Y_i) = 0$ iff $q_i y_i$ V all $p > 0$ So we write

$$-Z = \textstyle\sum . x_i \otimes y_i$$

$$= p(X_i)\, q(Y_i) \left[\frac{X_i}{P(X_i)} \otimes \frac{Y_i}{q(Y_i)}\right]$$

$$+ \delta q \textstyle\sum .(Y_k) \left[\frac{x_k}{\delta} \otimes \frac{Y_k}{q(Y_k)}\right]$$

$$+ \delta p \textstyle\sum .(X_j) \left[\frac{X_j}{P(X_j)} \otimes \frac{Y_j}{\delta}\right]$$

$$+ \delta^2{}_m \textstyle\sum . \left[\frac{x_m}{\delta} + \frac{Y_m}{\delta}\right]$$

In each of the four terms in the sum representing $-Z_i$ the quantity in the brackets [ ] is in $\tau(U\otimes V)$. Given $\epsilon > 0$, we may choose sufficiently small so that

(*) : $-Z \;\epsilon\; (\sum . p(x_i) q(y_i) + \epsilon\,)\, \tau(U\otimes V)$

So $\tau(U\otimes V)$ is absorbing.

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Now τ(U⊗V) is absorbing convex also , so it defines semi- norm τ(-Z) on E⊗Few now show τ(-Z) = p ⊗ q(-Z)

(i) τ(-Z) ⊆ p ⊗ (-Z) , τ(-Z) is defined by
τ(-Z) = inf λ, -z ∈ λ τ(U⊗V) , by (*) above
τ(-Z) ≤ inf $\sum$.p(x_i)q(y_i) + ϵ = p ⊗ q(-Z) + ϵ , ϵ arbitrary yields τ(-Z) ≤ p ⊗ q(-Z)

(ii) p ⊗ q(-Z) ≤ τ(-Z) suppose -Z ∈ λ τ(U⊗V), Then
-Z = $\sum . \propto_k (X_k^i \otimes \ Y_k^i)$ with p (X_k^i) ≤ 1 , q (Y_k^i) ≤ 1, $\sum$.I α_k I≤ λ and $\propto_k \geq 0$. For this particular representation of –Z , we see

$\sum$.p($\alpha_k$$x_k$) q($y_k$) = $\sum$.I α_k I≤ λ So.
p⊗ q(-Z) =inf $\sum$.p(x_i)q(y_i) ≤ λ .

This is true for every λ with –Z ∈ $\lambda\tau$(U⊗V).
Thus p⊗ q(-Z) ≤ inf λ, -z∈ λ τ(U⊗V)
= τ(-Z)
This completes the proof

Proposition 2 : The projective tensor product E⊗F of two normed space E, p and F. q is a normed space with norm p⊗q

If E and F are metrizable locally convex spaces with semi norms $p_1 \leq p_2 \leq$ ……… and $q_1 \leq q_2$ ....…. respectively, then E ⊗Fis metrizable with defining semi- norms $p_1 \otimes q_1 \leq p_2 \otimes q_2 \leq$ ………

Proof: Follows immediately from Proposition 1 and definition 2.

MAIN RESULT

Theorem: The projective topology π is the finest locally convex topology on E⊗F for which the canonical map Ψ : E X F → E⊗F is continuous.

Proof: Ψ is continuous with respect to π since Ψ : U X V → U⊗V ⊆ τ(U⊗V)Now let τ be any topology on E⊗F for which Ψ is continuous and let W be an absolutely convex closed τ neighborhood of 0. Then there exist U, V with Ψ (U x V) - U⊗ V ⊆ W .Since W is absolutely convex, τ(U⊗V) ⊆W. So π is finer than τ

This completes the proof of the hypothesis.

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