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A New Limit to the Core Mass in Stars with $M \geq 2M_{\Theta}$

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***Abstract*--According to the studies of (Schönberg & Chandrasekhar 1942; Henrich & Chandrasekhar 1941)[6,4], it exists an upper limit to the mass of the isothermal core for the stars situated on the post main sequence MS on the HR diagram with a mass $M \geq 2M_{\Theta}$. In the present work, and using another approach that I find more rigorous than the calculus done in the other works, I demonstrate the existence of an other value to this upper limit and I establish in function of this upper limit M_{iso} the formulae of the luminosity produced by these stars.**

I. INTRODUCTION

For stars of masses greater than $2M_{\Theta}$ and classified within the post main sequence on the HR diagram, the interior region or the core is under the rule of the gravitational contractions in the phase of hydrogen rarefaction. Because of the lack of the hydrogen, the luminosity produced by the core is null (L =0) and therfore the core is isothermal. In this phase, the gravitational energy generated from these contractions in the core heats the upper layers, and this increasing in temperature in these layers allows the nuclear reactions to take place in the so called circum-nuclear shell situated above the core. This shell feeds the core with the nuclear reaction products and contribuate in increasing the mass of this core. Henrich & Chandrasekhar (1941) [4]and Schönberg & Chandrasekhar (1942) [6] calculated the greatest mass M_{iso} supported by this isothermal core. In the present work, I find an other upper limit to this mass and in function of this mass I establish the formulae of the luminosity produced by these stars in the frame of the following approach:

II. CALCULUS OF THE MASS M_{iso} WHICH CORRESPONDS TO THE MAXIMUM PRESSURE IN THE CORE

In this calculus, the core is assumed to be a sphere of gas with a quasi constant density. Using the equations of the hydrostatic equilibrium and the equation of the mass in a star, one can find the equation relating

between the quantities $dP/dM(r), M(r)$ and r,

where P is the pressure, $M(r)$ is the mass of the star and r is the radial coordinate.

The hydrostatic equilibrium of a spherical star of a quasi constant density ρ and a mass $M(r)$ is expressed as in [5],

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2} \qquad (1)$$

where $M(r) = \frac{4}{3}\pi r^3 \rho$

and,

$G = 6.67 \times 10^{-8} erg\ cm\ g^{-2}$ is the gravitational constant.

However,

$$4\pi r^2 \rho dr = dM \qquad (2)$$

Combining the relations (1) and (2), one finds:

$$4\pi\ r^3 \frac{dP}{dM(r)} = -\frac{GM(r)}{r} \quad (3)$$

One can rewrite $4\pi\ r^3\, dP/dM(r)$ in the following form,

$$4\pi\ r^3 \frac{dP}{dM(r)} = \frac{d(4\pi\ r^3 P)}{dM(r)} - \frac{3P}{\rho} \quad (4)$$

If one inserts this last relation in the relation (3) and integrates over the whole isothermal core of mass M_{iso}, one obtains,

$$\int_0^{M\ iso} \frac{d\left(4\pi\ r^3 P\right)}{dM(r)}\, dM(r) - \int_0^{M\ iso} \frac{3P}{\rho}\, dM(r)$$

$$= -\int_0^{M\ iso} \frac{GM(r)}{r}\, dM(r) \quad (5)$$

The right term of the relation (5) represents the gravitational energy of the core, it's equal to $-3GM_{iso}^2/5R_{iso}$ if $M(r)$=0 when r=0. Using the following state equation relative to a perfect isothermal gas,

$P/\rho = kT/\mu m_H$,where T is the temperature,

T =*Tiso*=cte, μ is the molecular weight, m_H is the atomic mass of the hydrogen, and k is the Boltzman constant, the relation (5) becomes then,

$$4\pi R_{iso}^3 P_{iso} - \frac{3M_{iso}}{\mu_{iso} m_H} kT_{iso} = -\frac{3GM_{iso}^2}{5R_{iso}} \quad (6)$$

where R_{iso}, P_{iso}, M_{iso}, μ_{iso} and T_{iso} are respectively the radius, the pressure, the mass, the molecular weight and the temperature of the isothermal core. Then, the pressure of this core is given by,

$$P_{iso} = \frac{3}{4\pi R_{iso}^3}\left(\frac{M_{iso} kT_{iso}}{\mu_{iso} m_H} - \frac{1}{5}\frac{GM_{iso}^2}{R_{iso}}\right) \quad (7)$$

In order to find the maximum value of the pressure in the core, one proceeds by the variation of the mass M_{iso}. Since the density of the core ρ_{iso} is considered quasi-constant, the radius of the core R_{iso} varies when M_{iso} varies. R_{iso} and M_{iso} are so that,

$$R_{iso} = \left(\frac{3M_{iso}}{4\pi\rho_{iso}}\right)^{1/3} \quad (8)$$

and $M_{iso}\left(R_{iso}\right) = \frac{4}{3}\pi R_{iso}^3 \rho_{iso}$

For each mass M_{iso} correspond R_{iso}, T_{iso}, ρ_{iso} and μ_{iso} which are being now functions of M_{iso}. P_{iso} can be rewritten as,

$$P_{iso} = \frac{kT_{iso}\rho_{iso}}{\mu_{iso} m_H} - \frac{G}{5}\left(\frac{4\pi}{3}\right)^{1/3} \rho_{iso}^{4/3} M_{iso}^{2/3} \quad (9)$$

Hence the derivative of P_{iso} with respect to M_{iso} is given by,

$$\frac{dP_{iso}}{dM_{iso}} = \left[\frac{kT_{iso}}{\mu_{iso} m_H} - \frac{4G}{15}\left(\frac{4\pi}{3}\right)^{1/3} \rho_{iso}^{1/3} M_{iso}^{2/3}\right]\frac{d\rho_{iso}}{dM_{iso}}$$

$$+ \frac{k\rho_{iso}}{\mu_{iso} m_H}\frac{dT_{iso}}{dM_{iso}} - \frac{kT_{iso}\rho_{iso}}{m_H \mu_{iso}^2}\frac{d\mu_{iso}}{dM_{iso}} -$$

$$-\frac{2G}{15}\left(\frac{4\pi}{3}\right)^{1/3} \rho_{iso}^{4/3} M^{-1/3} \quad (10)$$

Assuming that μ_{iso} is quasi-constant in the core and doesn't vary appreciably with the small variation of the mass M_{iso}, one can neglect the derivative $d\mu_{iso}/dM_{iso}$ and the relation (10) becomes,

$$\frac{dP_{iso}}{dM_{iso}} = \left[\frac{kT_{iso}}{\mu_{iso} m_H} - \frac{4G}{15}\left(\frac{4\pi}{3}\right)^{1/3} \rho_{iso}^{1/3} M_{iso}^{2/3}\right]\frac{d\rho_{iso}}{dM_{iso}}$$

$$+ \frac{k\rho_{iso}}{\mu_{iso} m_H}\frac{dT_{iso}}{dM_{iso}} - \frac{2G}{15}\left(\frac{4\pi}{3}\right)^{1/3} \rho_{iso}^{4/3} \frac{1}{M_{iso}^{1/3}}$$

(11)

To evaluate dP_{iso}/dM_{iso}, we need to calculate the derivatives $d\rho_{iso}/dM_{iso}$ and dT_{iso}/dM_{iso} appearing in the relation (11).

A. Calculus of the derivative $d\rho_{iso}/dM_{iso}$

$d\rho_{iso}/dM_{iso}$ can be rewritten under the following form,

$$\frac{d\rho_{iso}}{dM_{iso}} = \frac{d\rho_{iso}}{dR_{iso}}\frac{dR_{iso}}{dM_{iso}} \quad (12)$$

we set $d\rho_{iso} = \rho'_{iso} - \rho_{iso}$ where ρ'_{iso} is the density of the core after the variation of the mass M_{iso} from M_{iso} to $M_{iso} + dM_{iso}$, this leads to a radius variation from R_{iso} to $R_{iso} + dR_{iso}$ (the density ρ'_{iso} is still considered quasi-constant in the core which is supposed to conserve its spherical form and has the new mass $M_{iso} + dM_{iso}$). Therefore ρ'_{iso} is given by,

$$\rho'_{iso} = \frac{M_{iso} + dM_{iso}}{\frac{4}{3}\pi\left(R_{iso} + dR_{iso}\right)^3} \quad (13)$$

Since $R_{iso} \rangle\rangle dR_{iso}$, one can do the following approximation,

$$\rho'_{iso} \approx \frac{M_{iso} + dM_{iso}}{\frac{4}{3}\pi R_{iso}^3} \quad (14)$$

$$\rho'_{iso} \approx \frac{M_{iso}}{\frac{4}{3}\pi R_{iso}^3} + \frac{dM_{iso}}{\frac{4}{3}\pi R_{iso}^3} \quad (15)$$

and since,

$$dM_{iso} = 4\pi R_{iso}^2 \rho_{iso} dR_{iso} \quad (16)$$

the relation (15) becomes,

$$\rho'_{iso} \approx \rho_{iso} + \frac{4\pi R_{iso}^2 \rho_{iso} dR_{iso}}{\frac{4}{3}\pi R_{iso}^3}$$

Where, $\rho_{iso} = \dfrac{M_{iso}}{\frac{4}{3}\pi R_{iso}^3}$

then, $d\rho_{iso} = \rho'_{iso} - \rho_{iso} \cong \dfrac{3\rho_{iso}}{R_{iso}} dR_{iso}$

we obtain,

$$\frac{d\rho_{iso}}{dR_{iso}} \cong \frac{3\rho_{iso}}{R_{iso}} \quad (17)$$

The relation (12) becomes,

$$\frac{d\rho_{iso}}{dM_{iso}} \cong \frac{3\rho_{iso}}{R_{iso}}\frac{dR_{iso}}{dM_{iso}} \quad (18)$$

From the relation (16), the derivative dR_{iso}/dM_{iso} is equal to,

$$\frac{dR_{iso}}{dM_{iso}} = \frac{1}{4\pi R_{iso}^2 \rho_{iso}}$$

and the relation (18) becomes:

$$\frac{d\rho_{iso}}{dM_{iso}} \cong \frac{3\rho_{iso}}{R_{iso}}\frac{1}{4\pi R_{iso}^2 \rho_{iso}}$$

we obtain,

$$\frac{d\rho_{iso}}{dM_{iso}} \cong \frac{3}{4\pi R_{iso}^3} \quad (19)$$

as R_{iso} is given by the relation (8), $d\rho_{iso}/dM_{iso}$ is finally given by,

$$\frac{d\rho_{iso}}{dM_{iso}} \cong \frac{\rho_{iso}}{M_{iso}} \quad (20)$$

we insert this expression of $d\rho_{iso}/dM_{iso}$ in the relation (11) and obtain the following relation for the derivative of the pressure,

$$\frac{dP_{iso}}{dM_{iso}} = \left[\frac{kT_{iso}}{\mu_{iso} m_H} - \frac{4G}{15}\left(\frac{4\pi}{3}\right)^{1/3} \rho_{iso}^{1/3} M_{iso}^{2/3}\right]\frac{\rho_{iso}}{M_{iso}} + $$

$$+ \frac{k\rho_{iso}}{\mu_{iso} m_H}\frac{dT_{iso}}{dM_{iso}} - \frac{2G}{15}\left(\frac{4\pi}{3}\right)^{1/3} \rho_{iso}^{4/3} \frac{1}{M_{iso}^{1/3}}$$

(21)

B. Calculus of the derivative dT_{iso}/dM_{iso} :

The relation (3) can be rewritten as,

$$4\pi R_{iso}^3 \frac{dP_{iso}}{dT_{iso}}\frac{dT_{iso}}{dM_{iso}} = -\frac{GM_{iso}(r)}{R_{iso}} \qquad (22)$$

therefore,

$$\frac{dT_{iso}}{dM_{iso}} = -\frac{GM_{iso}}{4\pi R_{iso}^4}\frac{dT_{iso}}{dP_{iso}} \qquad (23)$$

From the state equation used above which

is given by $P = \rho kT/\mu m_H$, we can write,

$$dP_{iso} = \frac{\rho_{iso} k}{\mu_{iso} m_H} dT_{iso} \qquad (24)$$

so,

$$\frac{dT_{iso}}{dP_{iso}} = \frac{\mu_{iso} m_H}{\rho_{iso} k} \qquad (25)$$

After replacing the expression of

dT_{iso}/dP_{iso} in the relation (23), we find,

$$\frac{dT_{iso}}{dM_{iso}} = -\frac{GM_{iso}}{4\pi R_{iso}^4}\frac{\mu_{iso} m_H}{\rho_{iso} k} \qquad (26)$$

Since the radius R_{iso} depends on M_{iso} and it's given by $R_{iso} = (3M_{iso}/4\pi\rho_{iso})^{1/3}$, the relation (26) becomes,

$$\frac{dT_{iso}}{dM_{iso}} = -\frac{G}{4\pi}\left(\frac{4\pi}{3}\right)^{4/3}\frac{\mu_{iso} m_H}{k}\frac{\rho_{iso}^{1/3}}{M_{iso}^{1/3}} \qquad (27)$$

we replace the expression of dT_{iso}/dM_{iso} in the relation (21) and we obtain the following expression for the derivative dP_{iso}/dM_{iso} ,

$$\frac{dP_{iso}}{dM_{iso}} = \left[\frac{kT_{iso}}{\mu_{iso} m_H} - \frac{4G}{15}\left(\frac{4\pi}{3}\right)^{1/3} \rho_{iso}^{1/3} M_{iso}^{2/3}\right]\frac{\rho_{iso}}{M_{iso}}$$

$$-\frac{7G}{15}\left(\frac{4\pi}{3}\right)^{1/3} \rho_{iso}^{4/3} \frac{1}{M_{iso}^{1/3}}$$

(28)

The value of M_{iso} for which $dP_{iso}/dM_{iso} = 0$ is given by,

$$M_{iso} = \left[\frac{3}{4\pi}\left(\frac{15}{11}\right)^3\right]^{\frac{1}{2}}\left(\frac{1}{Gm_H}\right)^{3/2}\left(\frac{kT_{iso}}{\mu_{iso}\rho_{iso}^{1/3}}\right)^{3/2}$$

$$\cong 0.778\left(\frac{1}{Gm_H}\right)^{3/2}\left(\frac{kT_{iso}}{\mu_{iso}\rho_{iso}^{1/3}}\right)^{3/2} \qquad (29)$$

The maximum value of pressure corresponding to this mass is then given by,

$$P_{iso|\max} = \frac{8}{11}\left(\frac{kT_{iso}\rho_{iso}}{\mu_{iso} m_H}\right) =$$

$$= \frac{8}{15}\left(\frac{4\pi}{3}\right)^{\frac{1}{3}} G\ \rho^{\frac{4}{3}}\ M_{iso}^{2/3} \qquad (30)$$

C. Calculus of the pressure at the interface between the envelope and the core P_{env}^{iso}

Integrating the equation of the hydrostatic equilibrium over the envelope and by taking the pressure null on the star surface, one obtains,

$$P_{env}^{iso} = \int_{M_{iso}}^{M} \frac{GM}{4\pi\ r^4} dM(r) \qquad (31)$$

where P_{env}^{iso} is the pressure at the interface between the isothermal core and the envelope, M is the whole mass of the star. Approximately, one finds,

$$P_{env}^{iso} \cong \frac{G}{8\pi\langle r^4\rangle}\left(M^2 - M_{iso}^2\right) \qquad (32)$$

where,

$$\langle r^4\rangle \cong \frac{R^4}{2} \qquad (33)$$

Since $M\rangle\rangle M_{iso}$ the relation (32) becomes,

$$P_{env}^{iso} \cong \frac{G}{4\pi}\frac{M^2}{R^4} \qquad (34)$$

Using the state equation of a perfect gas,

$$T_{env}^{iso} = \frac{\mu_{env} m_H\, p_{env}^{iso}}{k\rho_{env}^{iso}} \qquad (35)$$

where T_{env}^{iso}, μ_{env} and ρ_{env}^{iso} are respectively the temperature, the molecular weight in the envelope (supposed to be constant in the envelope), and the density at the interface between the isothermal core and the envelope. The density ρ_{env}^{iso} is assumed to be approximately equal to,

$$\rho_{env}^{iso} \cong \frac{M}{\frac{4\pi}{3}R^3} \qquad (36)$$

where M and R are respectively the mass and the radius of the whole star.

From the relations (34), (35), and (36) one can deduce the following expression of the radius of the star R,

$$R \cong \frac{1}{3}\frac{GM}{T_{iso}}\frac{\mu_{env} m_H}{k} \qquad (37)$$

This expression of R is inserted into the relation (34) and one finds the expression of the pressure P_{env}^{iso},

$$P_{env}^{iso} \cong \frac{81}{4\pi}\frac{1}{G^3 M^2}\left(\frac{kT_{iso}}{\mu_{inv} m_H}\right)^4 \qquad (38)$$

Confronting now the two pressures P_{env}^{iso} and $P_{iso|\max}$ to find the relation between the mass of the whole star M and the mass of the isothermal core of the same star M_{iso} which corresponds to the maximum of pressure $P_{iso|\max}$ in the core, we find,

$$P_{iso|\max} = P_{env}^{iso}$$

$$\Rightarrow \frac{8}{11}\frac{kT_{iso}\rho_{iso}}{\mu_{iso} m_H} = \frac{81}{4\pi}\frac{1}{G^3 M^2}\left(\frac{kT_{iso}}{\mu_{env} m_H}\right)^4 \qquad (39)$$

From the relation (29) we deduce the expression of $kT_{iso}/Gm_H\mu_{iso}$,

$$\frac{kT_{iso}}{Gm_H\mu_{iso}} = \frac{M_{iso}^{2/3}\rho_{iso}^{1/3}}{\frac{15}{11}\left(\frac{3}{4\pi}\right)^{\frac{1}{3}}}$$

Then from the relation (39), we obtain,

$$\frac{M_{iso}}{M} = \sqrt{\frac{24}{1215}\left(\frac{15}{11}\right)^2\left(\frac{\mu_{env}}{\mu_{iso}}\right)^2}$$

$$\cong 0.261\left(\frac{\mu_{env}}{\mu_{iso}}\right)^2 \qquad (40)$$

Therefore the allowed values of M_{iso} are given by,

$$M_{iso} \le 0.261\left(\frac{\mu_{env}}{\mu_{iso}}\right)^2 M \qquad (41)$$

which verify the condition $P_{iso|\max} \ge P_{env}^{iso}$.

This is the result obtained in this present work. The calculus of this upper mass by L. R. Henrich and S. Chandrasekhar [2,3,4] and Schönberg & Chandrasekhar [6] gives the following result,

$M_{iso} \leq 0.35(\mu_{env}/\mu_{iso})^2 M$ which is given by $M_{iso} \leq 0.35M$ for $\mu_{env}/\mu_{iso} = 1$.

It appears that the two approaches to estimate the value of M_{iso} don't diverge and although they give different values the results aren't very far from each other.

D. The calculus of the luminosity produced by the envelope

Since the luminosity produced in the isothermal core is null $(L(r,\ 0 \leq r \leq R_{iso}) = 0)$, we can assume the luminosity produced by such stars to be equal to,

$$L = \int_{R\ iso}^{R} 4\pi\ r^2 \rho_{env}^{iso}\ \varepsilon(r) dr \tag{42}$$

where $\varepsilon(r)$ is the energy produced by a mass unit in the envelope, ρ_{env}^{iso} is the density of the envelope assumed to be quasi constant as mentioned above,

$$\rho_{env}^{iso} \approx cte \tag{43}$$

R and R_{iso} are respectively the radius of the star and the radius of the isothermal core.

If $\varepsilon(r)$ can be supposed constant and approximated such that in [7], page 89,

$$\varepsilon(r) \approx \varepsilon \cong \frac{1}{4\pi R^2 \rho_{env}^{iso}} \left(\frac{dL}{dr}\right)_{r=R} \tag{44}$$

Replacing the expression of $\varepsilon(r)$ into (42), the luminosity of such stars is then given by,

$$L \cong \frac{1}{R^2}\left(\frac{R^3}{3} - \frac{R_{iso}^3}{3}\right)\left(\frac{dL}{dr}\right)_{r=R} \tag{45}$$

R_{iso} is related to the mass of the isothermal core M_{iso} by $R_{iso} = (3M_{iso}/4\pi\rho)^{1/3}$.

which can be approximated to be equal to,

$R_{iso} = (3M_{iso}/4\pi\bar{\rho})^{1/3}$ where $\bar{\rho}$ is the mean density of the star, it's given by $\bar{\rho} = M \Big/ \frac{4}{3}\pi R^3$

Then,

$$L \cong \left(1 - \frac{M_{iso}}{M}\right)\frac{R}{3}\left(\frac{dL}{dr}\right)_{r=R} \tag{46}$$

If the energy transport in the envelope is radiative, the luminosity $L(r)$ is so that in [7], page 89,

$$L(r) = -\frac{4acT^3}{3\kappa\rho} 4\pi\ r^2 \left(\frac{dT}{dr}\right)_{rad} \tag{47}$$

where a is the radiative constant, c is the light velocity, κ is the opacity, and $(dT/dr)_{rad}$ is the radiative temperature gradient. From the relation (47), we can deduce the expression of $(dL/dr)_{r=R}$ and replace its expression in the relation (46). Then,

$$\frac{dL(r)}{dr} = -\frac{4ac}{3\kappa\rho} 8\pi\ rT^3 \left(\frac{dT}{dr}\right)_{rad} - \frac{4ac}{3\kappa\rho} 4\pi\ r^2 \left[3T^2\left(\frac{dT}{dr}\right)_{rad} + T^3\left(\frac{d^2T}{dr^2}\right)_{rad}\right] \tag{48}$$

we can rewrite $(dT/dr)_{rad}$ in the form,

$$\left(\frac{dT}{dr}\right)_{rad} = \left(\frac{dT}{dM}\right)\left(\frac{dM}{dr}\right)$$

and $(d^2T/dr^2)_{rad}$ in the form,

$$\left(\frac{d^2T}{dr^2}\right)_{rad} = \left(\frac{d^2T}{drdM}\right)\left(\frac{dM}{dr}\right) + \left(\frac{dT}{dM}\right)\left(\frac{d^2M}{dr^2}\right)$$

From the relations (2) and (27), we find,

$$\left(\frac{dT}{dr}\right)_{rad} = -G\left(\frac{4\pi}{3}\right)^{4/3} \frac{\mu m_H}{k} \frac{\rho^{4/3}}{M^{1/3}} r^2 \quad (49)$$

and,

$$\left(\frac{d^2T}{dr^2}\right)_{rad} = -\frac{G}{4\pi}\left(\frac{4\pi}{3}\right)^{4/3} \frac{\mu\, m_H}{k} \rho^{1/3} \times \left[\frac{8\pi\; r\rho}{M^{1/3}} - \frac{\left(4\pi\; r^2\rho\right)^2}{3}\frac{1}{M^{4/3}}\right] \quad (50)$$

Replacing these two last relations into (48), we find,

$$\frac{dL(r)}{dr} = \frac{4ac}{3\kappa\rho}\frac{G}{4\pi}\left(\frac{4\pi}{3}\right)^{4/3}\left(\frac{\mu m_H}{k}\right)\rho^{1/3} \times\left[\left(\frac{32\pi^2r^3T^3\rho + 48\pi^2r^4T^2\rho + 32\pi^2r^3T^3\rho}{M^{1/3}}\right) - \left(\frac{64\pi^3r^6T^3\rho^2}{3M^{4/3}}\right)\right] \quad (51)$$

Then the expression of the luminosity given by (46) becomes:

$$L \cong \left(1 - \frac{M_{iso}}{M}\right)\frac{R}{3}\frac{4ac}{3\kappa\rho} \times \frac{G}{4\pi}\left(\frac{4\pi}{3}\right)^{4/3}\left(\frac{\mu m_H}{k}\right)\rho^{1/3} \times\left[\left(\frac{32\pi^2r^3T^3\rho + 48\pi^2r^4T^2\rho + 32\pi^2r^3T^3\rho}{M^{1/3}}\right) - \left(\frac{64\pi^3r^6T^3\rho^2}{3M^{4/3}}\right)\right] \quad (52)$$

This relation provide us with an estimation of the luminosity produced by a star from the POST MS region with a mass $M \geq 2M_{\Theta}$ in which the envelope is considered radiative. As expected, the luminosity of these stars is in function of M_{iso}.

E. Choice of the parameters:

→The molecular weight μ

To calculate the luminosity of the star given by the relation (52), we need to know the value of the molecular weight μ. To get the values of this parameter, we use the formulae given by [1], page 119:

$$\mu = \left[\frac{(1-\beta)}{0.00309\beta^4\left(\frac{M}{M_\Theta}\right)^2}\right]^{1/4}$$

where M and M_Θ are respectively the mass of the star and the mass of the sun. β is a constant relating between the total pressure P_{TOT}, the radiative pressure P_R ,and the gas pressure P_{GAS} of the star. They are defined by (see [1], page 116),

$$P_R = (1-\beta)P_{TOT} \text{ and } P_{GAS} = \beta\; P_{TOT}$$

→The opacity κ

For the calculus of the luminosity, we use the opacity formulae given by (see [1], page 119),

$$\kappa = \frac{4\pi\; c\; G\; M\; (1-\beta)}{L}$$

where β is the constant mentioned above, c the light velocity , G the gravitational constant, M is the mass of the star, and L its luminosity. In this work κ is calculated using the observational values of the luminosity L .

III. DISCUSSION OF THE RESULTS:

On the Table 1 and the Table 2, it's shown the results of the luminosity calculated for several stars from the POST MS zone which their masses exceed $2M_\Theta$. The values of the luminosity L of the present work aren't far from those obtained from the bolometric measures. The differences between the observational and the calculated values of the luminosity are very reasonable with respect to the approximations done in this work. So this work can be considered as a very appreciable approach to find the theoretical expression of the luminosity which fit the best the observation and in the same time takes into account the upper limit of the mass of the isothermal core.

IV. CONCLUSION

The present work is a contribution to the study of the internal structure of the stars with $M \geq 2M_\Theta$. According to the results obtained in the frame of this work ,one can deduce that it consists a good approach to establish the most accurate theoretical expression for the luminosity produced by this category of stars taking into account the upper limit of the isothermal core mass M_{iso}

Tableau 1:The calculated luminosities produced by stars with isothermal core and radiative envelope.

These results are obtained for $\mu_{iso} = \mu_{env}$ so for $M_{iso} = 0.261M$, $Teff$ is the effective temperature

References. (1) Eddington & Chandrasekhar 1988, page 145 [1]; (2) Eddington & Chandrasekhar 1988, page 182[1].

Stars	Mass $M\ (g)$	$(1-\beta)$	Radius $R\ (cm)$	Temperature $Teff\ (°c)$	Density ρ $(g\ cm^{-3})$	Molecular Mass μ	Luminosity $L\ (erg\ s^{-1})$	Reference
RR LYR.	$3.70M_\Theta$	0.260	4.32×10^{11}	7800	2.180×10^{-2}	2.127	4.793×10^{35}	(2)
CAPELLA B	$4.18M_\Theta$	0.283	9.55×10^{11}	5200	2.270×10^{-3}	2.110	4.045×10^{36}	(1)
β Cephei	$5.10M_\Theta$	0.330	1.52×10^{11}	19000	6.895×10^{-1}	2.125	1.053×10^{35}	(2)
SU.CAS	$5.30M_\Theta$	0.330	9.20×10^{11}	6350	3.232×10^{-3}	2.084	7.194×10^{36}	(2)
SZ.TAU	$6.60M_\Theta$	0.380	13.5×10^{11}	5850	1.273×10^{-3}	2.091	3.076×10^{37}	(2)
SU.CYG	$6.80M_\Theta$	0.390	12.0×10^{11}	6450	1.868×10^{-3}	2.107	2.557×10^{37}	(2)
RT.AUR	$6.90M_\Theta$	0.390	13.9×10^{11}	5950	1.219×10^{-3}	2.092	3.788×10^{37}	(2)
T.VUL	$7.70M_\Theta$	0.420	16.7×10^{11}	5750	7.850×10^{-4}	2.121	7.782×10^{37}	(2)
POLARIS	$7.80M_\Theta$	0.420	19.6×10^{11}	5250	4.919×10^{-4}	2.108	1.153×10^{38}	(2)

Tableau 2:The comparison between the observational luminosities and those calculated in the present work.

References. (1) Eddington & Chandrasekhar 1988, page 145[1]; (2) Eddington & Chandrasekhar 1988, page 182[1]

Stars	The Luminosity calculated using M_{iso} of the present work $L_{CALCULATED}\ (erg\ s^{-1})$	Bolometric Luminosity $L_{MEASURED}\ (erg\ s^{-1})$	$\frac{L_{CALCULATED}}{L_{MEASURED}}$	Reference
RR LYR.	4.793×10^{35}	5.030×10^{35}	0.953	(2)
CAPELLA B	4.045×10^{36}	4.800×10^{35}	8.427	(1)
β Cephei	1.053×10^{35}	2.215×10^{36}	0.047	(2)
SU.CAS	7.194×10^{36}	1.012×10^{36}	7.109	(2)
SZ.TAU	3.076×10^{37}	1.561×10^{36}	19.109	(2)
SU.CYG	2.557×10^{37}	1.809×10^{36}	14.135	(2)
RT.AUR	3.788×10^{37}	1.776×10^{36}	20.329	(2)
T.VUL	7.782×10^{37}	2.215×10^{36}	35.133	(2)
POLARIS	1.153×10^{38}	2.096×10^{36}	55.009	(2)

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