

A New Mathematical Model to Fully Fuzzy Project Crashing Problems

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Abstract: Optimization problems in project management have been traditionally solved by two distinctive approaches: heuristic methods and optimization techniques. Although heuristic methods can handle large-size projects; they do not guarantee optimal solutions. In this paper, a linear mathematical optimization model for fuzzy project time cost trade off problem is developed. An illustration is provided to demonstrate the efficiency of the proposed method.

Keywords: Fuzzy Project crashing - Triangular fuzzy number - Linear Programming - α - Cut Interval.

I. INTRODUCTION

Project Management is a very important field employed for scheduling activities and monitoring the progress, in competitive and fluctuating environments. The feasible duration required to perform a specific project is determined using critical path method. However, because of competitive priorities, time is important and the completion time of a project determined using critical path method should be reduced to meet a deadline requested. In scheduling a project, it is generally considered to expedite the duration of some activities through expanding extra budget in order to compress the project completion time. This procedure can be considered under either some fixed available budget or a threshold of project completion time. This problem is known as time cost trade off problem or project crashing problem in the project management literature.

The main objective of these kinds of problem is to determine the optimum duration and cost should be assigned to the activities such that the overall cost is minimized.

The project duration can be shortened by the acceleration of the critical activity times. The acceleration of the activity times can be achieved using more resources (using more productive equipment, material or hiring more workers) which means higher costs. Project crashing problem analyzes how to modify project activities so as to achieve the trade off between the project cost and the completion time.

A. LITERATURE REVIEW

By reviewing the literature, it is observed that there are several studies investigated and analyzed the project management problems. Mathematical and heuristic methods are the two major approaches used to solve the time cost trade off problems in project scheduling. Mathematical methods convert the project time cost trade

off problems to mathematical models and utilize linear programming, integer programming, dynamic programming, goal programming or multi-objective linear programming to solve the problems. However, formulating the objective function as well as the required constraints is time-consuming and prone to errors. Heuristic methods provide a way to obtain good solutions but do not guarantee optimality. However, they require less computational effort than mathematical methods.

The problem of project time cost trade off was first introduced by Kelly [9]. By assuming that direct cost of an activity changes with time, mathematical programming models were developed to minimize the project's direct cost [9]. Thereafter, many researchers have developed mathematical programming model for these kinds of problems. The time cost trade off problems have been extensively investigated. Many models have been proposed and they can be categorized into two types: Deterministic scheduling and non-deterministic scheduling. Recently, Yang [19] proposed a chance-constrained programming model to analyze the time cost trade off problem, where funding variability is considered. Yang [19] took budget uncertainty into account on project time cost trade off in a chance-constrained programming model. A hybrid intelligent algorithm integrating simulation and genetic algorithm was designed for solving the proposed models.

The above mentioned time cost trade off models mainly based on probability theory. As generally known, it requires a prior predictable regularity or a posterior frequency distribution to construct the probability distribution of activity times. However, in real world applications some activity times must be forecasted subjectively; for example, we have to use human judgment instead of stochastic assumptions to determine activity times. An alternative way to deal with imprecise data is to employ the concept of fuzziness, whereby the vague activity times can be represented by fuzzy sets. The main advantages of methodologies based on fuzzy theory are that they do not require prior predictable regularities or posterior frequency distributions, and they can deal with imprecise input information containing feelings and emotions quantified based on the decision-makers subjective judgment.

In the literature, there are several studies that have investigated the project management problems with fuzzy parameters. Liu et al. [14] proposed a fuzzy optimal construction time cost trade off method. In their study, the

activity duration is accepted as fuzzy number. An acceptable risk level is defined as the minimum concept of the fuzzy set theory; fuzzy durations are transformed into crisp sets. Then the genetic algorithm techniques are used to find the optimal or near optimal solutions. Arıkan and Gungor [2] applied fuzzy goal programming to the time cost trade off problem with two objectives which are minimum completion time and crashing costs. The aspiration levels of the objective are defined as fuzzy numbers. The goal programming is solved using max-min approach. Wang and Liang [15] solved project management decision problem with multiple fuzzy goals in their study. The goals of the problem are defined using linear membership functions, and the multiple fuzzy goal programming problem is solved after transforming into its crisp equivalent using Bellman and Zadeh's fuzzy decision concept. Eshtehardian et al. [4] presented a new approach for the solution of time cost trade off problems with uncertain costs. An appropriate genetic algorithm is used to find the solution of the multi objective fuzzy time cost problem. Lin [12] proposed an approach to solve project crashing problems with uncertain activity times and crash costs. The confidence-interval estimates and the previous statistical data are used to solve the fuzzy project crashing problem. In this study, level $(1-\alpha)$ fuzzy numbers were derived from $(1-\alpha) \times 100\%$ confidence interval estimates of the statistical data with a distance ranking which is used to define the fuzzy ordering. The activities execution times and costs, the daily costs are accepted as triangular fuzzy numbers. The proposed approach explicitly embeds the fuzzy set theory into the optimization procedure and then a multi objective genetic algorithm is used to solve the discontinuous and multi objective fuzzy time cost model. Maa and Ke [10] solved time cost trade off problem with fuzzy activity duration times. The fuzzy activity duration times defined as fuzzy variables based on self-dual credibility measure. Then, the obtained time cost trade off problem is solved with a hybrid intelligent algorithm integrating fuzzy simulation and genetic algorithm. Ghazanfari et al. [6] proposed a mathematical model to deal with fuzzy time cost trade off problem. The normal and crash durations of activities are considered as triangular fuzzy numbers. For the solution of the fuzzy problem, a ranking fuzzy numbers method used. Liang [16] proposed a possibilistic linear programming approach for the solution of fuzzy multi objective project management decision problem. The fuzzy parameters are defined using the triangular possibility distribution. In the proposed possibilistic linear programming approach, the fuzzy objectives and the fuzzy constraints are transformed into their crisp equivalents. Then, the obtained multi objective linear programming problem transformed into an equivalent linear programming problem using Zimmerman's fuzzy decision concept and the minimum operator. Chen and Tsai [3] proposed a new approach to solve time cost trade off problems. This approach described the minimum total crash cost of a project network via a membership function which completely conserves all the fuzziness of parameters, and the corresponding optimal activity time for each activity under different possibility

levels are obtained. Tolunay Gocken [18] proposed a solution process for the fuzzy multi objective project crashing problem.

In this paper, a mathematical optimization model for solving time cost trade off problem is presented. The model is formulated in the form of linear programming that produces optimal solution. The proposed model minimizes the total cost of the project and this model concerns both direct cost and indirect cost.

II. PRELIMINARIES

In this section, some basic definitions of fuzzy theory defined by Kaufmann, Gupta and Zimmermann, are presented.

Definition 2.1

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X . This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \rightarrow [0,1]$. The assigned values indicate the membership grade of the element in the set A .

The function $\mu_{\tilde{A}}$ is called the membership function and

the set $\tilde{A} = \{(A, \mu_{\tilde{A}}(x) : x \in X)$

defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2.2

A fuzzy set \tilde{A} defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}(x) : R \rightarrow [0,1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $(-\infty, a] \cup [c, \infty)$.
3. $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[b, c]$.
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in b$ where $a \leq b \leq c$.

Definition 2.3

Triangular fuzzy number is a fuzzy number represented with three points as follows: $A = (a_1, a_2, a_3)$ this representation is interpreted as membership functions.

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < a_1 \text{ and } x > a_3 \\ \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \leq x \leq a_3 \end{cases}$$

α - cut interval of TFN A_α shall be obtained as follows

$$\forall \alpha \in [0,1]$$

$$A_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}]$$

$$\text{Where } a_1^{(\alpha)} = (a_2 - a_1)\alpha + a_1 \quad \text{and}$$

$$a_3^{(\alpha)} = -(a_3 - a_2)\alpha + a_3$$

$$\text{Thus } A_\alpha = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3]$$

A. OPERATIONS ON α - cut INTERVAL

If $A_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}]$ and $B_\alpha = [b_1^{(\alpha)}, b_3^{(\alpha)}]$ are two α - cut intervals of triangular fuzzy numbers A and B, then the arithmetic operations between A_α and B_α is follows:

$$A_\alpha + B_\alpha = [a_1^{(\alpha)} + b_1^{(\alpha)}, a_3^{(\alpha)} + b_3^{(\alpha)}]$$

$$A_\alpha - B_\alpha = [a_1^{(\alpha)} - b_3^{(\alpha)}, a_3^{(\alpha)} - b_1^{(\alpha)}]$$

III. FUZZY LINEAR ROGRAMMING

On the basis of LP formulation, this paper proposes an approach to the fuzzy time cost trade off problem. Here crisp LP and one of fuzzy LP models are briefly introduced, together with discussion on relaxing the assumptions of LP into fuzzy LP for solving the case.

LP has been demonstrated to be one of the most frequently applied Operational Research/Management Science techniques in practical problems. LP is concerned with the effective and efficient allocation of limited resources to interrelated activities with the objective of achieving a pre-specific goal such as maximizing profit. The general LP model can be described in the canonical form using matrix notation as follows:

$$\text{Max } C^T x$$

$$\text{subject to } Ax \leq b$$

$$x \geq 0$$

Where C is the n-vector of profit coefficients of the objective function, A is the $m \times n$ constraint matrix, x is the n-vector of decision variables and b is the m-vector of total resources available. As generally known, four assumptions of LP are proportionality, additivity, divisibility and deterministic. In general, the first two assumptions are also called linearity. Note that in this model, all coefficients of A, b, and C are crisp numbers, and each constraint must be satisfied strictly. In practical situations, these input data for this model are usually imprecise because of incomplete information; in addition, vagueness in these input data may not be of a probabilistic type. As Rommelfanger pointed out that "LP requires much well-defined and precise data which involves high information costs," these four assumptions are often too strict to limit the applications. To deal quantitatively with imprecise information in making decisions, Bellman and Zadeh introduced the concept of fuzzy set theory. Since then, much research has developed specific fuzzy linear

programming models and proposed the associated solution methods.

Infact, there are many different types of FLP models, Lai and Hwang provided a systematical classification in detail. Instead of describing all types of FLP models, here the FLP model with fuzzy profit coefficients of the objective function and total resources available is exemplified as follows:

$$\text{Max } \tilde{C}^T x$$

$$\text{subject to } Ax \leq \tilde{b}$$

$$x \geq 0$$

Where \tilde{C} and \tilde{b} are the vectors of profit coefficients and resources available being fuzzy numbers, respectively. Clearly, the assumption of certainty of the crisp LP is relaxed to fit real-world situations. Moreover, the operations of addition and multiplication are operations of fuzzy arithmetic, and " \leq " denotes the ordering of fuzzy numbers.

A. PROBLEM DESCRIPTION

With the progress of the project, project managers always need to make tradeoff between the cost and the completion time. Sometimes managers may make decision in order to finish the project sooner with project cost augment by accelerating the project schedule, which is also named as project crashing in project management. In other cases, motivated by reducing the project cost, managers may be conscripted to sacrifice with prolonging the project completion time. Therefore, it is naturally desirable for managers to find a schedule to complete a project with the balance of the cost and completion time.

The total cost function of a project has two components: direct and indirect costs. Direct costs are incurred because of the performance of project activities, while indirect costs include those items that are not directly related to individual project activities and thus can be assessed for the entire project. In general, indirect cost increases almost linearly with the increase of project duration and usually assumed as a percentage of project direct cost. The project time cost trade off problem, thus, is reduced to determine project cost against project duration. A possible way to solve time cost trade off problem is to use a mathematical programming model whose objective function is constructed so that project direct cost is minimized and the imposed constraints guarantee a desired project deadline, while the precedence requirements of the network are maintained.

A project can be represented by an activity-on-arc network $G = (V, A)$, where $V = \{1, 2, \dots, n\}$ is the set of nodes representing the milestones and A is the set of arcs representing the activities. In the network, node 1 and n represent the start and end of the project respectively. In this paper, the normal activity durations are assumed to be uncertain variables.

B. FORMULATION OF THE DEVELOPED MODEL

To formulate the time cost trade off problem, one of the important issues is to access the value of that time, total cost of the project. Clearly, project completion time can be calculated by determining the total completion time for the critical path. A critical path in a project is the path with longest duration. Moreover the total cost is equal to the sum of cost used for all activities. The main goal in this problem is to minimize the total cost while the total time is minimized.

The primary information obtained from traditional scheduling is basically activities start and finish timings and floats. The duration and the corresponding cost for an activity are selected optimally from their utility data to satisfy the objective function and the imposed constraints. If the start time of an activity is determined, the finish time can be specified by adding the selected activity duration, and vice versa.

Parameters and decision variables of model are as follows:

n	Number of nodes
\tilde{T}_i	Starting time of event i
\tilde{C}_{ij}	Planned cost of activity $i \rightarrow j$ (ie. Normal cost + crash cost)
\tilde{s}_{ij}	Slope cost of activity $i \rightarrow j$
\tilde{I}	Indirect cost per day
\tilde{x}_{ij}	Planned duration of activity $i \rightarrow j$
\tilde{T}	Project completion time
$N\tilde{C}_{ij}$	Normal cost of activity $i \rightarrow j$
$N\tilde{T}_{ij}$	Normal time of activity $i \rightarrow j$
$C\tilde{T}_{ij}$	Crash time of activity $i \rightarrow j$

- **Precedence relationship constraint:**

The completion time of project could be constrained by one of two methods. The first approach is to allow for a precedence constraint for each immediate preceding relationship in the project network. This approach was used in almost all existing optimization techniques. The second is to allow for one constraint for each path from the first activity to the last one in the project network. In the present model, the first approach will be adopted.

The logical relationship between any two consecutive activities i and its immediate predecessor j , is expressed mathematically as

$$\tilde{T}_j - \tilde{T}_i - \tilde{x}_{ij} \geq 0$$

- **Project completion constraint:**

Project completion is controlled by the latest finish time of ending activities. If the number of ending activities is denoted by n , the project completion constraint is given by

the equation, in which T is the desired deadline of the project.

$$\tilde{T}_n \leq \tilde{T}$$

The upper and lower bounds on T are the normal project duration and crash project duration respectively.

- **Bounded constraint of duration:**

The planned duration of activity $i \rightarrow j$ (i.e. x_{ij}) is always lies between normal duration and crash duration of that activity. The mathematical form of this bounded constraint is

$$CT_{ij} \leq x_{ij} \leq NT_{ij}$$

- **Cost constraint:**

The cost constraint involves the normal cost, slope cost, normal time and planned duration of activity ij . Thus the project cost of each activity is written in the form of

$$\tilde{C}_{ij} = N\tilde{C}_{ij} + \tilde{s}_{ij} (N\tilde{T}_{ij} - \tilde{x}_{ij})$$

- **Objective function:**

Our main objective of this model is to minimize the cost of the project. The linear programming objective will be

$$\text{Min} \sum_i \sum_j \tilde{C}_{ij} + \tilde{I} (\tilde{T}_n - \tilde{T}_1)$$

Thus the **complete fuzzy mathematical model** can be summarized as follows:

$$\text{Min} \tilde{Z} = \sum_i \sum_j \tilde{C}_{ij} + I * (\tilde{T}_n - \tilde{T}_1)$$

Subject to

$$\tilde{T}_1 = 0$$

$$\tilde{T}_j - \tilde{T}_i - \tilde{x}_{ij} \geq 0$$

$$\tilde{T}_n \leq \tilde{T}$$

$$\tilde{C}_{ij} = N\tilde{C}_{ij} + \tilde{s}_{ij} (N\tilde{T}_{ij} - \tilde{x}_{ij})$$

$$C\tilde{T}_{ij} \leq \tilde{x}_{ij} \leq N\tilde{T}_{ij} \quad \forall (i, j) \in P$$

Theorem 4.1

If the optimal solution of the fully fuzzy linear programming problem (P₂) lies in the interval $[x_1^{(\alpha)}, x_3^{(\alpha)}]$, then $x_1^{(\alpha)}$ is the optimal solution of the lower linear programming model (P₄) and $x_3^{(\alpha)}$ is the optimal solution of the upper linear programming model (P₅) where $[x_1^{(\alpha)}, x_3^{(\alpha)}]$ is the α -cut interval of triangular fuzzy number \tilde{x} .

IV. ALGORITHM TO SOLVE THE FULLY FUZZY PROJECT CRASHING PROBLEMS

In this section, we have presented a new algorithm for solving fuzzy time cost trade off problems through fuzzy linear programming problems.

1. Formulate the chosen fuzzy time cost trade off problem (P) into the following fuzzy linear programming problem (P₁):

$$\text{Min } \tilde{Z} = \sum_i \sum_j \tilde{C}_{ij} + I * (\tilde{T}_n - \tilde{T}_1)$$

Subject to

$$\tilde{T}_1 = 0$$

$$\tilde{T}_j - \tilde{T}_i - \tilde{x}_{ij} \geq 0$$

$$\tilde{T}_n \leq \tilde{T}$$

$$\tilde{C}_{ij} = N\tilde{C}_{ij} + \tilde{s}_{ij} (N\tilde{T}_{ij} - \tilde{x}_{ij})$$

$$C\tilde{T}_{ij} \leq \tilde{x}_{ij} \leq N\tilde{T}_{ij} \quad \forall (i, j) \in P$$

(P₁)

2. Change the representation of fuzzy number (i.e. triangular fuzzy number into α -cut interval fuzzy number) in the above fuzzy linear programming problem (P₁). The representation of fuzzy linear programming problem (P₂) where the datas are in the form of α -cut interval of fuzzy number is the following:

3.

$$\text{Min } \sum_i \sum_j \tilde{C}_{ij}^\alpha + \tilde{I}^\alpha * (\tilde{T}_n^\alpha - \tilde{T}_1^\alpha)$$

Subject to

$$\tilde{T}_1^\alpha = 0$$

$$\tilde{T}_j^\alpha - \tilde{T}_i^\alpha - \tilde{x}_{ij}^\alpha \geq 0 \quad (P_2)$$

$$\tilde{T}_n^\alpha \leq \tilde{T}^\alpha$$

$$\tilde{C}_{ij}^\alpha = N\tilde{C}_{ij}^\alpha + \tilde{s}_{ij}^\alpha (N\tilde{T}_{ij}^\alpha - \tilde{x}_{ij}^\alpha)$$

$$C\tilde{T}_{ij}^\alpha \leq \tilde{x}_{ij}^\alpha \leq N\tilde{T}_{ij}^\alpha \quad \forall (i, j) \in P$$

4. The above mathematical formation of fuzzy linear programming problem (P₂) can be written in the following form (P₃):

$$\text{Min } \sum_i \sum_j [C_{ij1}^{(\alpha)}, C_{ij3}^{(\alpha)}] + [I_1^{(\alpha)}, I_3^{(\alpha)}] * ([T_{n1}^{(\alpha)}, T_{n3}^{(\alpha)}] - [T_{11}^{(\alpha)}, T_{13}^{(\alpha)}])$$

Subject to

$$[T_{11}^{(\alpha)}, T_{13}^{(\alpha)}] = 0$$

$$[T_{j1}^{(\alpha)}, T_{j3}^{(\alpha)}] - [T_{i1}^{(\alpha)}, T_{i3}^{(\alpha)}] - [x_{ij1}^{(\alpha)}, x_{ij3}^{(\alpha)}] \geq 0$$

$$5. [T_{n1}^{(\alpha)}, T_{n3}^{(\alpha)}] \leq [T_1^{(\alpha)}, T_3^{(\alpha)}]$$

$$[C_{ij1}^{(\alpha)}, C_{ij3}^{(\alpha)}] = [NC_{ij1}^{(\alpha)}, NC_{ij3}^{(\alpha)}] + [s_{ij1}^{(\alpha)}, s_{ij3}^{(\alpha)}] * ([NT_{ij1}^{(\alpha)}, NT_{ij3}^{(\alpha)}] - [x_{ij1}^{(\alpha)}, x_{ij3}^{(\alpha)}])$$

$$[CT_{ij1}^{(\alpha)}, CT_{ij3}^{(\alpha)}] \leq [x_{ij1}^{(\alpha)}, x_{ij3}^{(\alpha)}] \leq$$

$$[NT_{ij1}^{(\alpha)}, NT_{ij3}^{(\alpha)}] \quad \forall (i, j) \in P$$

The above fuzzy linear programming model (P₃) can be decompose into two crisp linear programming model,

namely lower linear programming model (P₄) and upper linear programming model (P₅).

$$\text{Min } \sum_i \sum_j C_{ij1}^{(\alpha)} + I_1^{(\alpha)} (T_{n1}^{(\alpha)} - T_{11}^{(\alpha)})$$

(P₄)

Subject to

$$T_{11}^{(\alpha)} = 0$$

$$T_{j1}^{(\alpha)} - T_{i1}^{(\alpha)} - x_{ij1}^{(\alpha)} \geq 0$$

$$T_{n1}^{(\alpha)} \leq T_1^{(\alpha)}$$

$$C_{ij1}^{(\alpha)} = NC_{ij1}^{(\alpha)} + s_{ij1}^{(\alpha)} (NT_{ij1}^{(\alpha)} - x_{ij1}^{(\alpha)})$$

$$CT_{ij1}^{(\alpha)} \leq x_{ij1}^{(\alpha)} \leq NT_{ij1}^{(\alpha)} \quad \forall (i, j) \in P$$

$$\text{Min } \sum_i \sum_j C_{ij3}^{(\alpha)} + I_3^{(\alpha)} * (T_{n3}^{(\alpha)} - T_{13}^{(\alpha)})$$

Subject to

$$T_{13}^{(\alpha)} = 0$$

$$T_{j3}^{(\alpha)} - T_{i3}^{(\alpha)} - x_{ij3}^{(\alpha)} \geq 0$$

$$T_{n3}^{(\alpha)} \leq T_3^{(\alpha)}$$

$$C_{ij3}^{(\alpha)} = NC_{ij3}^{(\alpha)} + s_{ij3}^{(\alpha)} (NT_{ij3}^{(\alpha)} - x_{ij3}^{(\alpha)})$$

$$CT_{ij3}^{(\alpha)} \leq x_{ij3}^{(\alpha)} \leq NT_{ij3}^{(\alpha)} \quad \forall (i, j) \in P$$

(P₅)

4. Solve the above lower linear programming problem (P₄) and upper linear programming problem (P₅) using LINGO software package for different values of α , where $\alpha \in [0,1]$
5. Substituting the obtained solutions of (P₄) and (P₅) in the problem (P₁), the solution of (P) for different values of α , where $\alpha \in [0,1]$ can be obtained.

V. NUMERICAL ILLUSTRATION

A. Example 5.1

Consider an AOA project network in fuzzy environments whose corresponding network is given in Fig. 1. In this section, first, parameter and decision variables are considered in triangular fuzzy number form and then reconsidered in α -cut interval form of triangular fuzzy number. Activities information is given in Table I.

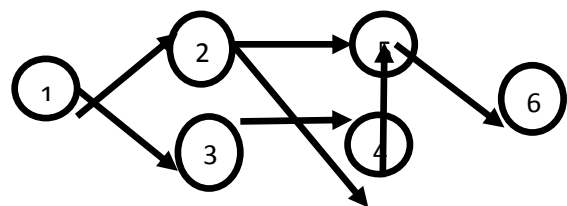


Table I. THE FUZZY DATA OF EXAMPLE 5.1

Activity	Normal time ($N\tilde{T}_{ij}$)	Crash time ($C\tilde{T}_{ij}$)
(1, 2)	(13, 14, 15)	(4, 6, 6)
(1, 3)	(10, 12, 13)	(7, 8, 9)
(2, 5)	(16, 18, 19)	(10, 14, 14)
(2, 4)	(5, 6, 8)	(3, 4, 5)
(3, 4)	(3, 4, 5)	(2, 2, 3)
(4, 5)	(7, 8, 10)	(4, 6, 7)
(5, 6)	(9, 12, 14)	(6, 8, 8)
Normal cost (\tilde{C}_{ij})		Slope cost (\tilde{S}_{ij})
(1200, 1400, 1600)		(80, 100, 120)
(1100, 1200, 1400)		(190, 200, 220)
(1600, 1700, 1800)		(90, 100, 130)
(700, 800, 900)		(160, 200, 210)
(400, 500, 600)		(180, 200, 210)
(600, 800, 1000)		(80, 100, 110)
(1000, 1100, 1200)		(90, 100, 120)

Using step 1, the complete fuzzy linear mathematical model of the example project has been formulated. The input data are dependency relationships (predecessors), activities' utility data.

$$\text{Min } Z = \sum_{i=1}^5 \sum_{j=2}^6 \tilde{C}_{ij} + \tilde{T} * \tilde{T}_6 \quad (P_1)$$

Subject to

$$\tilde{T}_1 = 0$$

$$\tilde{T}_j - \tilde{T}_i - \tilde{x}_{ij} \geq 0 \quad i, j = 1, 2, \dots, 6$$

$$\tilde{T}_6 \leq \tilde{T}$$

$$\tilde{C}_{ij} = N\tilde{C}_{ij} + \tilde{s}_{ij} (N\tilde{T}_{ij} - \tilde{x}_{ij}) \quad i, j = 1, 2, \dots, 6$$

$$C\tilde{T}_{ij} \leq \tilde{x}_{ij} \leq N\tilde{T}_{ij} \quad \forall (i, j) \in P$$

Now, change the form of triangular fuzzy number in the problem (P₁) into α -cut interval as given in step 2, in the procedure. The following is the α -cut interval form of the given information.

Table II. α -cut INTERVAL FORM OF $N\tilde{T}_{ij}$, $C\tilde{T}_{ij}$, \tilde{C}_{ij} , \tilde{S}_{ij}

Normal Time $N\tilde{T}_{ij}^{(\alpha)}$	Crash Time $C\tilde{T}_{ij}^{(\alpha)}$
($\alpha + 13, -\alpha + 15$)	($2\alpha + 4, 6$)
($2\alpha + 10, -\alpha + 13$)	($\alpha + 7, -\alpha + 9$)
($2\alpha + 16, -\alpha + 19$)	($4\alpha + 10, 14$)
($\alpha + 5, -2\alpha + 8$)	($\alpha + 3, -\alpha + 5$)
($\alpha + 3, -\alpha + 5$)	($2, -\alpha + 3$)
($\alpha + 7, -2\alpha + 10$)	($2\alpha + 4, -\alpha + 7$)
($3\alpha + 9, -2\alpha + 14$)	($2\alpha + 6, 8$)
Slope Cost $\tilde{S}_{ij}^{(\alpha)}$	Normal Cost $\tilde{C}_{ij}^{(\alpha)}$
($20\alpha + 80, -20\alpha + 120$)	($200\alpha + 1200, -200\alpha + 1600$)
($10\alpha + 190, -20\alpha + 220$)	($100\alpha + 1100, -200\alpha + 1400$)
($10\alpha + 90, -30\alpha + 130$)	($100\alpha + 1600, -100\alpha + 1800$)
($40\alpha + 160, -10\alpha + 210$)	($100\alpha + 700, -100\alpha + 900$)
($20\alpha + 180, -10\alpha + 210$)	($100\alpha + 400, -100\alpha + 600$)
($20\alpha + 80, -10\alpha + 110$)	($200\alpha + 600, -200\alpha + 1000$)
($10\alpha + 90, -20\alpha + 120$)	($100\alpha + 1000, -100\alpha + 1200$)

Procedure to solve fuzzy time cost trade off problems presented in section should be used in order to solve this problem. As mentioned in step 4, problem (P₃) can be decomposed in to (P₄) and (P₅). (i.e lower linear programming model and upper linear programming model) The values of minimum total cost and planned duration of the project have been determined using LINGO solver. A computer package called LINGO (LINGO 2000) is used on a personal computer to solve the mathematical model of the example project. LINGO is a commercial package using the power of linear and non-linear optimization to formulate large problems concisely, solve them, and analyze the solution. In all tested runs, the linear mathematical model of the example project requires less than one second on LINGO to obtain the optimal solution. The Fuzzy Mathematical model of the example project is solved for different values α , where $\alpha \in [0,1]$. Table III shows the lower and upper bounds of the cost values of the example project.

TABLE III. LOWER AND UPPER BOUNDS OF COST VALUES OF EXAMPLE 5.1.

α	Lower bound	Upper bound
0.0	11490	16160
0.1	11691.9	15928.6
0.2	11893.6	15698.4
0.3	12095.1	15469.4
0.4	12309.2	15241.6
0.5	12532.5	15015
0.6	12759.2	14806.4
0.7	12989.3	14565.4
0.8	13282.8	14342.4
0.9	13589.7	14120.6
1.0	13900	13900

TABLE IV. THE LOWER AND UPPER BOUNDS OF THE ACTIVITY TIMES OF EXAMPLE 5.1

α	x_{ij}	x_{12}	x_{13}	x_{25}	x_{24}	x_{34}	x_{45}	x_{56}
1.0	U	6	12	14	6	2	6	8
	L	6	12	14	6	2	6	8
0.9	U	6	12	14.2	6.2	2.1	6.1	8
	L	5.8	11.8	14.4	5.9	2.6	5.8	7.8
0.8	U	6	12	14.4	6.4	2.2	6.2	8
	L	5.6	11.6	14.8	5.8	3.2	5.6	7.6
0.7	U	6	12	14.6	6.6	2.3	6.3	8.0
	L	5.4	11.4	15.2	5.7	3.7	5.5	7.4
0.6	U	6	12	14.8	6.8	2.4	6.4	8
	L	5.2	11.2	15.6	5.6	3.6	6	7.2
0.5	U	6	12	15	7	2.5	6.5	8
	L	5	11	16	5.5	3.5	6.5	7
0.4	U	6	12	15.2	7.2	2.6	6.6	8
	L	4.8	10.8	16.4	5.4	3.4	7	6.8
0.3	U	6	12	15.4	7.4	2.7	6.7	8
	L	4.6	10.6	16.6	5.3	3.3	7.3	6.6
0.2	U	6	12	15.6	7.6	2.8	6.8	8
	L	4.4	10.4	16.4	5.2	3.2	7.2	6.4
0.1	U	6	12	15.8	7.8	2.9	6.9	8
	L	4.2	10.2	16.2	5.1	3.1	7.1	6.2
0.0	U	6	12	16	8	3	7	8
	L	4	10	16	5	3	7	6

B. RESULT ANALYSIS

As generally known, when the parameters are fuzzy numbers, the minimal total cost \tilde{C} is a fuzzy number as well. The α -cut of \tilde{C} represents the possibility that the total crashing time of this time cost trade off problem in this project network will appear in the associated range. Specially, the $\alpha = 1.0$ cut shows the crashing time that is most likely, and the α -cut that approaches 0 shows the range in which the crashing cost could appear. Table IV lists the optimal activity times corresponding to the lower and upper bounds of the minimum total cost at these 11 values of α , 0, 0.1, 0.2...1.0. Notably, the total cost associated with smallest possible values of parameters need not be lowest, even if they are cost coefficients in the objective function, such as CT_{ij} .

VI. CONCLUSION

A fuzzy mathematical model has been developed which links the CPM with least cost optimization, mathematical programming in order to optimize the traditional time cost trade off problem. The developed model is a stand-alone piece of generic technique which may well be applied to projects of any kind; provided the projects can be defined within the boundaries of the techniques used (i.e. project is being divided into precedence related activities, each with normal and crash time and cost data.)

This paper investigated the time cost trade off problem in the project network with several fuzzy parameters which makes this problem being complicated. The underlying idea is based on linear programming formulation and the concept of α -cut to transform the fuzzy time cost trade off problem to pair of mathematical programming. By enumerating different values of the possibility level α , the lower and upper bound of the α -cuts of the fuzzy minimum total cost are calculated to approximate the membership function; and the corresponding optimal activity times are also obtained. A numerical example solved by other researchers is being taken into consideration to illustrate the validity of the proposed model and approach.

It is clear that if the obtained minimum total cost is a crisp value, then it may lose some useful information. Compared with other studies, the proposed model and approach can obtain more reasonable solutions suitable for all cases ranging from the pessimistic case to the optimistic case, and so more information is provided for making project management decisions.

The proposed model is also applicable to more complicated project networks in real world. Clearly, the proposed approach is not confined to the fuzzy parameters of triangular type. This is illustrated by successfully solving an example with fuzzy parameters. Other types such as

trapezoidal type and interval type are also applicable. The proposed model suits well for the fuzzy time cost trade off problem involving both direct costs and indirect costs.

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