

A Note On Solutions To The Ambulance Facility Real-Time Relocation Models

M. Sofyan, Cut Latifah, Dewi Suryani Hanum, Herman Mawengkang
Graduate School of Mathematics,
University Sumatera Utara,
Medan, Indonesia

Abstract— Aiming to improve the efficiency and reliability of ambulance services, some allocation and location models has been developed in operations research literature. Coverage model for the locations to maximize the (deterministic or probabilistic) number of ambulance calls. This research will present a dynamic modeling in an ambulance emergency medical services system, where the research focus an emergency ambulance type. The main purpose of this relocation models is as an anticipation of the availability of ambulance vehicle to maximize the number of calls or covered demand of ambulance calls. This research represents the extended of MEXCLP model into linear programming with 0-1 constraint both of demand point of ambulance and allocation status any ambulance unit at location and obtained a model that can be used to determine departure and relocation of available ambulances at any demand location.

Keywords— ambulance, Emergency Medical Service (EMS), relocation, dynamic modeling.

I. INTRODUCTION

Ambulance is a component of Emergency Medical Services (EMS) facility that available 24 hours per day in a hospital. Erdojan et al. [1] considered that effectiveness of EMS's quality services component can be define into multiple criteria, including the average response time, type of EMS service of each hospital staff and the medical equipment that used. In this problem, the indicator that often used in determined the quality of EMS service is ratio between the total number of covered demand points in a given certain limit of time, eight to ten minutes. Thus, for planning model, the total demand points are given by using coverage concept where we assume that there is a demand point covered by ambulance only if the average response time has a given certain limit of time.

Zaharudin et al. [2] affixed the rate of patient's death risk is depend on response time to the demand of ambulance services. The response time is defined as a time where operator received a call for demand of ambulance services, so that the response time be a necessary component in determined the performance of EMS. The EMS components originally is a kind of *out-of-hospital* service and provides

transportations for the patients that needs a medical treatment by the hospital.

In developing the improvement of efficiency and reliability of ambulance services, several allocation models of ambulance services was developed in operation research literature, is coverage model and median model. Coverage model knows as a model that can be used to maximize total number of covered demand points of ambulance and well-oriented to deterministic or probabilistic model. Whereas the object of median model is to minimize total distance between ambulance to hospital and demand points. Then, this model gives the efficiency score of ambulance services as a solution [3].

Several of these models were based on location problem under static and deterministic assumptions without considering any stochastic factors [4]. It is also explained the difference among EMS and others facility. Ambulance is allocated at a certain location such as parking lot by considering that ambulance is used periodically where the goal is to cover the total number of demand point at a certain location. Thus, this allocation problem need implementation of computation framework that helps in decision making process. There are three classification of relocation models that has been suggested: (i) the solution of real-time integer program to ambulance relocation decision making process (see Kolesar and Walker [5], Gendreau et al. [6], Brotcorne et al. [4], and Nair and Miller-Hooks [7]); (ii) the model helps in determine optimal location for ambulance in order to cover the total number of demand points by using integer program formula (see Gendreau et al. [8], Ingolfsson [9], and Goldberg [10]); and (iii) the model changes the random variable into explicit system.

An estimated model have been developed to formulate ambulance relocation model as a Markov decision process and used to obtain an optimal solution by using a dynamic programming (see Berman [11] and Zhang et al. [12]). Another heuristic models also have been developed to determine relocation decision making process based on an approximation of a certain configuration system. Andersson [13] and Andersson and Vaerband [14] proposed an estimated model of "function of availability" with the aim is to determine the capacity of a certain configuration so that the total number of demand points can be covered.

Much of the research described above focused on covering models where the demand points is considered to be covered if an ambulance is available within the given time. Daskin's model of maximum expected coverage location problem (MEXCLP) was among the earliest to incorporate the probabilistic dimension of the ambulance location scenario [15]. This paper propose a new model of ambulance relocation model based on extended of dynamic programming of MEXCLP model that combined with a real-time algorithm of relocation model. This paper is organized as follows. Section II we review some background information on the study area and outline that adopted the relocation model of EMS facility. The relocation model is proposed in Section III with a real-time algorithm of model is discussed in Section IV. Finally, the conclusion are provided in the last section.

II. RECENT DEVELOPMENTS IN RELOCATION MODELS

Facility location problem is one of NP-complete problem with the solution space of m locations that allocated to n unit facilities, denoted by n^m . Due to problem, some approximation solutions can be determined by using a metaheuristic methods such as tabu search, simulated annealing and evolutionary algorithms. Karaman [16] suggested two approximation method to determine location problem of EMS based on the objective, is (1) total of distances or times from or to ambulance location and (2) total of sum distances or times that must be passed through to reach the ambulance location. Mandell [17] considered that EMS facility was based on two kind of facility providers: Basic Life Support (BLS) units such as fire and police facility and Advanced Life Support (ALS) such as EMS components includes ambulance facility. Both of these facilities works in the same scope, life support, but with a differ standard time of each.

Daskin [15] proposed the first ambulance facility location model by using a deterministic model and without considering the probability of the ambulance condition or the number of available ambulance that allocated in a certain region. The model used queueing theory under assumptions the probability of each ambulance that to be allocated in a certain region is q . If there are j demand points covered by k units ambulance, then the expectation score is $p_j(1-q^k)$. Assume that K is being relocated to some of region. Thus, the total expected of demand points that can be covered in j is

(MEXCLP)

$$\max \sum_{j=1}^n \sum_{i=1}^M (1-p)^{i-1} h_j y_{ij} \quad (1.1)$$

s.t

$$\sum_{i=1}^M y_{ij} - \sum_{i=1}^n a_{ij} x_i \leq 0 \quad (1.2)$$

$$\sum_{i=1}^n x_i \leq M \quad (1.3)$$

where $x_i = 0, 1, 2, \dots, M, \forall i$; $y_{ij} = 0, 1, \forall i, j$; M is total maximum of ambulances that to be allocated; n is total number of demand points; x_i is total number of available ambulances that allocated at i ; and the decision making process

$$y_{ij} = \begin{cases} 1; & \text{if } j \text{ is covered by at least } i \text{ ambulances} \\ 0; & \text{if } j \text{ is covered by less than } i \text{ ambulances} \end{cases}$$

with h_j is total number of demand points at j . The objective function in Eq. (1.1) is to maximize the total number of covered demand points with constraint (1.2) to estimate demand rates at j location that covered in which based on decision variable y_{ij} to decision variables set, x_i . Constraint (1.3) gives specification to total number of available ambulance that to be allocated where the model allows total of allocated ambulances in a certain region more than the expected.

Goldberg et al. [18] suggested the relocation model that considered a given travel time as a parameter which defined in stochastic. The aim of this extended model is to maximize total number of covered demand points with a given standard time, eight minutes. In determining the solution, the probability is estimated from total demand points that covered by the ambulance under the assumptions as follows.

- probability ambulances that allocated at k is covered the demand point in eight minutes;
- probability that ambulance is available at a certain location;
- probability ambulances that allocated at $k-1$ is unavailable

Using data in Tucson, Arizona, the result show the increasing performance of ambulance facility at the location from 24% to 53.1%. Repede and Bernando [19] then extended this model with assumption the velocity of the ambulance to cover the demand points per day in Louisville, Kentucky. The main result from the model show the increasing performance of demand points within ten minutes from 84% to 95%, but decreasing of response time to 36%.

III. MODELS

This paper is based on the model simulation in emergency medic facility system by using a discrete-time system to emergency calls or demands as follows.

- (1) assume that there is an emergency call;
- (2) ambulance arrives at the location;
- (3) medic team gives proceeds at the location;
- (4) ambulance brings the patient to the hospital;
- (5) medic team bring the patient to the hospital to receive the medical treatment; and
- (6) ambulance back to the source location

In this simulation, two of random data are given, the total number of demand points and the location of demand points in a certain time. In relocation problem, the decision maker choose the main location, relocation time and facility location that given in our models. In our simulation of relocation models, we consider several components that influence the relocation models such as the source location of ambulances, hospital location, transportation network in a certain region, the location of demand points, scheduling and the status of the ambulance whether the ambulance is available to cover the emergency calls.

Define the relocation problem of the ambulance facility by representing a graph $G=(V \cup W, E)$ where $V = \{v_1, v_2, \dots, v_n\}$ and $W = \{v_{n+1}, \dots, v_{n+m}\}$ are two of vertex sets that define the location of demand points and the potential location region, respectively. Then, assume that $E = \{(v_i, v_j) : v_i, v_j \in V \cup W, i < j\}$ is set of edge in the graph. The location of demand points are given by a vertex, λ_i , where each edge is connected with the parameter of travel time, t_{ij} . For $v_i \in V$ and $v_j \in V$, given

$$\gamma_{ij} = \begin{cases} 1; & t_{ij} \leq r_1 \\ 0; & \text{otherwise} \end{cases} \quad (3.1)$$

$$\delta_{ij} = \begin{cases} 1; & t_{ij} \leq r_2 \\ 0; & \text{otherwise} \end{cases} \quad (3.2)$$

Given the total of available ambulances is p where $p \leq m$, thus the total maximum number of "idle-time" status of ambulances is p_j at $v_j \in W$. Set M_{jl}^t as a penalty coefficient that related to the relocation ambulance, $l = 1, \dots, p$ from the current demand point in time t lo the other destination demand point, $v_j \in W$. Clearly that the coefficient M_{jl}^t changes for each period t . Consequently, α is the proportion of the total number of demand points that needs to be cover by the allocated ambulance, r .

For the decision making process, we use a binary variable as follows.

$$y_{jl} = \begin{cases} 1; & \text{if and only if the ambulance } l \text{ is allocated to } v_j \in W \\ 0; & \text{otherwise} \end{cases} \quad (3.3)$$

$$x_i^k = \begin{cases} 1; & \text{if and only if } v_i \text{ is relocated } k \text{ times} \\ 0; & \text{otherwise} \end{cases} \quad (3.4)$$

Thus, our propose relocation model can be formulated as follows.

$$\max \sum_{i=1}^n \lambda_i x_i^2 - \sum_{j=1}^m \sum_{l=1}^p M_{jl}^t y_{jl} \quad (3.5)$$

$$\text{s.t} \quad \sum_{j=1}^m \sum_{l=1}^p \gamma_{ij} y_{jl} \geq 1, \forall v_i \in V \quad (3.6)$$

$$\sum_{i=1}^n \lambda_i x_i^1 \geq \alpha \sum_{i=1}^n \lambda_i \quad (3.7)$$

$$\sum_{j=1}^m \sum_{l=1}^p \gamma_{ij} y_{jl} \geq x_i^1 + x_i^2, \forall v_i \in V \quad (3.8)$$

$$x_i^2 \leq x_i^1, \forall v_i \in V \quad (3.9)$$

$$\sum_{j=1}^m y_{jl} = 1, l = 1, \dots, p \quad (3.10)$$

$$\sum_{l=1}^p y_{jl} \leq p_j, \forall v_j \in W \quad (3.11)$$

$$x_i^1 = 0 \text{ atau } 1 \quad (3.12)$$

$$x_i^2 = 0 \text{ atau } 1 \quad (3.13)$$

$$y_{jl} = 0 \text{ atau } 1 \quad (3.14)$$

In our relocation model, constraint (3.6) and (3.7) showed the covered demand rates by r_2 . Constraint (3.8) and (3.9) showed the relative needs of ambulance demand rates that can be covered where the proportion α of the covered demand points. Eq. (3.10) is a binary variable where Eq. (3.11) showed an upper limit of the total number of the "idle-time" ambulance in a certain demand location. For some solutions, we use genetics algorithm concept that developed by Aytug and Saydam [20] as follows.

a. Average Response Time (ART)

One of important parameter of EMS component is the average response time to the demand points of ambulance that defined as an interval time between the demand calls and time when the ambulance arrives at the demand location. As an illustration, assume that we have 3-months of period with total number of calls is 300. The total time, $T = 4050$ minutes, to cover all the demand points. Thus, the ART is

$$\text{ART} = \frac{\text{total number of calls}}{\text{total time}} = \frac{4050}{300} = 13.5 \text{ minutes}$$

It means that the average response time to the demand calls of ambulance in 3-months of period is 13.5 minutes.

b. Coverage radius

Assume that the first coverage radius, r_1 , is calculated from the 3-month of period with ART is 13.5 minutes. The second coverage radius, r_2 , will be based on the standard

response time on the 10 minutes for all emergency calls. Given the known allowable constant ambulance speed of $V = 40$ km/h. For each r_1 and r_2 we determined

$$r_1 = \frac{V \times \text{ART}}{60} = \frac{40 \times 13.5}{60} = 9 \text{ km}$$

$$r_2 = \frac{V \times \text{ART}}{60} = \frac{40 \times 10}{60} = 6.67 \text{ km}$$

c. System-wide probability

Based on the illustration above, the total period of 3-months is 300 days that converting to minutes and equal to 43400 minutes. Thus, the probability for each ambulance in this relocation model is

$$p = \frac{T}{D_i \times NA} = \frac{4050}{300 \times 450} = 0.03$$

with assumption that $D_i = 300$ and $NA = 450$ is denoted as the given total number of available ambulance. It means the probability for each available number of ambulance is 0.03 for the relocation in a certain location.

IV. PROCEDURES OF ALGORITHM

We used the procedures concepts of algorithm that suggested by Farahani et al. [21] in determining the relocation ambulance. This algorithm then we modified based on our model Eq. (3.5) – (3.14) as follows.

Step 1. Set $w_i(t)$ as a time function based on the period. Set $t_1 = 0$ and $t_n = T$ as location-relocation points of n total of demand points. Calculate

$$w_i^j = \int_{t_j}^{t_{j+1}} w_i(t) dt$$

Step 2. Set $w_i^{jk} = \int_{t_j}^{t_k} w_i(t) dt$, $j < k$. For each demand point

i , calculate w_i^{jk} for all j and k that integrated with the weight of the graph at i demand location in interval $[t_j, t_k)$.

Step 3. For each interval $[b_j, b_k)$, calculate the optimal score at facility (x_{jk}, y_{jk}) with w_i^{jk} and coordinate of ambulance location is (a_i, b_i) . Thus, we determine the optimal solution.

Step 4. If the current relocation point is the same demand location at interval $[t_j, t_k)$, we calculate C_{jk} , cost of relocation ambulance that formulated by

$$C_{jk} = \sum_{i=1}^m w_i^{jk} d(X^{jk}, P_i)$$

where $d(X^{jk}, P_i)$ is distance between optimal location and the new relocation of demand points i for $[t_j, t_k)$ and X^{jk} is calculated by Step 3.

Step 5. Then, we calculate the relocation ambulance in time t , based on Eq. (3.5) – (3.14).

V. CONCLUSIONS

In this paper we propose a new relocation model of ambulance by modifying the MEXCLP model. The model is formulated by considering the probability of each demand points that covered by the ambulance facility. It also based on a simulation model of emergency medic facility system, that is total number of emergency calls and unit of available ambulances that have been located. Then, the extended MEXCLP represented into linear program with 0-1 constraint.

REFERENCES

- [1] Erdogan, G., Erkut, E., Ingolfsson, A. and Laporte, G. *Scheduling ambulance crews for maximum coverage*, Canadian Natural Sciences, and Engineering Research Council, (2008).
- [2] Zaharudin, Z.A, Shuib, A., Shahidin, A.M. and Nordin, N.A. *A goal programming model for ambulance location problem - a preliminary study*, Proceeding Seminar Kebangsaan Sains Matematika, Vol. 17 (2009), pp. 463-469.
- [3] Amponsah, S.K, Amoako, G., Darkwah, K.F. and Agyeman, E. *Location of ambulance emergency medical service in the Kumasi metropolis, Ghana*, African Journal of Mathematics and Computer Science Research, Vol. 41 (2010), pp. 18-26.
- [4] Brotcorne, L, Laporte, G. dan Semet, F. *Ambulance location and relocation models*. European Journal of Operational Research, Vol. 147 (2003), pp. 451-463.
- [5] Kolesar, P. and Walker, W.E. *An algorithm for the dynamic relocation of fire companies*, Operation Research, Vol. 22 (1974), pp. 249-274.
- [6] Gendreau, M. and Laporte, G. *A dynamic model and parallel tabu search heuristic for real-time ambulance relocation*, Parallel Computing, Vol. 27 (2001), pp. 1641-1653.
- [7] Nair, R. and Miller-Hooks, E. *A case study of ambulance location and relocation*, Presentation at 2006 INFORMS conference, (2006).
- [8] Gendreau, M. Laporte, G., and Semet, S. *The maximal expected coverage relocation problem for emergency vehicles*, Journal of the Operational Research Society, Vol.57 (2006), pp. 22-28.
- [9] Ingolfsson, A. *The impact of ambulance system status management*, Presentation at 2006 INFORMS conference, (2006)
- [10] Goldberg, J.B. *Personal communication*, (2007).
- [11] Berman, O. *Dynamic repositioning of indistinguishable service units on transportation networks*, Transportation Science, Vol. 15 (1981), pp. 115-136.

- [12] Zhang, O., Mason, A.J., Philpott, A.B. *Simulation and optimization for ambulance logistics and relocation*, Presentation at the INFORMS 2008 conference, (2008).
- [13] Andersson, T. *Decision support tools for dynamic fleet management*, PhD thesis, Department Science and Technology Linköpings Universiteit, Norrköping, Sweden.
- [14] Andersson, T. and Vaerband, P. *Decision support tools for ambulance dispatch and relocation*, Journal of Operational Research Society, Vol. 58 (2007), pp. 195-201.
- [15] Daskin, M.S. *A maximum expected location model: formulation, properties and heuristic location*, Transportation Science, Vol. 7 (1983), pp. 48-70.
- [16] Karaman, M. *A genetic algorithm for the multi-level maximal covering ambulance location problem*, PhD thesis, Middle East Technical University (METU), Turkey, (2008).
- [17] Mandell, M.B. *Covering models for two-tiered emergency medical service systems*, Location Science, Vol. 6 (1998), pp.355-368.
- [18] Goldberg, J., Dietrich, R., Chen, J.M. and Mitwasi, M.G. *Validating and applying a model for location emergency medical services in Tuscon, AZ*, European Journal of Operational Research, Vol. 49 (1990), pp. 308-324.
- [19] Repede, J.F. and Bernardo, J.J. *Developing and validating a decision support system for location emergency medical vehicles in Louisville, Kentucky*, European Journal of Operational Research, Vol. 75 (1994), pp. 567-581.
- [20] Aytug, H. and Saydam, C. *Solving large-scale maximum expected covering location problems by genetic algorithms: A comparative study*, European Journal of Operational Research, Vol. 141 (2002), pp. 480-494.
- [21] Farahani, R.Z. and Hekmatfar, M. *Facility location: concepts, models, algorithms and case studies*, Contribution to Management Science, Physica-Verlag Heidelberg, (2009).

IJERT