

# A Novel Approach on Performance Analysis of MIMO Using Space Time Block Coded Spatial Domain

R.Venkatesh<sup>1</sup>, P.N.V.Ramana<sup>2</sup>, V.Rama Krishna<sup>3</sup>

<sup>1</sup>B.Tech (ECE) Student, Department of ECE, St Ann's engineering college, chirala, A.P., India

<sup>2</sup>B.Tech (ECE) Student, Department of ECE, St Ann's engineering college, chirala, A.P., India

<sup>3</sup>Assistant prof, Department of ECE, St Ann's engineering college, chirala, A.P., India

**Abstract**—Space-time block codes have been shown to perform well with Multiple-Input Multiple Output (MIMO) systems. STBC is a MIMO transmit strategy which exploits transmit diversity and high reliability. Multiple-input multiple-output (MIMO) transmission scheme, called space-time block coded spatial modulation (STBC-SM), is proposed. It combines spatial modulation (SM) and space-time block coding (STBC) to take advantage of the benefits of both while avoiding their drawbacks. In the STBCSM scheme, the transmitted information symbols are expanded not only to the space and time domains but also to the spatial (antenna) domain which corresponds to the on/off status of the transmit antennas available at the space domain, and therefore both core STBC and antenna indices carry information. A general technique is presented for the design of the STBC-SM scheme for any number of transmits antennas. Besides the high spectral efficiency advantage provided by the antenna domain, the proposed scheme is also optimized by deriving its diversity and coding gains to exploit the diversity advantage of STBC. A low-complexity maximum likelihood (ML) decoder is given for the new scheme which profits from the orthogonality of the core STBC. The performance advantages of the STBC-SM over simple SM and over V-BLAST are shown by simulation results for various spectral efficiencies and are supported by the derivation of a closed form expression for the union bound on the bit error probability.

**Keywords**—Multiple-Input Multiple-Output(MIMO),Maximum likelihood decoding(ML),Space-time block codes/coding, Spatial modulation

## I.INTRODUCTION

MIMO technology means multiple antennas at both the ends of a communication system, that is, at the transmitting end and receiving end. The idea behind MIMO is that the transmit antennas at one end and the receive antennas at the other end are connected and combined in such a way that the bit error rate (BER), or the data rate for each user is improved. MIMO has the capacity of producing independent parallel channels and transmitting multipath data streams and thus meets the demand for high data rate wireless transmission. This system can provide high frequency spectral efficiency and is a promising approach with tremendous potential. The use of multiple antennas at both transmitter and receiver has been shown to be an effective way to improve capacity and reliability over those achievable with single antenna wireless systems [1]. Consequently, multiple-input multiple-output (MIMO) transmission techniques have been comprehensively studied over the past decade by numerous researchers, and two general MIMO transmission strategies, a space-time block coding (STBC) and spatial multiplexing, have been proposed. The increasing demand for high data rates and, consequently,

high spectral efficiencies has led to the development of spatial multiplexing systems such as V-BLAST (Vertical-Bell Lab Layered Space-Time) [2]. In V-BLAST systems, a high level of inter-channel interference (ICI) occurs at the receiver since all antennas transmit their own data streams at the same time. This further increases the complexity of an optimal decoder exponentially, while low-complexity sub optimum linear decoders, such as the minimum mean square error (MMSE) decoder, degrade the error performance of the system significantly. On the other hand, STBCs offer an excellent way to exploit the potential of MIMO systems because of their implementation simplicity as well as their low decoding complexity [3], [4]. A special class of STBCs, called orthogonal STBCs (OSTBCs), has attracted attention due to their single-symbol maximum likelihood (ML) receivers with linear decoding complexity. However it has been shown that the symbol rate of an OSTBC is upper bounded by  $\frac{3}{4}$  symbols per channel use (PCU) for more than two transmit antennas [5]. Several high rate STBCs have been proposed in the past decade (see [6]-[8] and references therein), but their ML decoding complexity grows exponentially with the constellation size, which makes their implementation difficult and expensive for future wireless communication systems. The basic idea of SM is an extension of two dimensional signal constellations (such as  $M$ -ary phase shift keying ( $M$ -PSK) and  $M$ -ary quadrature amplitude modulation ( $M$ -QAM), where  $M$  is the constellation size) to a third dimension, which is the spatial (antenna) dimension. Therefore, the information is conveyed not only by the amplitude/phase modulation (APM) techniques, but also by the antenna indices. An optimal ML decoder for the SM scheme, which makes an exhaustive search over the aforementioned three dimensional space has been presented in [11]. It has been shown in [11] that the error performance of the SM scheme [9] can be improved approximately in the amount of 4 dB by the use of the optimal detector under conventional channel assumptions and that SM provides better error performance than V-BLAST and maximal ratio combining (MRC). More recently, Jeganathan *et al.* have introduced a so-called space shift keying (SSK) modulation scheme for MIMO channels in [12]. In SSK modulation, APM is eliminated and only antenna indices are used to transmit information, to obtain further simplification in system design and reduction in decoding complexity. This leads to the introduction here of Space Time Block Coded Spatial Modulation (STBCSM), designed to take advantage of both SM and STBC.

The main contributions of this paper can be summarized as follows:

- A new MIMO transmission scheme, called STBC-SM, is proposed, in which information is conveyed with an STBC

matrix that is transmitted from combinations of the transmit antennas of the corresponding MIMO system. The Alamouti code [3] is chosen as the target STBC to exploit. As a source of information, we consider not only the two complex information symbols embedded in Alamouti's STBC, but also the indices (positions) of the two transmit antennas employed for the transmission of the Alamouti STBC.

- A general technique is presented for constructing the STBC-SM scheme for any number of transmit antennas. Since our scheme relies on STBC, by considering the general STBC performance criteria proposed by Tarokh *et al.* [14], diversity and coding gain analyses are performed for the STBC-SM scheme to benefit the second order transmit diversity advantage of the Alamouti code.

- A low complexity ML decoder is derived for the proposed STBC-SM system, to decide on the transmitted symbols as well as on the indices of the two transmits antennas that are used in the STBC transmission.

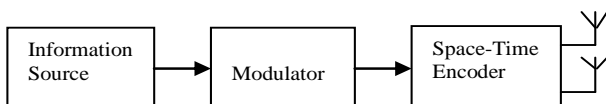
- It is shown by computer simulations that the proposed STBC-SM scheme has significant performance advantages over the SM with an optimal decoder, due to its diversity advantage. A closed form expression for the union bound on the bit error probability of the STBCSM scheme is also derived to support our results. The derived upper bound is shown to become very tight with increasing signal-to-noise (SNR) ratio.

The organization of the paper is as follows. In Section II, we introduce our STBC-SM transmission scheme via an example with four transmit antennas, give a general STBC-SM design technique for  $m$  transmit antennas, and formulate the optimal STBC-SM ML detector. In Section III we introduce Alamouti STBC. In Section IV, the performance analysis of the STBC-SM system is presented. Simulation results and performance comparisons are presented in Section V. Finally, Section VI includes the main conclusions of the paper.

*Notation:* Bold lowercase and capital letters are used for column vectors and matrices, respectively.  $(\cdot)^*$  and  $(\cdot)^H$  denote complex conjugation and Hermitian transposition, respectively. For a complex variable  $x$ ,  $\Re\{x\}$  denotes the real part of  $x$ .  $\mathbf{0}_{m \times n}$  denotes the  $m \times n$  matrix with all-zero elements.  $\|\cdot\|$ ,  $\text{tr}(\cdot)$  and  $\det(\cdot)$  stand for the Frobenius norm, trace and determinant of a matrix, respectively. The probability of an event is denoted by  $P(\cdot)$  and  $E\{\cdot\}$  represents expectation. The union of sets  $A_1$  through  $A_n$  is written as  $\cup_{i=1}^n A_i$ . We use  $\binom{n}{k}$ ,  $\lfloor x \rfloor$ , and  $\lceil x \rceil$  for the binomial coefficient, the largest integer less than or equal to  $x$ , and the smallest integer larger than or equal to  $x$ , respectively. We use  $\lfloor x \rfloor_2$  for the largest integer less than or equal to  $x$ , that is an integer power of 2.  $\gamma$  denotes a complex signal constellation of size  $M$ .

## II. SPACE-TIME BLOCK CODED SPATIAL MODULATION (STBC-SM)

In the STBC-SM scheme, both STBC symbols and the indices of the transmit antennas from which these symbols are transmitted, carry information.



**FIG. 1: BLOCK DIAGRAM OF SPACE-TIME CODING**

We choose Alamouti's STBC, which transmits one symbol PCU, as the core STBC due to its advantages in terms of spectral efficiency and simplified ML detection. In Alamouti's STBC, two complex information symbols ( $x_1$  and  $x_2$ ) drawn from an  $M$ -PSK or  $M$ -QAM constellation are transmitted from two transmit antennas in two symbol intervals in an orthogonal manner by the codeword

$$\mathbf{X} = (\mathbf{x}_1 \ \mathbf{x}_2) = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} \quad (1)$$

where columns and rows correspond to the transmit antennas and the symbol intervals, respectively. For the STBC SM scheme we extend the matrix in (1) to the antenna domain.

### III. Alamouti's STBC

Alamouti's scheme was the first STBC that provides full diversity at full data rate for two transmit antennas. This scheme has full rate (i.e. a rate of 1) since it transmits two symbols every two time intervals. The information bits are first modulated using a digital modulation scheme, and then the encoder takes the block of two modulated symbols  $s_1$  and  $s_2$  in each encoding operation. Here we adopt multilevel modulation. First, we modulate  $m$  ( $m = \log_2 M$ ) bits as a group, then the channel encoder will get two modulated signals  $s_1, s_2$  as a group each time when encoding, and map the two signals into the transmit antennas.

Let us introduce the concept of STBC-SM via the following simple example.

*Example (STBC-SM with four transmits antennas, BPSK modulation):*

Consider a MIMO system with four transmit antennas which transmits the Alamouti STBC using one of the following four codeword's:

$$\begin{aligned} \chi_1 &= \{\mathbf{X}_{11}, \mathbf{X}_{12}\} = \left\{ \begin{pmatrix} x_1 & x_2 & 0 & 0 \\ -x_2^* & x_1^* & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & x_1 & x_2 \\ 0 & 0 & -x_2^* & x_1^* \end{pmatrix} \right\} \\ \chi_2 &= \{\mathbf{X}_{21}, \mathbf{X}_{22}\} = \left\{ \begin{pmatrix} 0 & x_1 & x_2 & 0 \\ 0 & -x_2^* & x_1^* & 0 \end{pmatrix}, \begin{pmatrix} x_2 & 0 & 0 & x_1 \\ x_1^* & 0 & 0 & -x_2^* \end{pmatrix} \right\} e^{j\theta} \end{aligned} \quad (2)$$

Where  $\chi_i, i = 1, 2$  are called the STBC-SM codebooks each containing two STBC-SM codeword's  $\mathbf{X}_{ij}, j = 1, 2$  which do not interfere to each other. The resulting STBC-S code is  $\chi = \cup_{i=1}^2 \chi_i$ . A non-interfering codeword group having  $a$  elements is defined as a group of codeword's satisfying  $\mathbf{X}_{ij} \mathbf{X}_{ik}^H = \mathbf{0}_{2 \times 2}, j, k = 1, 2, \dots, a, j \neq k$ ; that is they have no overlapping columns. In (2),  $\theta$  is a rotation angle to be optimized for a given modulation format to ensure maximum diversity and coding gain at the expense of expansion of the signal constellation. However, if  $\theta$  is not considered, overlapping columns of codeword pairs from different codebooks would reduce the transmit diversity order to one. Assume now that we have four information bits ( $u_1, u_2, u_3, u_4$ ) to be transmitted in two consecutive symbol intervals by the STBCSM technique. The mapping rule for 2 bits/s/Hz transmission is given by Table I for the codebooks of (2) and for binary phase-shift keying (BPSK) modulation, where a realization of any codeword is called a transmission matrix. In Table I, the first two information bits ( $u_1, u_2$ ) are used to determine the antenna-pair position  $\theta$  while the last

two ( $u_3, u_4$ ) determine the BPSK symbol pair. If we generalize this system to  $M$ -ary signaling, we have four different codeword's each having  $M/2$  different realizations. Consequently, the spectral efficiency of the STBC-SM scheme for four transmit antennas becomes  $m = (1/2) \log_2 4M/2 = 1 + \log_2 M$  bits/s/Hz, where the factor  $1/2$  normalizes for the two channel uses spanned by the matrices in (2). For STBCs using larger numbers of symbol.

TABLE I  
STBC-SM MAPPING RULE FOR 2 BITS/s/Hz TRANSMISSION USING BPSK, FOUR TRANSMIT ANTENNAS AND ALAMOUTI'S STBC

	Input Bits	Transmission Matrices		Input Bits	Transmission Matrices	
$\chi_1$	$\ell = 0$	0000	$\begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}$	$\ell = 2$	1000	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix} e^{j\theta}$
		0001	$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$		1001	$\begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} e^{j\theta}$
		0010	$\begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix}$		1010	$\begin{pmatrix} 0 & -1 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{pmatrix} e^{j\theta}$
		0011	$\begin{pmatrix} -1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$		1011	$\begin{pmatrix} 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} e^{j\theta}$
	$\ell = 1$	0100	$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$	$\ell = 3$	1100	$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix} e^{j\theta}$
		0101	$\begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$		1101	$\begin{pmatrix} -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} e^{j\theta}$
		0110	$\begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$		1110	$\begin{pmatrix} 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 \end{pmatrix} e^{j\theta}$
		0111	$\begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$		1111	$\begin{pmatrix} -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix} e^{j\theta}$

Intervals such as the quasi-orthogonal STBC [15] for four transmit antennas which employs four symbol intervals, the spectral efficiency will be degraded substantially due to this normalization term since the number of bits carried by the antenna modulation ( $\log_2 c$ ), (where  $c$  is the total number of antenna combinations) is normalized by the number of channel uses of the corresponding STBC. A. *STBC-SM System Design and Optimization* In this subsection, we generalize the STBC-SM scheme for MIMO systems using Alamouti's STBC to  $n_T$  transmit antennas by giving a general design technique. An important design parameter for quasi-static Rayleigh fading channels is the minimum coding gain distance (CGD) [15] between two STBC-SM codeword's  $\mathbf{X}_{ij}$  and  $\hat{\mathbf{X}}_{ij}$ , where  $\mathbf{X}_{ij}$  is transmitted and  $\hat{\mathbf{X}}_{ij}$  is erroneously detected, is defined as

$$\delta_{\min}(\mathbf{X}_{ij}, \hat{\mathbf{X}}_{ij}) = \min_{\mathbf{X}_{ik}, \hat{\mathbf{X}}_{jl}} \det(\mathbf{X}_{ij} - \hat{\mathbf{X}}_{ij})(\mathbf{X}_{ij} - \hat{\mathbf{X}}_{ij})^H. \tag{3}$$

The minimum CGD between two codebooks  $\chi_i$  and  $\chi_j$  is defined as

$$\delta_{\min}(\chi_i, \chi_j) = \min_{k,l} \delta_{\min}(\mathbf{X}_{ik}, \mathbf{X}_{jl}) \tag{4}$$

and the minimum CGD of an STBC-SM code is defined by

$$\delta_{\min}(\chi) = \min_{i,j,i \neq j} \delta_{\min}(\chi_i, \chi_j). \tag{5}$$

Note that,  $\delta_{\min}(\chi)$  corresponds to the determinant criterion given in [14] since the minimum CGD between non interfering codeword's of the same codebook is always greater than or equal to the right hand side of (5). Unlike in the SM scheme, the number of transmit antennas in the STBC-SM scheme need not be an integer power of 2, since the pair wise combinations are chosen from  $n_T$  available transmit antennas for STBC transmission. This provides design flexibility.

However, the total number of codeword combinations considered should be an integer power of 2. In the following, we give an algorithm to design the STBC-SM scheme:

- 1) Given the total number of transmit antennas  $n_T$ , calculate the number of possible antenna combinations for the transmission of Alamouti's STBC, i.e., the total number of STBC-SM codeword's from  $c = \binom{n_T}{2} 2^p$ , where  $p$  is a positive integer.
- 2) Calculate the number of codeword's in each codebook  $\chi_i, i = 1, 2, \dots, n-1$  from  $a = \lfloor n_T/2 \rfloor$  and the total number of codebooks from  $n = \lceil c/a \rceil$ . Note that the last codebook  $\chi_n$  does not need to have  $a$  codeword's, i.e., its cardinality is  $a = c - a(n-1)$ .
- 3) Start with the construction of  $\chi_1$  which contains  $a$  no interfering codeword's as

$$\chi_1 = \left\{ \begin{pmatrix} \mathbf{X} & \mathbf{0}_{2 \times (n_T-2)} \\ \mathbf{0}_{2 \times 2} & \mathbf{X} & \mathbf{0}_{2 \times (n_T-4)} \\ \mathbf{0}_{2 \times 4} & \mathbf{X} & \mathbf{0}_{2 \times (n_T-6)} \\ \vdots \\ \mathbf{0}_{2 \times 2(a-1)} & \mathbf{X} & \mathbf{0}_{2 \times (n_T-2a)} \end{pmatrix} \right\} \tag{6}$$

where  $\mathbf{X}$  is defined in (1). 4) Using a similar approach, construct  $\chi_i$  for  $2 \leq i \leq n$  by considering the following two important facts: · Every codebook must contain non-interfering codeword's chosen from pair wise combinations of  $n_T$  available transmit antennas. · Each codebook must be composed of codeword's with antenna combinations that were never used in the construction of a previous codebook.

5) Determine the rotation angles  $\theta_i$  for each  $\chi_i, 2 \leq i \leq n$ , that maximize  $\delta_{\min}(\chi)$  in (5) for a given signal constellation and antenna configuration; that is  $\theta_{opt} = \arg \max_{\theta} \delta_{\min}(\chi)$ , where  $\theta = (\theta_2, \theta_3, \dots, \theta_n)$ . As long as the STBC-SM codeword's are generated by the algorithm described above, the choice of other antenna combinations is also possible but this would not improve the overall system performance for uncorrelated channels Since we have  $c$  antenna combinations, the resulting spectral efficiency of the STBC-SM scheme can be calculated as.

$$m = \frac{1}{2} \log_2 c + \log_2 M \text{ [bits/s/Hz].}$$

The block diagram of the STBC-SM transmitter is shown in Fig. 2

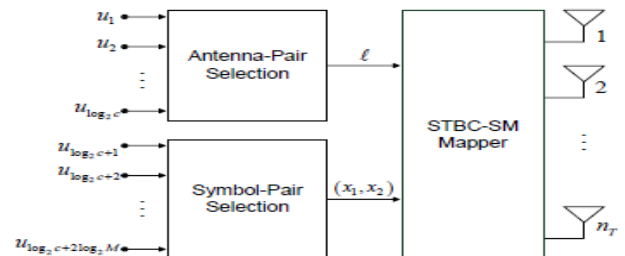


Fig. 2 Block diagram of the STBC-SM transmitter.

· During each two consecutive symbol intervals,  $2m$  bits  $u = (u_1, u_2, \dots, u_{\log_2 c}, u_{\log_2 c+1}, \dots, u_{\log_2 c+2\log_2 M})$

enter the STBC-SM transmitter, where the first  $\log_2 c$  bits determine the antenna-pair position  $\ell = u12^{\log_2 c-1} + u22^{\log_2 c-2} + \dots + u\log_2 c 2^0$  that is associated with the corresponding antenna pair, while the last  $2\log_2 M$  bits determine the symbol pair  $(x_1, x_2) \in \mathcal{X}_2$ . If we compare the spectral efficiency (7) of the STBC-SM scheme with that of Alamouti's scheme ( $\log_2 M$  bits/s/Hz), we observe an increment of  $1/2\log_2 c$  bits/s/Hz provided by the antenna modulation. We consider two different cases for the optimization of the STBC-SM scheme.

**Case 1 -  $nr \leq 4$ :** We have, in this case, two codebooks  $\mathcal{X}_1$  and  $\mathcal{X}_2$  and only one non-zero angle, say  $\theta$ , to be optimized. It can be seen that  $\delta_{\min}(\mathcal{X}_1, \mathcal{X}_2)$  is equal to the minimum CGD between any two interfering code words from  $\mathcal{X}_1$  and  $\mathcal{X}_2$ . Without loss of generality, assume that the interfering Code words are chosen as

$$\begin{aligned} \mathbf{X}_{1k} &= (\mathbf{x}_1 \ \mathbf{x}_2 \ \mathbf{0}_{2 \times (nr-2)}) \\ \mathbf{X}_{2l} &= (\mathbf{0}_{2 \times 1} \ \hat{\mathbf{x}}_1 \ \hat{\mathbf{x}}_2 \ \mathbf{0}_{2 \times (nr-3)}) e^{j\theta} \end{aligned} \quad (8)$$

where  $\mathbf{X}_{1k} \in \mathcal{X}_1$  is transmitted and  $\mathbf{X}_{2l} \in \mathcal{X}_2$  is erroneously detected. We calculate the minimum CGD between  $\mathbf{X}_{1k}$  and  $\mathbf{X}_{2l}$  from (3) as  $\delta_{\min}(\mathbf{X}_{1k}, \mathbf{X}_{2l})$

$$\begin{aligned} &= \min_{\mathbf{X}_{1k}, \mathbf{X}_{2l}} \det \begin{pmatrix} x_1 & x_2 - e^{j\theta} \hat{x}_1 & -e^{j\theta} \hat{x}_2 & \mathbf{0}_{1 \times (nr-3)} \\ -x_2^* & x_1^* + e^{j\theta} \hat{x}_1^* & -e^{j\theta} \hat{x}_2^* & \mathbf{0}_{1 \times (nr-3)} \\ x_1^* & -x_2 & -e^{-j\theta} \hat{x}_1 & \mathbf{0}_{(nr-3) \times 1} \\ x_2^* - e^{-j\theta} \hat{x}_1^* & x_1 + e^{-j\theta} \hat{x}_2 & -e^{-j\theta} \hat{x}_1 & \mathbf{0}_{(nr-3) \times 1} \end{pmatrix} \\ &= \min_{\mathbf{X}_{1k}, \mathbf{X}_{2l}} \{ (\kappa - 2\Re\{\hat{x}_1^* x_2 e^{-j\theta}\}) (\kappa + 2\Re\{x_1 \hat{x}_2 e^{j\theta}\}) \\ &\quad - |x_1|^2 |\hat{x}_1|^2 - |x_2|^2 |\hat{x}_2|^2 + 2\Re\{x_1 \hat{x}_1 x_2^* \hat{x}_2^* e^{j2\theta}\} \} \quad (9) \end{aligned}$$

Where  $\kappa = \sum_{i=1}^2 (|x_i|^2 + |\hat{x}_i|^2)$ . Although maximization of  $\delta_{\min}(\mathbf{X}_{1k}, \mathbf{X}_{2l})$  with respect to  $\theta$  is analytically possible for BPSK and quadrature phase-shift keying (QPSK) constellations, it becomes unmanageable for 16-QAM and 64-QAM which are essential modulation formats for the next generation wireless standards such as LTE-advanced and WiMAX. We compute  $\delta_{\min}(\mathbf{X}_{1k}, \mathbf{X}_{2l})$  as a function of  $\theta \in [0, \pi/2]$  for BPSK, QPSK, 16-QAM and 64-QAM signal constellations via computer search and plot them in Fig. 2. These curves are denoted by  $f_M(\theta)$  for  $M = 2, 4, 16$  and  $64$ , respectively.  $\theta$  values maximizing these functions can be determined from fig 3 follows:

$$\max_{\theta} \delta_{\min}(\mathcal{X}) = \begin{cases} \max_{\theta} f_2(\theta) = 12, & \text{if } \theta = 1.57 \text{ rad} \\ \max_{\theta} f_4(\theta) = 11.45, & \text{if } \theta = 0.61 \text{ rad} \\ \max_{\theta} f_{16}(\theta) = 9.05, & \text{if } \theta = 0.75 \text{ rad} \\ \max_{\theta} f_{64}(\theta) = 8.23, & \text{if } \theta = 0.54 \text{ rad.} \end{cases}$$

**Case 2 -  $nr > 4$ :** In this case, the number of codebooks,  $n$ , is greater than 2. Let the corresponding rotation angles to be optimized be denoted in ascending order by  $\theta_1 = 0 < \theta_2 < \theta_3 < \dots < \theta_n < p\pi/2$ , where  $p = 2$  for BPSK and  $p = 1$  for QPSK. For BPSK and QPSK signaling, choosing

$$\theta_k = \begin{cases} \frac{(k-1)\pi}{n}, & \text{for BPSK} \\ \frac{(k-1)\pi}{2n}, & \text{for QPSK} \end{cases} \quad (10)$$

for  $1 \leq k \leq n$  guarantees the maximization of the minimum CGD for the STBC-SM scheme. This can be explained as follows. For any  $n$ , we have to maximize  $\delta_{\min}(\mathcal{X})$  as

$$\begin{aligned} \max \delta_{\min}(\mathcal{X}) &= \max_{i,j,i \neq j} \min \delta_{\min}(\chi_i, \chi_j) \\ &= \max_{i,j,i \neq j} \min f_M(\theta_j - \theta_i) \end{aligned} \quad (11)$$

where  $\theta_j > \theta_i$  for  $j > i$  and the minimum CGD between codebooks  $\chi_i$  and  $\chi_j$  is directly determined by the difference between their rotation angles. This can be easily verified from (9) by choosing the two interfering codeword's as  $\mathbf{X}_{ik} \in \chi_i$  and  $\mathbf{X}_{jl} \in \chi_j$  with the rotation angles  $\theta_i$  and  $\theta_j$ , respectively. Then, to maximize  $\delta_{\min}(\mathcal{X})$ , it is sufficient to maximize the minimum CGD between the consecutive codebooks  $\chi_i$  and  $\chi_{i+1}$ ,  $i = 1, 2, \dots, n-1$ . For QPSK signaling, this is accomplished by dividing the interval  $[0, \pi/2]$  into  $n$  equal sub-intervals and choosing, for  $i = 1, 2, \dots, n-1$ ,

$$\theta_{i+1} - \theta_i = \pi/2n \quad (12)$$

The resulting maximum  $\delta_{\min}(\mathcal{X})$  can be evaluated from (11)

$$\begin{aligned} \max \delta_{\min}(\mathcal{X}) &= \min \{f_4(\theta_2), f_4(\theta_3), \dots, f_4(\theta_n)\} \\ &= f_4(\theta_2) = f_4\left(\frac{\pi}{2n}\right). \end{aligned} \quad (13)$$

Similar results are obtained for BPSK signaling except that  $\pi/2n$  is replaced by  $\pi/n$  in (12) and (13). We obtain the corresponding maximum  $\delta_{\min}(\mathcal{X})$  as  $f_2(\theta_2) = f_2(\pi/n)$ . On the other hand, for 16-QAM and 64-QAM signaling, the selection of  $\{\theta_k\}$ 's in integer multiples of  $\pi/2n$  would not guarantee to maximize the minimum CGD for the STBC-SM scheme since the behavior of the functions  $f_{16}(\theta)$  and  $f_{64}(\theta)$

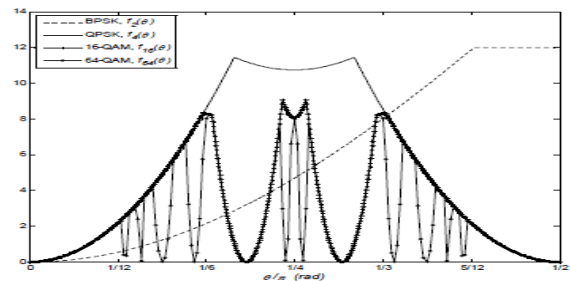


Fig. 3. Variation of  $\delta_{\min}(\mathcal{X})$  given in (9) for BPSK, QPSK, 16-QAM and 64-QAM ( $f_2(\theta)$ ,  $f_4(\theta)$ ,  $f_{16}(\theta)$  and  $f_{64}(\theta)$ )

TABLE II  
BASIC PARAMETERS OF THE STBC-SM SYSTEM FOR DIFFERENT NUMBER OF TRANSMIT ANTENNAS

$n_T$	$c$	$a$	$n$	$\delta_{\min}(\mathcal{X})$			$m$ [bits/s/Hz]
				$M = 2$	$M = 4$	$M = 16$	
3	2	1	2	12	11.45	9.05	$0.5 + \log_2 M$
4	4	2	2	12	11.45	9.05	$1 + \log_2 M$
5	8	2	4	4.69	4.87	4.87	$1.5 + \log_2 M$
6	8	3	3	8.00	8.57	8.31	$1.5 + \log_2 M$
7	16	3	6	2.14	2.18	2.18	$2 + \log_2 M$
8	16	4	4	4.69	4.87	4.87	$2 + \log_2 M$

is very non-linear, having several zeros in  $[0, \pi/2]$ . However, our extensive computer search has indicated that for 16-QAM with  $n \leq 6$ , the rotation angles chosen as  $\theta_k = (k-1)\pi/2n$  for  $1 \leq k \leq n$  are still optimum. But for 16-QAM signaling with  $n > 6$  as well as for 64-QAM signaling with  $n > 2$ , the optimal  $\{\theta_k\}$ 's must be determined by an exhaustive computer search. In Table II, we summarize the basic parameters of the STBC-SM system for  $3 \leq nr \leq 8$ . We observe that increasing the number of transmit antennas results in an increasing number of antenna combinations and, consequently, increasing spectral efficiency achieved by the STBC-SM scheme. However, this requires a larger number of angles to be optimized and causes some reduction in the minimum CGD. On the other hand, when the same number of combinations can be supported by different numbers of transmit antennas, a higher number of transmit antennas requires fewer angles to be optimized resulting in higher minimum CGD (for an example, the cases  $c = 8$ ,  $nr = 5$  and 6 in Table II). We now give two examples for the codebook generation process of the STBC-SM design algorithm, presented above. *Design Example 1:* From Table II, for  $nr = 6$ , we have  $c = 8$ ,  $a = n = 3$  and the optimized angles are  $\theta_2 = \pi/3$ ,  $\theta_3 = 2\pi/3$  for BPSK and  $\theta_2 = \pi/6$ ,  $\theta_3 = \pi/3$  for QPSK and 16-QAM. The maximum of  $\delta_{\min}(\chi)$  is calculated for BPSK, QPSK and 16-QAM constellations as

$$\max_{\theta} \delta_{\min}(\chi) = \begin{cases} f_2(\pi/3) = 8.00, & \text{for BPSK} \\ f_4(\pi/6) = 8.57, & \text{for QPSK} \\ f_{16}(\pi/6) = 8.31, & \text{for 16-QAM.} \end{cases}$$

According to the design algorithm, the codebooks can be constructed as below,

$$\begin{aligned} \chi_1 &= \{(x_1 \ x_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}), (\mathbf{0} \ \mathbf{0} \ x_1 \ x_2 \ \mathbf{0} \ \mathbf{0}), (\mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_1 \ x_2)\} \\ \chi_2 &= \{(\mathbf{0} \ x_1 \ x_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}), (\mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_1 \ x_2 \ \mathbf{0}), (x_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_1)\} e^{j\theta_2} \\ \chi_3 &= \{(x_1 \ \mathbf{0} \ x_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}), (\mathbf{0} \ x_1 \ \mathbf{0} \ x_2 \ \mathbf{0} \ \mathbf{0})\} e^{j\theta_3} \end{aligned}$$

w ( here  $\mathbf{0}$  denotes the  $2 \times 1$  all-zero vector. Since there are  $\binom{6}{2} = 15$  possible antenna combinations, 7 of them are discarded to obtain 8 codeword's. Note that the choice of other combinations does not affect  $\delta_{\min}(\chi)$ . In other words, the codebooks given above represent only one of the possible realizations of the STBC-SM scheme for six transmit antennas.

*Design Example 2:* From Table II, for  $nr = 8$ , we have  $c = 16$ ,  $a = n = 4$  and optimized angles are  $\theta_2 = \pi/4$ ,  $\theta_3 = \pi/2$ ,  $\theta_4 = 3\pi/4$  for BPSK and  $\theta_2 = \pi/8$ ,  $\theta_3 = \pi/4$ ,  $\theta_4 = 3\pi/8$  for QPSK and 16-QAM. Similarly,  $\max \delta_{\min}(\chi)$  is calculated for BPSK, QPSK and 16-QAM constellations as

$$\max_{\theta} \delta_{\min}(\chi) = \begin{cases} f_2(\pi/4) = 4.69, & \text{for BPSK} \\ f_{4/16}(\pi/8) = 4.87, & \text{for QPSK \& 16-QAM.} \end{cases}$$

According to the design algorithm, the codebooks can be constructed as follows:

$$\begin{aligned} \chi_1 &= \{(x_1 \ x_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}), (\mathbf{0} \ \mathbf{0} \ x_1 \ x_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}), \\ & \quad (\mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_1 \ x_2 \ \mathbf{0} \ \mathbf{0}), (\mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_1 \ x_2)\} \\ \chi_2 &= \{(\mathbf{0} \ x_1 \ x_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}), (\mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_1 \ x_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}), \\ & \quad (\mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_1 \ x_2 \ \mathbf{0} \ \mathbf{0}), (x_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_1)\} e^{j\theta_2} \\ \chi_3 &= \{(x_1 \ \mathbf{0} \ x_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}), (\mathbf{0} \ x_1 \ \mathbf{0} \ x_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}), \\ & \quad (\mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_1 \ \mathbf{0} \ x_2 \ \mathbf{0}), (\mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_1 \ \mathbf{0} \ x_2)\} e^{j\theta_3} \\ \chi_4 &= \{(x_1 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}), (\mathbf{0} \ x_1 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_2 \ \mathbf{0} \ \mathbf{0}), \\ & \quad (\mathbf{0} \ \mathbf{0} \ x_1 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_2 \ \mathbf{0}), (\mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_1 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ x_2)\} e^{j\theta_4}. \end{aligned}$$

### B. Optimal ML Decoder for the STBC-SM System

In this subsection, we formulate the ML decoder for the STBC-SM scheme. The system with  $nr$  transmit and  $nr$  receive antennas is considered in the presence of a quasi-static Rayleigh flat fading MIMO channel. The received  $2 \times nr$  signal matrix  $\mathbf{Y}$  can be expressed as

$$\mathbf{Y} = \sqrt{\frac{\rho}{\mu}} \mathbf{X}_{\chi} \mathbf{H} + \mathbf{N} \quad (14)$$

where  $\mathbf{X}_{\chi} \in \chi$  is the  $2 \times nr$  STBC-SM transmission matrix, transmitted over two channel uses and  $\mu$  is a normalization factor to ensure that  $\rho$  is the average SNR at each receive antenna.  $\mathbf{H}$  and  $\mathbf{N}$  denote the  $nr \times nr$  channel matrix and  $2 \times nr$  noise matrixes, respectively. The entries of  $\mathbf{H}$  and  $\mathbf{N}$  are assumed to be independent and identically distributed (i.i.d.) complex Gaussian random variables with zero means and unit variances. We assume that  $\mathbf{H}$  remains constant during the transmission of a codeword and takes independent values from one codeword to another. We further assume that  $\mathbf{H}$  is known at the receiver, but not at the transmitter.

Assuming  $nr$  transmit antennas are employed, the STBCSM code has  $c$  codeword's, from which  $cM_2$  different transmission matrices can be constructed. An ML decoder must make an exhaustive search over all possible  $cM_2$  transmission matrices, and decides in favor of the matrix that minimizes the following metric:

$$\hat{\mathbf{X}}_{\chi} = \arg \min_{\mathbf{X}_{\chi} \in \chi} \left\| \mathbf{Y} - \sqrt{\frac{\rho}{\mu}} \mathbf{X}_{\chi} \mathbf{H} \right\|^2. \quad (15)$$

The minimization in (15) can be simplified due to the orthogonality of Alamouti's STBC as follows. The decoder can extract the embedded information symbol vector from (14), and obtain the following equivalent channel model:

$$\mathbf{y} = \sqrt{\frac{\rho}{\mu}} \mathcal{H}_{\chi} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \mathbf{n} \quad (16)$$

where  $\mathcal{H}_{\chi}$  is the  $2nr \times 2$  equivalent channel matrix [16] of the Alamouti coded SM scheme, which has  $c$  different realizations according to the STBC-SM codeword's. In (16),  $\mathbf{y}$  and  $\mathbf{n}$  represent the  $2nr \times 1$  equivalent received signal and noise vectors, respectively. Due to the orthogonality of Alamouti's STBC, the columns of  $\mathcal{H}_{\chi}$  are orthogonal to each other for all cases and, consequently, no ICI occurs in our scheme as in the case of SM. Consider the STBC-SM transmission model as described in Table I for four transmit antennas. Since there are  $c = 4$  STBC-SM codeword's, as seen from Table II, we have four different realizations for  $\mathcal{H}_{\chi}$ , which are given for  $nr$  receive antennas as

$$\begin{aligned} \mathcal{H}_0 &= \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{1,2}^* & -h_{1,1}^* \\ h_{2,1} & h_{2,2} \\ h_{2,2}^* & -h_{2,1}^* \\ \vdots & \vdots \\ h_{nr,1} & h_{nr,2} \\ h_{nr,2}^* & -h_{nr,1}^* \end{bmatrix}, \mathcal{H}_1 = \begin{bmatrix} h_{1,3} & h_{1,4} \\ h_{1,4}^* & -h_{1,3}^* \\ h_{2,3} & h_{2,4} \\ h_{2,4}^* & -h_{2,3}^* \\ \vdots & \vdots \\ h_{nr,3} & h_{nr,4} \\ h_{nr,4}^* & -h_{nr,3}^* \end{bmatrix}, \\ \mathcal{H}_2 &= \begin{bmatrix} h_{1,2\varphi} & h_{1,3\varphi} \\ h_{1,3\varphi}^* & -h_{1,2\varphi}^* \\ h_{2,2\varphi} & h_{2,3\varphi} \\ h_{2,3\varphi}^* & -h_{2,2\varphi}^* \\ \vdots & \vdots \\ h_{nr,2\varphi} & h_{nr,3\varphi} \\ h_{nr,3\varphi}^* & -h_{nr,2\varphi}^* \end{bmatrix}, \mathcal{H}_3 = \begin{bmatrix} h_{1,4\varphi} & h_{1,1\varphi} \\ h_{1,1\varphi}^* & -h_{1,4\varphi}^* \\ h_{2,4\varphi} & h_{2,1\varphi} \\ h_{2,1\varphi}^* & -h_{2,4\varphi}^* \\ \vdots & \vdots \\ h_{nr,4\varphi} & h_{nr,1\varphi} \\ h_{nr,1\varphi}^* & -h_{nr,4\varphi}^* \end{bmatrix} \quad (17) \end{aligned}$$

where  $h_{i,j}$  is the channel fading coefficient between transmit antenna  $j$  and receive antenna  $i$  and  $\varphi = e^{j\theta}$ . Generally, we have  $c$  equivalent channel matrices  $\mathcal{H}_{\ell}$ ,  $0 \leq \ell < c-1$ , and for the  $\ell$ th combination, the receiver determines the ML estimates

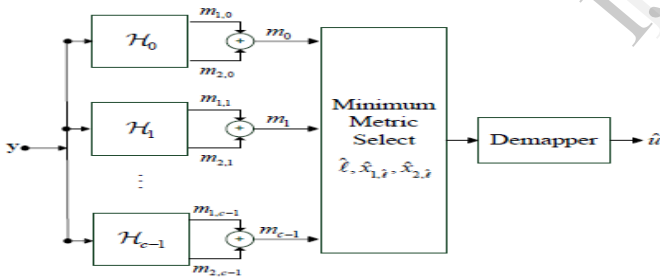
of  $x_1$  and  $x_2$  using the decomposition as follows [17], resulting from the orthogonality of  $\mathbf{h}_{\ell,1}$  and  $\mathbf{h}_{\ell,2}$ :

$$\begin{aligned} \hat{x}_{1,\ell} &= \arg \min_{x_1 \in \gamma} \left\| \mathbf{y} - \sqrt{\frac{\rho}{\mu}} \mathbf{h}_{\ell,1} x_1 \right\|^2 \\ \hat{x}_{2,\ell} &= \arg \min_{x_2 \in \gamma} \left\| \mathbf{y} - \sqrt{\frac{\rho}{\mu}} \mathbf{h}_{\ell,2} x_2 \right\|^2 \end{aligned} \quad (18)$$

where  $\mathcal{H}_\ell = [\mathbf{h}_{\ell,1} \ \mathbf{h}_{\ell,2}]$ ,  $0 \leq \ell \leq c-1$ , and  $\mathbf{h}_{\ell,j}$ ,  $j = 1, 2$ , is a  $2nr \times 1$  column vector. The associated minimum ML metrics  $m_{1,\ell}$  and  $m_{2,\ell}$  for  $x_1$  and  $x_2$  are

$$\begin{aligned} m_{1,\ell} &= \min_{x_1 \in \gamma} \left\| \mathbf{y} - \sqrt{\frac{\rho}{\mu}} \mathbf{h}_{\ell,1} x_1 \right\|^2 \\ m_{2,\ell} &= \min_{x_2 \in \gamma} \left\| \mathbf{y} - \sqrt{\frac{\rho}{\mu}} \mathbf{h}_{\ell,2} x_2 \right\|^2 \end{aligned} \quad (19)$$

respectively. Since  $m_{1,\ell}$  and  $m_{2,\ell}$  are calculated by the ML decoder for the  $\ell$ th combination, their summation  $m_\ell = m_{1,\ell} + m_{2,\ell}$ ,  $0 \leq \ell \leq c-1$  gives the total ML metric for the  $\ell$ th combination. Finally, the receiver makes a decision by choosing the minimum antenna combination metric as  $\hat{\ell} = \arg \min_{\ell} m_\ell$  for which  $(\hat{x}_1, \hat{x}_2) = (\hat{x}_1, \hat{\ell}, \hat{x}_2, \hat{\ell})$ . As a result, the total number of ML metric calculations in (15) is reduced from  $cM_2$  to  $2cM$ , yielding a linear decoding complexity as is also true for the SM scheme, whose optimal decoder requires  $Mnr$  metric calculations. Obviously, since  $c \geq nr$  for  $nr \geq 4$ , there will be a linear increase in ML decoding complexity with STBC-SM as compared to the SM scheme. However, as we will show in the next section, this insignificant increase in decoding complexity is rewarded with significant performance improvement provided by the STBC-SM over SM. The last step of the decoding process is the de-mapping operation based on the look-up table used at the transmitter, to recover the input bits  $\hat{u} = (\hat{u}_1, \dots, \hat{u}_{2\log_2 c}, \hat{u}_{2\log_2 c+1}, \dots, \hat{u}_{2\log_2 c+2\log_2 M})$  from the determined spatial position (combination)  $\hat{\ell}$  and the information symbols  $\hat{x}_1$  and  $\hat{x}_2$ . The block diagram of the ML decoder described above is given in Fig. 4



**Fig. 4. Block diagram of the STBC-SM ML receiver.**  
IV. PERFORMANCE ANALYSIS OF THE STBC-SM SYSTEM

In this section, we analyze the error performance of the STBC-SM system, in which  $2m$  bits are transmitted during two consecutive symbol intervals using one of the  $cM_2 = 22m$  different STBC-SM transmission matrices, denoted by  $\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_{22m}$  here for convenience. An upper bound on the average bit error probability (BEP) is given by the well known union bound [18]:

$$P_b \leq \frac{1}{2^{2m}} \sum_{i=1}^{2^{2m}} \sum_{j=1}^{2^{2m}} \frac{P(\mathbf{X}_i \rightarrow \mathbf{X}_j) n_{i,j}}{2m} \quad (20)$$

Where  $P(\mathbf{X}_i \rightarrow \mathbf{X}_j)$  is the pair wise error probability (PEP) of deciding STBC-SM matrix  $\mathbf{X}_j$  given that the STBC-SM matrix  $\mathbf{X}_i$  is transmitted, and  $n_{i,j}$  is the number of bits in error

between the matrices  $\mathbf{X}_i$  and  $\mathbf{X}_j$ . Under the normalization  $\mu = 1$  and  $E \{ \text{tr}(\mathbf{X}_i^H \mathbf{X}_j) \} = 2$  in (14), the conditional PEP of the STBC-SM system is calculated as

$$P(\mathbf{X}_i \rightarrow \mathbf{X}_j | \mathbf{H}) = Q \left( \sqrt{\frac{\rho}{2}} \|(\mathbf{X}_j - \mathbf{X}_i) \mathbf{H}\| \right) \quad (21)$$

Where  $Q(x) = (1/\sqrt{2\pi}) \int_{-\infty}^x e^{-y^2/2} dy$ . Averaging (21) over the channel matrix  $\mathbf{H}$  and using the moment generating function(MGF) approach [18], the unconditional PEP is obtained

$$P(\mathbf{X}_i \rightarrow \mathbf{X}_j) = \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{1}{1 + \frac{\rho \lambda_{i,j,1}}{4 \sin^2 \phi}} \right)^{nr} \left( \frac{1}{1 + \frac{\rho \lambda_{i,j,2}}{4 \sin^2 \phi}} \right)^{nr} d\phi \quad (22)$$

where  $\lambda_{i,j,1}$  and  $\lambda_{i,j,2}$  are the eigenvalues of the distance matrix  $(\mathbf{X}_i - \mathbf{X}_j)(\mathbf{X}_i - \mathbf{X}_j)^H$ . If  $\lambda_{i,j,1} = \lambda_{i,j,2} = \lambda_{i,j}$ , (22) simplifies to

$$P(\mathbf{X}_i \rightarrow \mathbf{X}_j) = \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{1}{1 + \frac{\rho \lambda_{i,j}}{4 \sin^2 \phi}} \right)^{2nr} d\phi \quad (23)$$

which is the PEP of the conventional Alamouti STBC [15]. Closed form expressions can be obtained for the integrals in (22) and (23) using the general formulas given in Section 5 and Appendix A of [18]. In case of  $c = an$ , for  $nr = 3$  and for an even number of transmit antennas when  $nr \geq 4$ , it is observed that all transmission matrices have the uniform error property due to the symmetry of STBC-SM codebooks, i.e., have the same PEP as that of  $\mathbf{X}_1$ . Thus, we obtain a BEP upper bound for STBC-SM as follows:

$$P_b \leq \sum_{j=2}^{2^{2m}} \frac{P(\mathbf{X}_1 \rightarrow \mathbf{X}_j) n_{1,j}}{2m} \quad (24)$$

Applying the natural mapping to transmission matrices,  $m_{i,j}$  can be directly calculated as  $m_{i,j} = w [(j-1)_2]$ , where  $w[x]$  and  $(x)_2$  are the Hamming weight and the binary representation of  $x$ , respectively. Consequently, from (24), we obtain the union bound on the BEP as

$$P_b \leq \sum_{j=2}^{2^{2m}} \frac{w [(j-1)_2]}{2m\pi} \int_0^{\pi/2} \left( \frac{1}{1 + \frac{\rho \lambda_{1,j,1}}{4 \sin^2 \phi}} \right)^{nr} \left( \frac{1}{1 + \frac{\rho \lambda_{1,j,2}}{4 \sin^2 \phi}} \right)^{nr} d\phi, \quad (25)$$

### V. SIMULATION RESULTS AND COMPARISONS

We present simulation results for the STBCSM system with different numbers of transmit antennas and make comparisons with SM, V-BLAST, rate-3/4 OSTBC for four transmit antennas [15], Alamouti's STBC, the Golden Code [19] and double space-time transmit diversity (DSTTD) scheme [20]. The bit error rate (BER) performance of these systems was evaluated by Monte Carlo simulations for various spectral efficiencies as a function of the average SNR per receive antenna ( $\rho$ ) and in all cases we assumed four receive antennas. All performance comparisons are made for a BER value of  $10^{-5}$ . The SM system uses the optimal decoder derived in [11]. The V-BLAST system uses MMSE detection with ordered successive interference cancellation (SIC) decoding where the layer with the highest post detection SNR is detected first, then nulled and the process is repeated for all layers, iteratively [21]. We employ ML decoders for both the Golden code and the DSTTD scheme. We first present the

BER performance curves of the STBCSM scheme with three and four transmit antennas for BPSK and QPSK constellations in Fig. 5. As a reference, the BEP upper bound curves of the STBC-SM scheme are also evaluated from (25) and depicted in the same figure. From Fig. 5 it follows that the derived upper bound becomes very tight with increasing SNR values for all cases and can be used as a helpful tool to estimate the error performance behavior of the STBC-SM scheme with different setups. Also note that the BER curves in Fig. 5 are shifted to the right while their slope remains unchanged and equal to  $2nr$ , with increasing spectral efficiency.

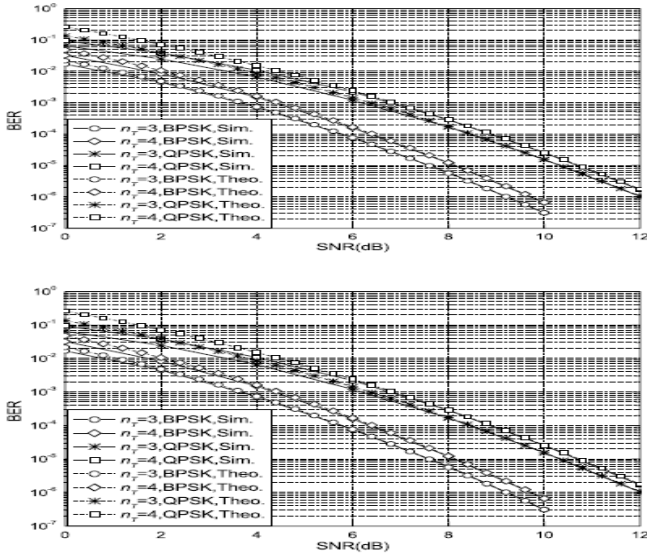


Fig. 5. BER performance of STBC-SM scheme for BPSK and QPSK compared with theoretical upper bounds.

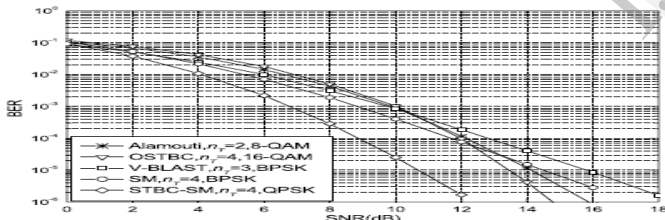


Fig. 6. BER performance at 3 bits/s/Hz for STBC-SM, SM, V-BLAST, OSTBC and Alamouti's STBC schemes

A. Comparisons with SM, V-BLAST, rate-3/4 OSTBC and Alamouti's STBC

In Fig. 6, the BER curves of STBC-SM with  $nr = 4$  and QPSK, SM with  $nr = 4$  and BPSK, V-BLAST with  $nr = 3$  and BPSK, OSTBC with 16-QAM and Alamouti's STBC with 8-QAM are evaluated for 3 bits/s/Hz transmission. We observe that STBC-SM provides SNR gains of 3.8 dB, 5.1 dB, 2.8 dB and 3.4 dB over SM, V-BLAST, OSTBC and Alamouti's STBC, respectively.

In Fig. 6, we employ two different STBC-SM schemes with  $nr = 8$  and QPSK, and  $nr = 4$  and 8-QAM (for the case  $nr \leq 4$ , the optimum rotation angle for rectangular 8-QAM is found from (9) as equal to 0.96 rad for which  $\delta_{\min}(\chi) = 11.45$ ) for 4 bits/s/Hz, and make comparisons with the following schemes: SM with  $nr = 8$  and BPSK, V-BLAST with  $nr = 2$  and QPSK, OSTBC with 32-QAM, and Alamouti's STBC

with 16-QAM. It is seen that STBCSM with  $nr = 8$  and QPSK provides SNR gains of 3.5 dB, 5 dB, 4.7 dB and 4.4 dB over, SM, V-BLAST, OSTBC and Alamouti's STBC, respectively. On the other hand, we observe 3 dB SNR gap between two STBC-SM schemes in favor of the one that uses a smaller constellation and relies more heavily on the use of the spatial domain to achieve 4 bits/s/Hz. This gap is also verified by the difference between normalized minimum CGD values of these two schemes. We conclude from this result that one can optimize the error performance without expanding the signal constellation but expanding the spatial constellation to improve spectral efficiency. However the number of required metric calculations for ML decoding of the first STBC-SM scheme is equal to 128 while the other one's is equal to 64, which provides an interesting trade-off between complexity and performance. Based on these examples, we conclude that for a given spectral efficiency, as the modulation order  $M$  increases, the number of transmit antennas  $nr$  should decrease, and consequently the SNR level needed for a fixed BER will increase while the overall decoding complexity will be reduced. On the other hand, as the modulation order  $M$  decreases, the number of transmit antennas  $nr$  should increase, and as a result the SNR level needed for a fixed BER will decrease while the overall decoding complexity increases.

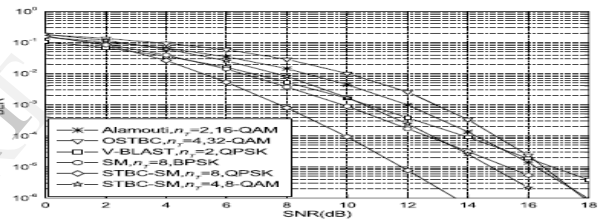


Fig. 7. BER performance at 4 bits/s/Hz for STBC-SM, SM, V-BLAST, OSTBC and Alamouti's STBC schemes

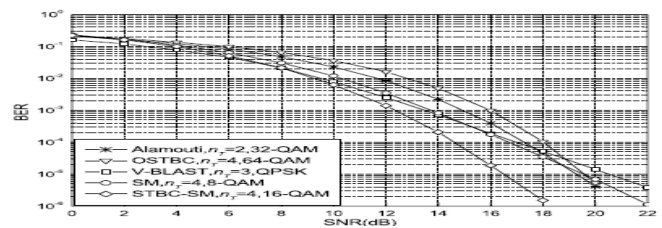


Fig. 8. BER performance at 5 bits/s/Hz for STBC-SM, SM, V-BLAST, OSTBC and Alamouti's STBC schemes

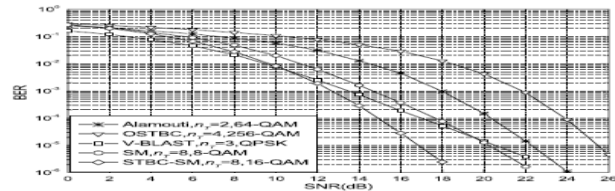


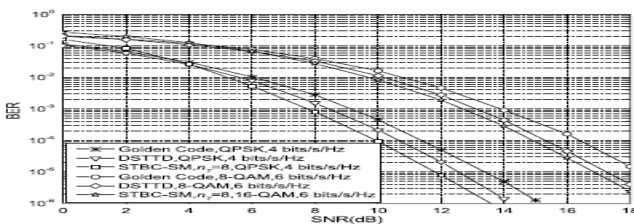
Fig. 9. BER performance at 6 bits/s/Hz for STBC-SM, SM, V-BLAST, OSTBC and Alamouti's STBC schemes.

SM scheme. We also observe that the BER performance of Alamouti's scheme can be greatly improved (approximately 3-5 dB depending on the transmission rate) with the use of the

spatial domain. Note that although having a lower diversity order, STBC-SM outperforms rate-3/4 OSTBC, since this OSTBC uses higher constellations to reach the same spectral efficiency as STBC-SM. Finally, it is interesting to note that in some cases, SM and V-BLAST systems are outperformed by Alamouti's STBC for high SNR values even at a BER of  $10^{-5}$ .

### B. Comparisons with the Golden code and DSTTD scheme

In Fig. 9, we compare the BER performance of the STBCSM scheme with the Golden code and DSTTD scheme which are rate-2 (transmitting four symbols in two time intervals) STBCs for two and four transmit antennas, respectively, at 4 and 6 bits/s/Hz. Although both systems have a brute-force ML decoding complexity that is proportional to the fourth power of the constellation size, by using low complexity ML decoders recently proposed in the literature, their worst



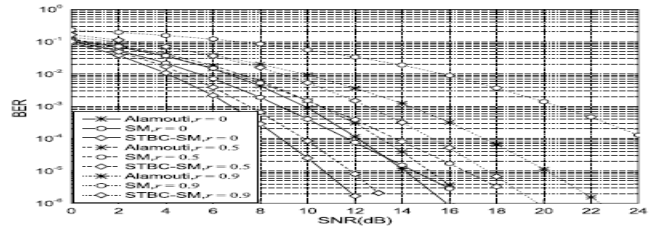
**Fig. 10. BER performance for STBC-SM, the Golden code and DSTTD schemes at 4 and 6 bits/s/Hz spectral efficiencies.**

Case ML decoding complexity can be reduced to  $2M/3$  from  $M/4$  for general  $M$ -QAM constellations, which we consider in our comparisons. MMSE decoding is widely used for the DSTTD scheme, however, we use an ML decoder to compare the pure performances of the considered schemes. From Fig. 10, we observe that STBC-SM offers SNR gains of 0.75 dB and 1.6 dB over the DSTTD scheme and the Golden code, respectively, at 4 bits/s/Hz, while having the same ML decoding complexity, which is equal to 128. On the other hand, STBC-SM offers SNR gains of 0.4 dB and 1.5 dB over the DSTTD scheme and the Golden code, respectively, at 6 bits/s/Hz, with 50% lower decoding complexity, which is equal to 512.

### C. STBC-SM Under Correlated Channel Conditions

Inadequate antenna spacing and the presence of local scatterers lead to spatial correlation (SC) between transmit and receive antennas of a MIMO link, which can be modeled by a modified channel matrix [22]  $\mathbf{H}_{corr} = \mathbf{R}_1^{1/2} \mathbf{H} \mathbf{R}_2^{1/2}$  where  $\mathbf{R}_t = [r_{ij}]_{n_t \times n_t}$  and  $\mathbf{R}_r = [r_{ij}]_{n_r \times n_r}$  are the SC matrices at the transmitter and the receiver, respectively. In our simulations, we assume that these matrices are obtained from the exponential correlation matrix model [23], i.e., their components are calculated as  $r_{ij} = r^*_{ji} = r^{|j-i|}$  for  $i \leq j$  where  $r$  is the correlation coefficients of the neighboring transmits and receive antennas' branches. This model provides a simple and efficient tool to evaluate the BER performance of our scheme under SC channel conditions. In Fig. 10, the BER curves for the STBC-SM with  $n_T = 4$  and QPSK, the SM with  $n_T = 4$  and BPSK, and the Alamouti's STBC with 8-QAM are shown for 3 bits/s/Hz spectral efficiency by  $r = 0, 0.5$  and  $0.9$ . As seen from Fig. 11, the BER performance of all schemes is degraded substantially by these correlations.

However, we observe that while the degradation of Alamouti's STBC and our scheme are comparable, the degradation for SM is higher. Consequently, we conclude that our scheme is more robust against spatial correlation than pure SM.



**Fig. 11. BER performance at 3 bits/s/Hz for STBC-SM, SM, and Alamouti's STBC schemes for SC channel with  $r = 0, 0.5$  and  $0.9$ .**

## VI. CONCLUSIONS

In this paper, we have introduced a novel high-rate, low complexity MIMO transmission scheme, called STBC-SM, as an alternative to existing techniques such as SM and VBLAST. The proposed new transmission scheme employs both APM techniques and antenna indices to convey information and exploits the transmit diversity potential of MIMO channels. A general technique has been presented for the construction of the STBC-SM scheme for any number of transmit antennas in which the STBC-SM system was optimized by deriving its diversity and coding gains to reach optimum performance. It has been shown via computer simulations and also supported by a theoretical upper bound analysis that the STBC-SM offers significant improvements in BER performance compared to SM and V-BLAST systems (approximately 3-5 dB depending on the spectral efficiency) with an acceptable linear increase in decoding complexity. From a practical implementation point of view, the RF (radio frequency) front-end of the system should be able to switch between different transmit antennas similar to the classical SM scheme. We conclude that the STBC-SM scheme can be useful for high-rate, low complexity, emerging wireless communication systems such as LTE and WiMAX. Our future work will be focused on the integration of trellis coding into the proposed STBC-SM scheme.

## REFERENCES

- [1] V. Tarokh, H. Jafarkhanl, "Space Time Block Codes from Orthogonal Design", IEEE Transaction on Information Theory, vol. 45, pp. 1456-1467, 1999.
- [2] H. Sampath, S. Talwar, J. Tellado, V. Erceg & A. Paulraj, "A fourth generation MIMO-OFDM broadband wireless system: design, performance, and filed trial results," IEEE Communication Magazine, vol. 40. no. 9, pp. 143-149, Sep 2002.
- [3] G. J. Foschini & M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," Wireless Personal Communication, vol.6, pp.311-335, March 1998.
- [4] I. E. Telatar, "Capacity of multi antenna gaussian channels," AT & T Bell Labs, Tech. Rep., Jun 1995.
- [5] E.S. Hassan et.al, "Enhanced performance of OFDM and single-carrier systems using frequency domain equalization and phase modulation," IEEE National Radio Science Conf., pp. 1-10, NewCairo, Egypt, 2009.