

A Review of Stress Analysis of Functionally Graded Material Plate with Cut-out

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Abstract— Here is a review about the investigations carried out on the topic of "Stress analysis of functionally graded materials with cut-outs". This Paper contains the information about the functionally graded Materials (FGMs), Types of FGM, manufacturing techniques of FGM different areas of applications of FGM, analytical and experimental solution techniques for FGMs, and different stress measuring techniques. Study of stress analysis will continue with changes in the parameters like, effect of stress analysis of FGM plate with isotropic and orthotropic material, change in loading conditions, effects of changes in radius of circular cutout.

Keywords—Circular cut-out, complex variable approach, Functionally Graded materials (FGMs), Stress concentration Factor

I. INTRODUCTION

Functionally Graded materials (FGM) is the class of composite materials, in which two or more than two materials are composited as per the function or application of the materials where it is going to be used. The composition of the materials is in continuously graded form. For example, a material required with high thermal resistance capacity at one side and high strength and toughness at another side. In this condition the required properties are very different. There is no material in nature which has good strength with brittleness. A material with good thermal resistance (like ceramics) having brittleness which can face the temperature resistance but, it cannot face the high mechanical loading. On the other side, the material with good toughness and strength (like metal) is with good ductility, which can face the mechanical loading conditions but for temperature resistance it is not suitable. In this case we can form a composite material which will participate as a metal at one side and as a ceramic at another side. In this case the FGM composed will be transformed from ceramic to metal from one end to another end in continuously graded form. So FGM is a composite material which is made up of two or more than two material combination with desired physical and chemical properties [1]. The main purpose of avoiding the abrupt changes in functionally graded material is to avoid the stress concentration factor [2]. The most common functionally graded materials are made of ceramic/metal non homogeneous structure, in which ceramic provides good thermal resistance and metal roles as a superior toughness and hardness.

A. History of FGM

First practical application of FGM was in 1984, during a space plane project was carried out in Japan. During the design of space plane body, requirement of a material to create 1000°F temperature difference within 10 mm plate [3]. In this case scientists required a material which can work as a good thermal resistor at the side nearer to the engine wall and as a good mechanical load resistor at the outer side. More investigation and research conducted for FGM in 1986, with special coordination funds for promoting science and technology. A national project was conducted in 1987, which named as 'FGM part I' which was ended in 1991. In 1990, the first international symposium on functionally graded materials was arranged in Sendai, Japan [9].

B. Types of FGM

FGM are classified according to different criteria like,

- (1) According to the structure, two types, one is continuously structured FGM and another is discontinues (Layered) FGM. Difference is clarified by the figure 1-(a) and figure 1-(b).

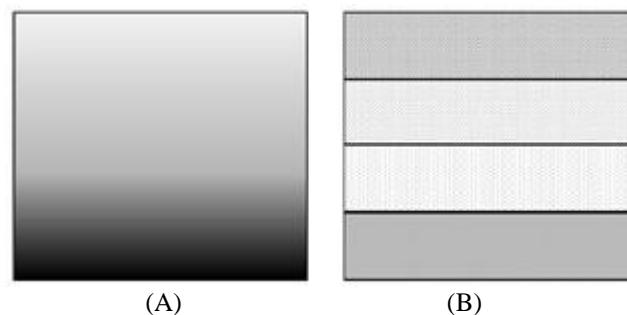


Figure 1 Continuous (A) and layered (B) type FGM

- (2) According to process of manufacturing, one type is Thin FGM and another type is Bulk FGM.

Thin FGM are manufactured with different methods like Physical Vapor Deposition (PVD), Chemical Vapor Deposition (CVD), Self propagating High temperature Synthesis (SHS) method etc. While the Bulk FGMs are

manufactured by the methods like, Powder Metallurgical Technique, Centrifugal Casting method, Solid Free Foam technique, etc [3]. Here we concentrate on bulk FGM only, so three main manufacturing techniques are described briefly.

In powder Metallurgy method for making bulk fgm, basic steps are weighing and mixing of powder, stacking and ramming of premixed powder, and finally sintering [1].

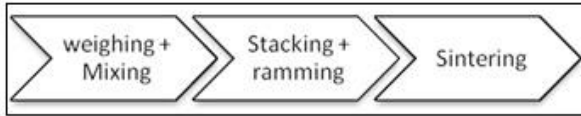


Figure 2 Powder Metallurgy Process

In Centrifugal method, gravitational force is used through spinning of the mould to form continuous bulk fgm [1].

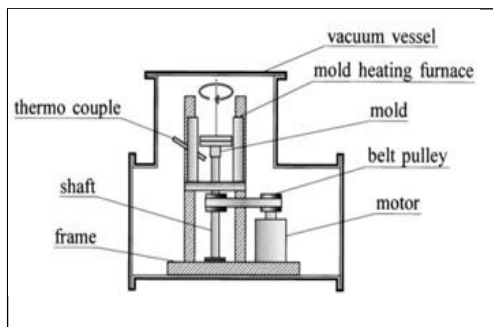


Figure 3 Centrifugal Method [4]

By the change in mould spinning speed and change in density of powder materials of metal will produces continuously graded structure. A limitation of the method is that, the gradient of materials cannot be tolerated as per requirement, because it depends naturally.

Solid Free Foam Fabrication method is a method in which first the modeling of the part is done, then converting the modeled file in .stl (standard triangulation language) file. Now slices of the .stl file model in to two dimensional cross sections. The next step is component building, and finally removal of unnecessary part & finishing. Generally in solid free foam fabrication method, laser based methods are used. Like, laser cladding based method, selective laser sintering (SLS), 3D printing method, Selective laser melting (SLM) etc [1].

This method for manufacturing fgm is most advantageous because it works at high speed production, less energy intensive, maximum material utilization, flexibility in modeling because we can directly convert the model from cad software, and complex shape formulation is also becomes easy by solid free foam fabrication method [5]. Limitation of this method is that the rough surface finishing.

A. Applications

Now a day's use of functionally graded materials is very popular, because of the flexibility to make the composite

material as per application requirement and functional suitability. We can see different areas of application of fgm in figure 5. Generally the area of application of functionally graded material is very large but, main application areas are aerospace projects, medicals, nuclear projects, energy sector, communication field and other miscellaneous uses.



Figure 4 Areas of applications for fgm

1. Aerospace projects: - FGM are used to make thermal barrier as a wall of plane (space plane frame), to resist the heat, generated at the outer surface in plane due to air friction. Here the reason to select FGM is because of a good thermal resistor. FGM are also useful to make rocket components and antifriction coating on turbine blade, to avoid crack, corrosion etc [1].
2. Medical Field: - With Medical way of view, FGM is very useful tool in dental and orthopedic fields. In dental, artificial teeth are used which are FGM, and in orthopedic artificial bones are used which are FGM as well. Even skin of a human body is also a functionally graded material [6].
3. Nuclear Projects :- In nuclear projects, a good thermal resisting material required, also a leak proof structure to avoid industrial accidents, which occurs due to fusion reaction process, so FGM is used. More use of FGM in fuel pallets and plasma wall of fusion reactor [3].
4. Energy sector: - FGM used in energy sector for the requirement of a good thermal barrier and protective coatings on turbine blades. Other usefulness of FGM in thermo electric generator, solar cell, sensors etc [7].
5. Communication field: - Optical fiber wires, which require a good electrical resistor at outer cover and a good electric transistor material property in inner side so, FGM is used. Even in lenses and semiconductors FGM become very useful.[3]

6. Other miscellaneous use: - Other uses of FGM are in cutting tool inserts coating, in heat exchanger, in tribology, in making of fire retardant doors, in defense pad making for inhibit crack propagation etc [1].

1.2 Effects of Stress concentration in FGM plate with cutout

Stress concentrations are deviations from the nominal stress on a part. What causes a stress concentration is an abrupt change in the flow of stress through a part. This abrupt change can cause a higher stress or a lower stress by a certain factor in comparison to the nominal stress. To study the effect of stress concentration and magnitude of localized stresses, a dimensionless factor called Stress Concentration Factor (SCF), K_t as defined by Eq. (1) is used.

$$K_t = \sigma_{\max} / \sigma_{\text{nom}} \quad (1)$$

Where, σ_{\max} is maximum stress at the continuity and σ_{nom} is nominal or background stress.

The stress concentration factor can be determined analytically by applying elasticity theory. For a large thin plate with a small circular hole at the center, subjected to uniaxial tension, σ , acting along the x-axis, the radial, circumferential and tangential stresses around the vicinity of the hole are given in polar coordinates (r, θ) as shown in figure 5,

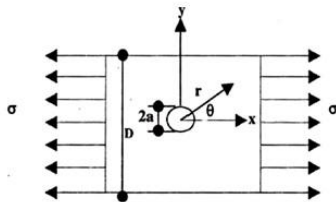


Figure 5 A thin large thin plates with circular hole at center under uni-axial loading

1.3 Measurement of stress

Researchers apply different value of forces on part in different loading conditions, to check the capacity of part. Resulted stress can be measured by different techniques of two main methods experimental and analytical. In experimental methods photo elasticity method, brittle coating method and electrical strain gauge methods are main, and for analytical methods Finite Element Method, Boundary Element Method and Complex Variable Approach are main [8].

II. LITERATURE REVIEW

There have been numerous research published in the area of stress analysis functionally graded materials. In this section, the review on stress analysis on FGM is focused on analytical studies and Complex variable approach. In an early study on review of FGM by Rashedat Mohammad [1] focused on the introduction about FGM, its types, its manufacturing techniques with areas of applications. William G. Cooley [2] guided the specialised application of FGM in Aerospace Engineering, with effect of stress concentration in FGM.

Fernando Ramirez, Paul R. Heyliger, and Ernian Pan [9] presented approximate solution for the static analysis of three-

dimensional, anisotropic, elastic plates composed of functionally graded materials (FGM). The solution is obtained by using a discrete layer theory in combination with the Ritz method in which the plate is divided into an arbitrary number of homogeneous and/or FGM layers. Two types of functionally graded materials are considered: an exponential variation of the mechanical properties through the thickness of the plate, and mechanical properties as a function of the fiber orientation, which varies quadratically through the laminate thickness. The method was validated by solving the problem of a single simply supported FGM plate, for which excellent agreement with the exact solution was obtained. Homogeneous, graded, and bi-layer plates examined in order to study potential advantages of using FGM.

Quanquan Yang, Cun-Fa Gao, and Wentao Chen [10] studied the stress distribution of a functional graded material plate (FGMP) with a circular hole under arbitrary constant loads. With using the method of piece-wise homogeneous layers, the stress distribution of the functional graded material plate having radial arbitrary elastic properties is derived based on the theory of the complex variable functions. It was concluded that the stress is greatly reduced as the radial Young's modulus increased, but it was less influenced by the variation of the Poisson's ratio. Also found that the stress level varies when the radial Young's modulus increased in different ways. Thus, it can be concluded that the stress around the circular hole in the FGMP can be effectively reduced by choosing the proper change ways of the radial elastic properties.

J. N. Reddy [11] presented theoretical formulation, Navier's solutions of rectangular plates, and finite element models based on the third- order shear deformation plate theory for the analysis of through-thickness functionally graded plates.

J. Sladeka, V. Sladeka, Ch. Zhangb [12] investigated, a meshless method based on the local Petrov-Galerkin approach is proposed for stress analysis in two-dimensional (2D), anisotropic and linear elastic/viscoelastic solids with continuously varying material properties. The correspondence principle is applied for nonhomogeneous, anisotropic and linear viscoelastic solids where the relaxation moduli are separable in space and time. A unit step function is used as the test functions in the local weak-form. It leads to local boundary integral equations (LBIEs). The analyzed domain is divided into small sub-domains with a circular shape. The moving least squares (MLS) method is adopted for approximating the physical quantities in the LBIEs. For time-dependent problems, the Laplace-transform technique is utilized.

D.V.Kubair, B.Bhanuchandra [13] numerically investigated the effect of the material property inhomogeneity on the stress concentration factor (SCF) due to a circular hole in functionally graded panels. The multiple isoparametric finite element formulation is used to simulate the elastostatic boundary value problem. A parametric study is performed by varying the functional form and the direction of the material property gradation. The material property inhomogeneity is

characterized by the intrinsic inhomogeneity length scale, modulus ratio and the power-law index. The results from parametric study showed that the SCF is reduced when Young's modulus progressively increased away from the hole. The angular position of the maximum tensile stress on the surface of the hole remains unaffected by the material property inhomogeneity. The SCF is seen to be most influenced by the power-law index, followed by the variation of the inhomogeneity length scale. The SCF is least affected by the modulus ratio.

Dharmendra S. Sharma [14] conducted general stress functions for determine the stress concentration around circular, elliptical and triangular cut-outs in laminated composite infinite plate subjected to arbitrary biaxial loading at infinity are obtained using Muskhelishvili's complex variable method. Victor Birman and Larry w. byrd [15] they include homogenization of particulate FGM, heat transfer issues, stress, stability and dynamic analyses, testing, manufacturing and design, applications, and fracture. The critical areas where further research is needed for a successful implementation of FGM in design are outlined in the conclusions.

D.K. Jha, Tarun Kant, R.K. Singh [16] attempted to identify and highlight the topics that are most relevant to FGMs and structures and review representative journal publications that are related to FGM topics. A critical review of the reported studies in the area of thermo-elastic and vibration analyses of functionally graded (FG) plates with an emphasis on the recent works published since 1998. The review carried out here, is concerned with deformation, stress, vibration and stability problems of FG plates. This review is intended to give the readers a feel for the variety of studies and applications related to graded composites. An effort has been made by researchers, to include all the important contributions in the current area of interest. The critical areas regarding future research needs for the successful implementation of FGM in design are outlined in the conclusions. By this paper conclusions came out that,

- 3D analytical solutions for FG plates are very useful since they provide benchmark results to assess the accuracy of various 2D plate theories and finite element formulations, but their solution methods involve mathematical complexities and are very difficult and tedious to solve.

- In most of the 2D theories developed to predict the global responses of FG plates, only the transverse shear deformation effect has been considered and very few theories consider the effect of both transverse shear and transverse normal deformations effect.

- In most of the 2D shear deformation theories developed till date, the validation and accuracy of the global responses of FG plates are done by comparing the results with 3D elasticity solutions. Very limited studies are reported on comparison of the accuracy with analytically predicted global responses of FG plates using various higher order theories.

- Having reviewed a large segment of the FGM research available it is apparent that nearly all the research conducted has been purely analytical or with numerical simulation.

- Use of improved 2D theoretical models which are now seem to provide accuracy as good as the 3D models should be pursued in the interest of computational cost and efficient analyses.

Mohammad Abedi and GH.Rahimi [17] introduced analytical solution of stress distribution around circular cutout in orthotropic Functionally Graded Composite (FGC) plate. By using the theory of complex variable functions, stress distributions around circular cutout of orthotropic FGC plate under plane stress conditions has been obtained. Variations of mechanical properties through the thickness have been considered.

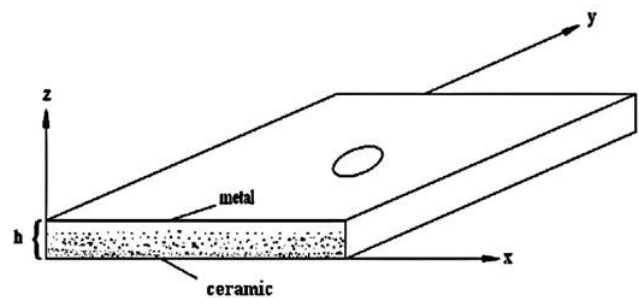


Figure 6:- Orthotropic plate with change in material properties in thickness

Also, variation of stresses through the cutout in orthotropic FGC plate has been presented. In the two cases, it is shown that the stress concentration in case of biaxial loading is less than that in case of uniaxial loading.

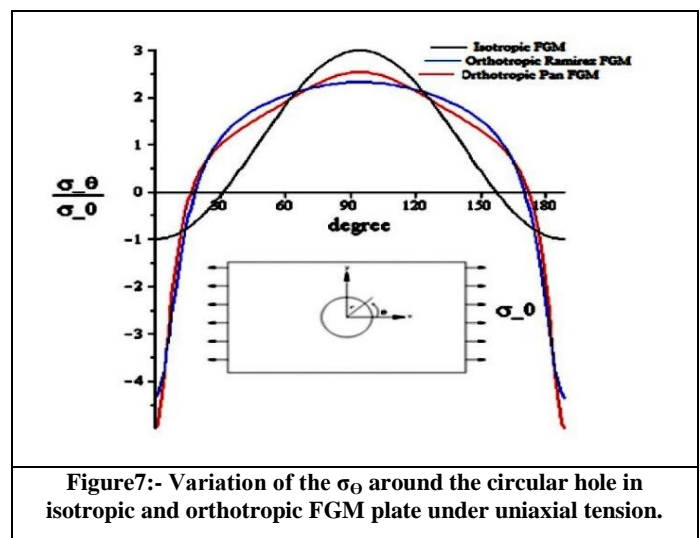


Figure7:- Variation of the σ_{θ} around the circular hole in isotropic and orthotropic FGM plate under uniaxial tension.

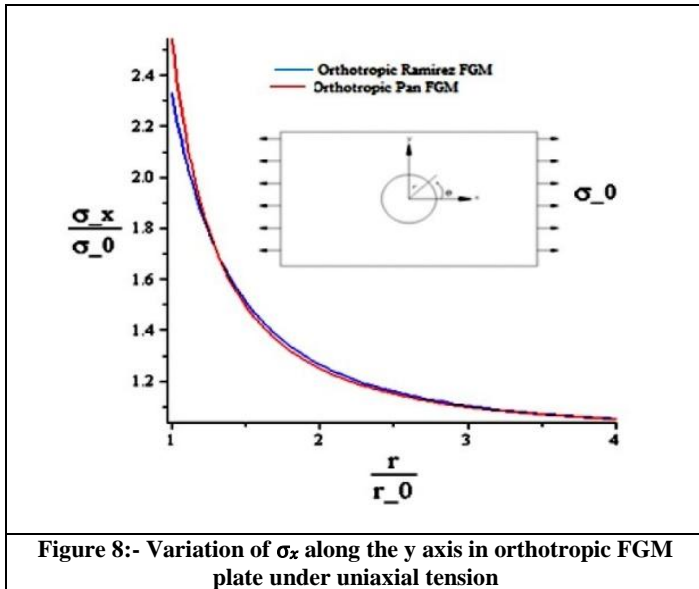


Figure 8:- Variation of σ_x along the y axis in orthotropic FGM plate under uniaxial tension

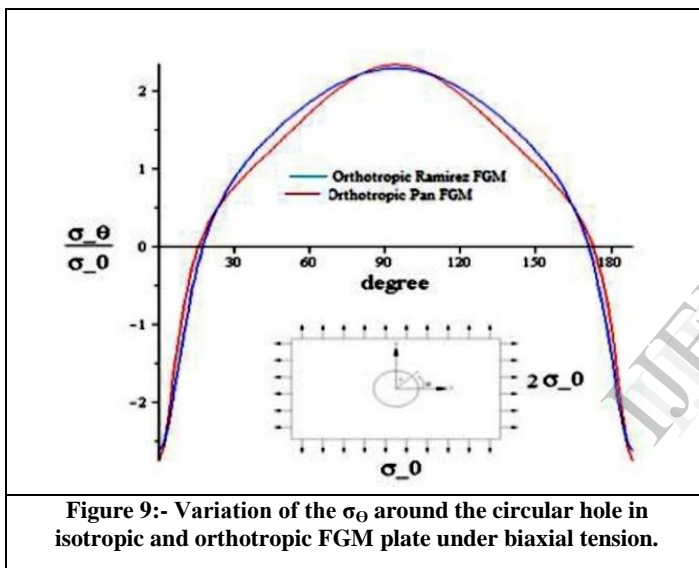


Figure 9:- Variation of the σ_θ around the circular hole in isotropic and orthotropic FGM plate under biaxial tension.

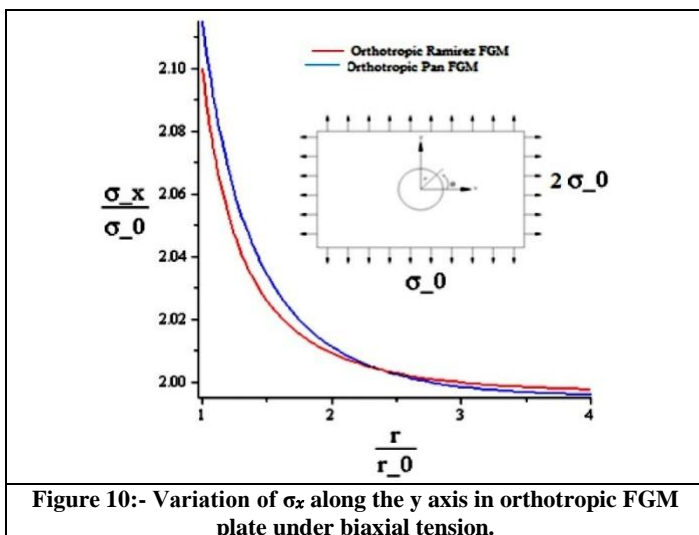


Figure 10:- Variation of σ_x along the y axis in orthotropic FGM plate under biaxial tension.

Moreover, the effect of material distribution on stress concentration was also considered, through which it was found that increase of coefficients of stiffness matrix, leads to decrease of the coefficient of stress concentration. Also, for orthotropic FGC plate, the effect of size of circular cutout on the stress distribution was examined.

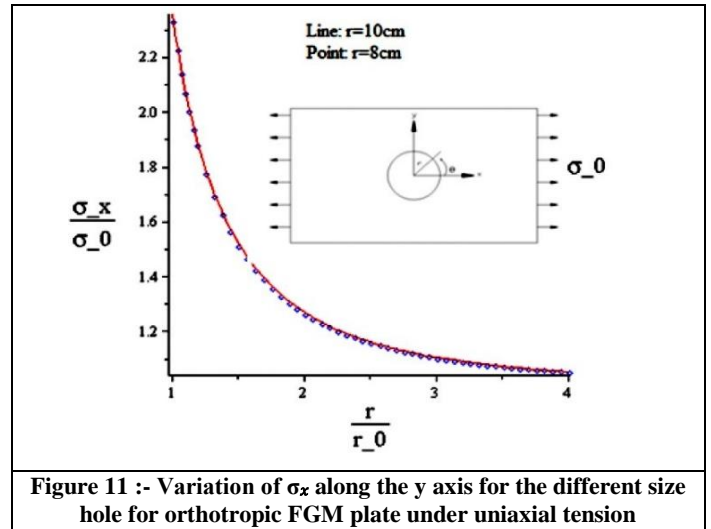


Figure 11 :- Variation of σ_x along the y axis for the different size hole for orthotropic FGM plate under uniaxial tension

Mohammad Abedi concluded that the stress concentration in case of biaxial loading is less than that in case of uniaxial loading and by considering the effect of material distribution on stress concentration, through which concluded that increment of coefficients of stiffness matrix, leads to declining the coefficient of stress concentration.

III. SUMMARY

In introduction of the paper we have seen what the Functionally Graded Materials (FGMs) is, and the history of FGMs with its types, different manufacturing techniques for FGMs, areas of application of FGMs, and effects of Stress concentration in FGM plate with cutout. Study continues with comparison of unidirectional and bidirectional stress in metal-ceramic type orthotropic FGM plate with hole. In future we can change the loading parameters like analysis for shear at infinity and internal pressure consideration at cut-out for different FGM compositions.

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