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A Solution for the Analysis of RC Framed Structure with Infill Frames under Dynamic Loads

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Abstract **- Keeping in line with the increased need for developing guidelines for the analysis and design of infilled frames in multistoried buildings under the effect of dynamic loads, we discuss in this manuscript an analysis technique for the seismic design of infilled frames, which has practical utilities. Infilled frames should be designed to withstand lateral forces that can result from seismic ground motion and the accuracy of the predicted forces depends on the calculated dynamic characteristics of the structures, namely natural frequencies, free vibration modes, and damping. Here we have modelled the infill in the reinforced concrete frame as diagonal strut or Finite Element model, the mass matrix and stiffness matrix is assembled and by the Eigen value analysis the time period could be found out. This presents a simple practical analysis technique for the seismic design of infilled frames, which can be used by practicing engineers.**

Key words: Infilled frames, RC framed structure, Finite Element model, Eigen value analysis, Natural time period

1. INTRODUCTION

The availability of improved analytical and experimental techniques in recent years have led to an increased recognition among structural engineers that elements of a building,which are generally considered "non-structural", may affect its characteristics especially under the effect of dynamic load. For eg., lateral loads can produce critical stress in a multistoried structure leading to undesirable vibrations during earth quakes[1-3]. Such lateral forces are a matter of great concern and need special considerations in the design of buildings. Reinforced concrete skeleton frames are filled with brick or concrete block masonry wallsin multistoried buildings to meet the architectural and functional requirements. In suchsituations, combination of frame and filler walls forms an 'infilled frame'. These infill walls,though constructed as secondary structural elements, behave as a constituent part of thestructural system and determine overall behavior of
the structure, especially when it issubjected to lateral loads[4]. However, designers tend to treat these infill walls as "non-structural" elements and treat the frames as conventional bare frames in practice, which isfar from representing the true behavior.

The neglect of infills in seismic design can be attributed to the common misconception thatmasonry infill in frames can only increase the overall lateral load capacity, and therefore,must always be beneficial to seismic performance[1]. However, a number of structuraldamages, as recorded in some of the recent earthquakes were traced to modifications of thestructural frame due to presence of infilled walls. Hence, there is increasing recognitionamong structural engineers that the infilled walls affect the characteristics of the buildingunder the effect of dynamic loads, specifically, wind and earthquake loads and should betaken into consideration in the analysis and design. Infilled frames, like any other structures should be designed to withstand lateral forces thatcan results from seismic ground motion[3]. The accuracy of the predicted forces depends onthe calculated dynamic characteristics of the structures, namely natural frequencies, freevibration modes, and damping[5]. The dynamic properties are significantly influenced by thepresence of infill. Empirical relationships, as proposed in design codes do not properlyaccount for the effect of infill. Furthermore, errors greater than ± 50% were observed when the natural frequencies, computed using empirical expressions, werecompared with measured natural frequencies [1]. Hence, the dynamic characteristics of aninfilled frame should be determined by accounting for the effect of infill.
The problem of building frames having infill panels with different materials and configurations has henceforthgenerated interest among researchers in this field. Frames could be steel, reinforced concrete or concrete encased steel frames and the materials of infill panels could be solid or hollow bricks, reinforced or non-reinforced concrete blocks, lightweight concrete, composite materials or reinforcedconcrete[6].A review of the literature on the behavior of infilled frames indicated that the addition of infills may cause significant changes in the dynamic characteristics of building and influence their behavior during earthquakes. An infill that is properly designed and connected to frame offers conceptual and practical advantages, particularly if the basic structural system is a moment resisting frame. Also, seismic resistant design should be based on the principle of avoiding unnecessary

masses and using necessary masses structurally to resist seismic effects. Thus, when walls and partitions are needed,attempts should be made to use masonry of walls and partitions as structural elements. Unfortunately, the lack of simple practical method for the analysis of infilled frame has led, in some cases, to the design of buildings without accounting for the effect of infill. Literature review on infilled frames revealed the need for developing guidelines for analysis and design of infilled frame. In this paper we present an analysis technique for the seismic design of infilled frames, which can be used by practicing engineers.

2. THEORETICAL FORMULATION

We have formulated a theoretical analysis where a diagonal strut-frame combination represents the infilledframe[4], beam and truss elements idealise the structure for the elastic analysis[7]. The analysis accounts forinfills in computingseismic load, determining forces in members and determining the strength of different components of the composite system.Preliminary design gives an estimate of member sizes of composite system. Modal analysis establishes natural periods and mode shapes, and static analysis yields member forces and displacements. Wehave compared these forces with the strength of infill and frame corresponding to possible modes of failure.

Dynamic analysis has been used to predict seismic loads, and truss elements idealise infilled frame and a frame analysis gives computed dynamic properties. A modal analysis procedure uses these dynamic properties to calculate the base shear. Maximum base shear is distributed as lateral forces in different storeys, based on the relative masses of each storey, and eventually the corresponding lateral force is distributed to each column of a storey. Total load acting on infilled frame system is established with due consideration to load factors and load combination factors given in the code of practice. Unsymmetrical distribution of infills leads to additional loads from torsion. Using all predicted loads, a static analysis for frame can give an estimate of the member forces. These member forces are used to design columns, beams and infills.

In the present work the infills are idealised as equivalent diagonal strut having two degrees of freedom at each node[4] i.e. it will act as a truss element. The degrees of freedom for frame element are three for each node. The mass degrees of freedom are only translational; the rotational degrees of freedom are ignored in the analysis. First the stiffness matrix for the frame elements and the brick infills are derived, and then Eigen vector analysis (free vibrational analysis) is performed to get the natural frequencies and period of vibration. The modal analysis is also carried out by using SAP 2000, using that natural period and by the IS code expressions, the dynamic analysis is performed[3].

2.1.Stiffness matrix for plane frame elements

In preparation for the analysis of plane frames, the member stiffness matrix for a typicalplane frame member is developed. The matrix is formulated with respect to member axesand then transformed to structure axes by the method of rotation of axes. The frameelements are standard prismatic bending elements having three degrees of freedom at eachnode. The material is idealized as elastic and isotropic. Depending on their angle (θ) with$X$-axis, they are divided as Beam-Frame elements and Column-Frame elements for $\theta = 0^o$and $\theta = 90^o$, respectively.

The element stiffness matrix for general Frame elements is calculated as presented in Figure 1.

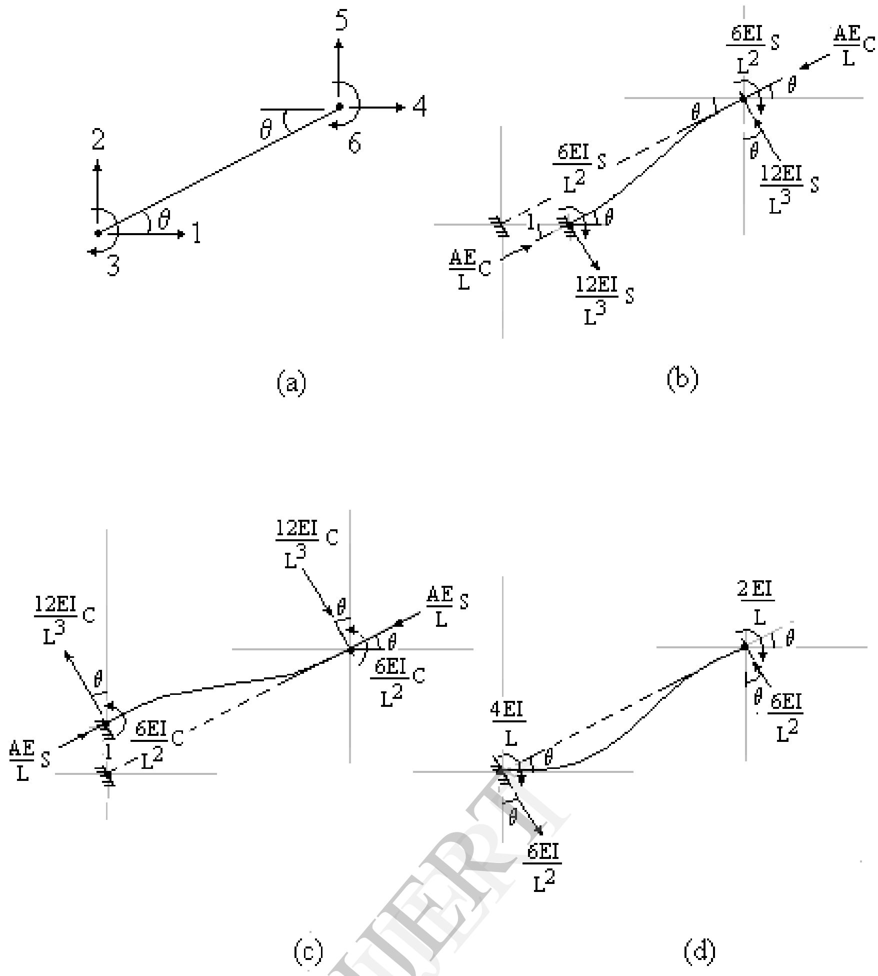

Figure 1.Element stiffnesses for general frame element.

The frame elements shown in Figure 1 (a) shows the deformation directions required forderiving stiffness matrix. Figures 1 (b), (c) and (d) represents forces produced due to unitdeformation at first joint along direction 1, 2 and 3 directions, respectively. Similarly one caneasily find stiffnesses for second joint. The resulting 6 x 6 member stiffness matrix for memberaxes is given in Eqn. (1).

The member stiffness matrix $S_M$is then transformedto the stiffnessmatrix for structure axes S_{MS}. In order to transform the stiffness matrix from member axesto structure axes, rotation transformation matrix $R_T$for a plane frame member is required.

$$S_M = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & \frac{-AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ \frac{-AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \quad (1)$$

As a first step the 3 x 3 rotational matrix$R$ will be expressed in terms of the directioncosines of the member. This may be accomplished by expressing the direction cosine λ of themember axes in terms of the angle θ and then substituting the direction cosine C and S forthe member, as follows

$$R = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C & S & 0 \\ -S & C & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The rotation transformation matrix $R_T$for a plane frame member can be shown to take thesame form as Eqn. (1).

$$R_T = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \quad (3)$$

Having the rotation transformation matrix on hand, one may then calculate the memberstiffness matrix for structure axes using equation.

$$S_{MS} = R_T{}^T S_M R_T \qquad (4)$$

Thus final stiffness matrix for frame element is,

$$[S_{MS}]_{6X6} = \begin{bmatrix} K_1C^2+K_2S^2 & (K_1-K_2).CS & K_3.S & -(K_1C^2+K_2S^2) & -(K_1-K_2).CS & K_3.S \\ (K_1-K_2).CS & K_1S^2+K_2C^2 & -K_3.C & -(K_1-K_2).CS & -(K_1S^2+K_2C^2) & -K_3.C \\ K_3.S & -K_3.C & K_4 & -K_3.S & K_3.C & K_5 \\ -(K_1C^2+K_2S^2) & -(K_1-K_2).CS & -K_3.S & K_1C^2+K_2S & (K_1-K_2).CS & -K_3.S \\ -(K_1-K_2).CS & -(K_1S^2+K_2C^2) & K_3.C & (K_1-K_2).CS & K_1S^2+K_2C^2 & K_3.C \\ K_3.S & -K_3.C & K_5 & -K_3.S & K_3.C & K_4 \end{bmatrix} \quad (5)$$

Where,

$$K_1 = \frac{AE}{L}, K_2 = \frac{12EI}{L^3}, K_3 = \frac{6EI}{L^2}, K_4 = \frac{4EI}{L}, K_5 = \frac{2EI}{L}, C = \cos\theta, S = \sin\theta$$

From the matrix [S_{MS}], stiffness matrix for Beam-frame element and Column-frameelement can be derived by substituting corresponding values of θ.[6,7]

2.2.Stiffness matrix for infill panel element

The contribution of infills to the composite system can be accounted for in this analysis byusing the equivalent diagonal strut method[4]. When the frame-diagonal strut idealization isused, the columns and beams of the frame are modelled as frame elements, with three degreeof freedom at each node, namely rotation and two displacements, while for the diagonalstrut, the truss element will be used[7].For the truss element the value of moment of inertia becomes zero in equation 5, hencethe stiffness matrix for truss elements becomes

$$S_{MS} = \frac{AE}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & -CS & S^2 \end{bmatrix} \qquad (6)$$

2.3.Element mass matric for frame and infill element

Modal analysis is used to determine the vibration modes of a structure in order to understand their structural behaviour[9,10]. The determination of mass influence coefficients for axial effects of a beam element may becarried out by any of two methods indicated for the flexural effects, namely (i) The lumped mass model and (ii) The consistent mass model.

In the lumped mass method, the mass allocation to the nodes of the frame element is foundfrom static a consideration, which for a uniform beam gives half of the total mass of theframe segment allocated at each node. Then for a prismatic beam segment, the relationbetween nodal axial forces and nodal acceleration is given by

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \frac{mL}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} \qquad (7)$$

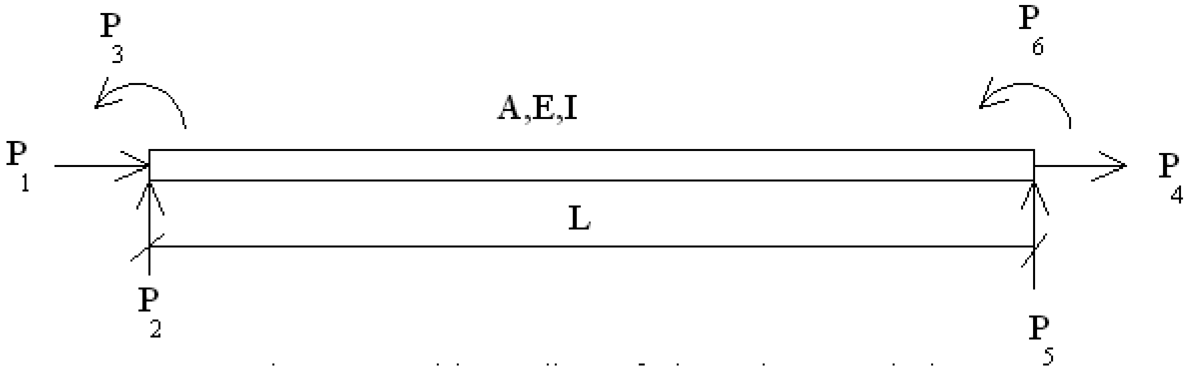

Figure 2:Nodal co-ordinates for lumped mass method

where*m*is the mass per unit of length. The combination of the flexural lumped masscoefficient and axial mass coefficient gives, in reference to the nodal co-ordinates in Fig. 2,the diagonal matrix given in equation (8).

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix} = \frac{mL}{2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \\ \ddot{u}_4 \\ \ddot{u}_5 \\ \ddot{u}_6 \end{bmatrix} \quad (8)$$

In the formation of mass matrix the rotational moment of inertia of the members areneglected and only translational inertia is considered. Hence the $P_3$and P_6 becomes zero.

2.4.Natural period

Consider a single degree of freedom (SDOF) system. When some initial disturbance(displacement/ or velocity) is given to this SDOF system, it undergoes a free vibration and time required to complete one oscillation of free vibration is defined as thenatural period of SDOF system.In a multi degree freedom system, masses at different locations can undergo free vibrationoscillations in different normal mode shapes of the deformation. In each of these normalmodes of vibration, the structure takes a definite amount of time to complete one cycle ofmotion; this time to complete one cycle of motion is called natural period of motion of thatnormal mode of vibration.This phenomenon is particularly important for determining seismic forces. In fact, without dragging it back and forth, it is not possible to make an object vibrate at anything other thanits natural period. Natural period primarily is a function of building height. Other factorssuch as the building construction materials, which affect the stiffness of the structure andbuilding geometric proportions, also affect the period, but height is the most importantconsideration.
The approximate fundamental natural period of vibration in seconds, *Ta*, for moment resistingframe buildings without brick infill panels can be estimated by using following expression.

For RC frame buildings-
$Ta = 0.075\ h^{0.75}$ (9)
For Steel frame buildings-
$Ta = 0.085\ h^{0.75}$ (10)
And the approximate fundamental natural period of vibration in secondsfor other typesof buildings including moment resisting building with infill can be estimated as

$$Ta = \frac{0.09h}{\sqrt{d}} \quad (11)$$

where, h = Height of building in meters. This excludes the basement storeys wherebasement walls are connected with ground floor deck or fitted with buildingcolumns, however, it includes the basement when they are not connected, d = base dimension of the building at the plinth level in meters along considereddirection of the lateral force[3].

2.5.Modal analysis

Modal analysis is used to determine the vibration modes of a structure. These modes areuseful to understand the behavior of structure. Mode shapes depend on distribution of massandstiffness in building. In tall buildings, higher modes can be quite significant and inirregular buildings mode shapes may be somewhat irregular. Hence, for tall and irregularbuildings, dynamic analysis is generally preferred.There are two types of modal analysis to choose from when defining a modal analysis (i) Eigenvectoranalysiswhich determines the undamped free-vibration mode shapes andfrequencies of the system. These natural modes provide an excellent insight into thebehavior of the structure. They can also be used as the basis for

response-spectrum or time-history analyses. (ii) Ritz-vectoranalysis which find modes that are excited by a particular loading. Ritzvectors can provide a better basis than do eigenvectors when used forresponse-spectrum or time-history analyses that are based on modal superposition. Modal analysis is always linear. It may be based on the stiffness of thefull-unstressed structure, or upon the stiffness at the end of a nonlinear analysis case. We have performedeigen vector analysis in the work discussed here.
Eigenvector analysis involves thesolution of the generalized eigenvalue problem

$$\left[[K]-\omega^2[M]\right]\{\phi\}=0 \qquad (12)$$

where*[K]* is the stiffness matrix, *[M]* is the diagonal mass matrix, ω^2representseigenvalues, and$\{\phi\}$ is the matrix of corresponding eigenvectors (mode shapes). Eacheigenvalue-eigenvector pair is called a natural vibration mode of the structure[10]. The modesare identified by numbers from 1 to *n* in the order in which the modes are found by theprogram.

The eigenvalue is the square of the circular frequency for that mode (unless a frequencyshift is used, see below). The cyclic frequency *f* and period *T* of the Mode are related toω by:

$$T=\frac{2\Pi}{\omega} and\, f=\frac{1}{T} \qquad (13)$$

2.6.Equivalent static analysis for evaluation of lateral loads as per IS-1893 (Part-I): 2002

For the purpose of determining seismic force, the country is classified into four seismiczones, which are presented in Figure 1, of IS 1893(part-I): 2002. The total design lateral force (design seismic base shear) along any principal direction shallbe calculated by using following expression

$V_B = A_h W$ (14)

where,V_B= design seismic base shear, A_h= design horizontal seismic coefficient for various structures and *W*= seismic weight of building.
The design horizontal seismic coefficient for a structure shall be determined by the followingexpression.

$$Ah=\frac{Z}{2}.\frac{I_m}{R}.\frac{S_a}{g} \qquad (15)$$

Where *Z*=zone factor, I_m=importance factor,*R*=response reduction factor, S_a/g= average response acceleration coefficient. These parameters are determined as follows.
(i)Zone factor (Z): It is a factor used to obtain the design acceleration spectrum depending upon perceived seismic hazard in the zone in which structure is located. The basic zone factors included in I.S. code are reasonable estimate of peak ground acceleration. Zone factor given in Table 2 of I.S. 1893: 2002 shows the values of zone factor depending upon the seismic intensity.
(ii)Importance factor (I_m): Seismic design philosophy assumes that a structure may undergo some damage duringsevere shaking. However critical and important facilities must respond better during anearthquake than an ordinary structure. Importance factor is meant to account for this byincreasing the design force level for critical and important structures. Importance factordepends upon functional use of structures; characterized by hazardous consequences of itsfailure, post-earthquake functional needs, historical value or economic importance. Theimportance factor is given in Table. 6 of I.S. 1893 (part I): 2002 depending on theimportance of structure.

(iii)Response reduction factor (R): The structure is allowed to be damaged in case of severe shaking. Hence the structure isdesigned for seismic force much less than what is expected under ground shaking ifstructure were to remain linearly elastic.The Response reduction factor depends upon the perceived seismic damage performance ofthe structure characterized by ductile and brittle deformations. However, the ratio *(I/R)* shallnot be greater than 1.0. The values of *R* for buildings are given in Table 7 of I.S. 1893 (partI): 2002 depending on the type of structure.

(iv)Average response acceleration coefficient (Sa/g): Average response acceleration for Rock and soil sites as given in Figure 2 of I.S. 1893 (part I):2002 based on appropriate natural periods and damping of structure. These curves representfree field ground motion. Here, Figure 2 shows proposed 5% spectra for different soil sitesand Table 3 of the I.S. gives multiplying factors for obtaining spectral values for variousother damping.
Provided that for any structure with approximate natural period of vibration, $T \leq 0.1s$, the value of $A_h$will not be taken less than *Z/2*whatever the value of *Im /R*.
Seismic weight of each floor is its full dead load plus appropriate amount of imposed load.While computing seismic weight of each floor, the weight of columns and walls shall beequally distributed to the floors above and below the storey. Any weight supported inbetween storeys shall be distributed to the floors above and below in inverse proportion toits distance from the floors.The percentage of imposed load to be considered in seismic weight calculations is given inTable 8 of I.S. 1893. As uniformly distributed imposed load up to 3.0 kN/m^2 percentage ofimposed load is 25% and above 3.0 kN/m^2 it is 50%.

In the limit state design of reinforced concrete structures, following load combinations shall be accounted as per I.S. 1893 (Part I)- 2002, where the terms DL, IL and EL stand for the response quantities due to dead load, imposed load and earthquake load, respectively.

1) 1.5 (DL + IL)
2) 1.2 (DL + IL± EL)
3) 1.5(DL ± EL)
4) 0.9 DL± 1.5 EL

2.7.Dynamic analysis

Dynamic analysis shall be performed to obtain the design seismic force, and its distributionto different levels along height of a building and to the various lateral load-resistingelements. To perform dynamic analysis most important thing is to carry out free vibrationanalysis of a frame, to obtain dynamic properties i.e. natural periods and mode shapes offrame.Expressions for design load calculation and load distribution with height are based on thefollowing assumptions: (i) Fundamental mode dominates the response (ii) Mass and stiffness are evenly distributed with building height, thus giving a regular modeshape.Mode shapes depend on the distribution of mass and stiffness in the building. In tallbuildings, higher modes can be quite significant and in irregular buildings mode shapes maybe somewhat irregular. Hence, for tall and irregular buildings, dynamic analysis is generallypreferred. Industrial buildings may also require dynamic analysis because they may havelarge spans, large heights, and considerable irregularities.

The dynamic analysis procedure for the calculation of design seismic force is valid when a building can be modelled as a lumped mass modelwith one degree of freedom per floor. This method of analysis does not imply that(a) Structure deforms only in the shear mode with no rotations or vertical translations atthe floor levels, and(b) Beams in a structure are flexurally rigid and hence undergo no rotations[9].

(i) Free vibration analysis: Undamped free vibration analysis of the entire building shall be performed as per establishedmethods of mechanics using the appropriate masses and elastic stiffness of the structuralsystem, to obtain natural periods (T) and mode shapes {φ} of those of its modes of vibrationthat need to be considered[9].

(ii) Modes to be considered: Number of modes to be used in analysis for a considered direction of earthquake shakingshould be such that sum total of modal masses of all modes considered is at least 90 percentof the total seismic mass and missing mass correction beyond 33 percent. If modes withnatural frequencies beyond 33 Hz are to be considered, the modal combination shall becarried out only for modes up to 33 Hz and the effect of higher modes with naturalfrequencies beyond 33Hz shall be included by considering the missing mass correctionprocedure.

In a multi-degree-of-freedom system, when the ground shakes in a particular direction, onlya part of the total mass of the whole structure vibrates in each mode of vibration. Thus, thenet mass accounted for in the modes of vibration considered may be less than the total massof the structure. Difference between total mass of the structure and net masses accountedfor in the modes considered is called missing mass. Often, this missing mass corresponds tomodes of vibration whose natural periods are very small (or whose natural frequencies arevery large). Thus, in missing mass correction procedure, it is assumed that the missing masscorresponds to modes of vibration that have natural periods close to zero.

In dynamic analysis, it is sufficient to consider only first three modes, since higher modes donot significantly alter the response of a frame for buildings lessthan ten storeys high [6].

3. CONCLUSIONS

There has been an increased realization among the structural engineers that the presence of infills influences the behavior of frame system in multistoried buildings andcan be either beneficial or detrimental, depending on the design situation. An increased need for developing guidelines for the analysis and design of infilled frames under the effect of dynamic loads is felt and in this manuscript we discuss an analysis technique for the seismic design of infilled frames, which has practical utilities. The infill in the reinforced concrete frame can be modelled as diagonal strut or Finite Element model.In the Finite Element model the frame is modelled as frame elements having three degrees of freedom.The stiffness matrix and the mass matrix can be formulated by knowing the size and the properties of the model.The infill is modelled as frame element having two degrees of freedom. Finally the mass matrix and stiffness matrix is assembled and by the Eigen value analysis the time period can be found out.

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