A Study on Slope Strengthening Works by LRFD based Reliability Analysis Approach

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***Abstract*—The problem of landslide is being encountered quite frequently nowadays due to number of reasons which includes technical and human induced. Therefore, the mitigation measures for the impending catastrophe must be identified and worked out before its emergence. Rectification of an existing slope or the preclusion of an imminent landslide mainly deals with curbing the driving factors that triggers the failure. This study aims to employ load resistance factor design (LRFD) approach in Geotechnical industry to work out the probability of failures of the structure and estimate the reliability through statistical knowledge. The increasing occurrences of landslides are becoming a serious concern. To overcome the risks and the uncertainties related to the slope failures, the concept of LRFD is authentic to determine the reliability of structures. In terms of providing solutions to strengthen the slopes, reliability based LRFD method is a meaningful tool which employs First Order Second Moment method or First Order Reliability Method (FORM) for determination of probabilities. The resistance factors can also be determined by varying the slope heights. Here, the site of Kuala Lumpur (Malaysia) has been considered to determine the probability of failure with respect to different failure modes**

Keywords—* Load Resistance Factor Design, First Order Reliability Method, Limit States, Reliability Analysis, Resistance Factors

I. INTRODUCTION

Landslides/associated slope instability have become progressively more common in many parts of the world and are liable for substantial losses in terms of both compensatory and non-compensatory. Exclusively, with reference to Malaysia, landslide problems are augmenting as a result of rapid economic development especially on hilly land during last 20 years. Numbers of landslides are tremendously increasing due to slope failures. Many townships, industrial areas, housing schemes have been developed without estimating the probability of slope failures and its consequences. Advancements on hillsides are totally out of control. Infrastructure facilities like highways, expressways, light rail transit, etc also contributed in queue along with other development schemes.

Rectification of an existing slope or the preclusion of an imminent landslide mainly deals with curbing the driving factors that triggers the failure.

Active design methodologies are carrying an essence of limit equilibrium principles that together with a single factor of safety or a set of partial factors on the material parameters and loads, report for uncertainties coupled with input parameter values. Relying on existing practices is not sufficient as the existing practices counter the uncertainties in a very superficial manner, that’s why recent changes in design methodologies of earth retaining structures are towards limit state design/ LRFD. As reliability based LRFD approach quantitatively tackle the uncertainties in regards to the materials and the loads. Therefore, this study aims to introduce the Load Resistance Factor Design in Malaysian Geotechnical industry. As LRFD is reliability based method, it can work out the probability of failures of the structure and estimate the reliability through statistical knowledge.

Frequent Landslides and slope failures are a major issue. To overcome the approaching risks and the uncertainties related to slope instability, slope strengthening measures are required. In terms of providing solution of how to strengthen the slopes successfully, reliability based LRFD method is proposed and carried out by calculating resistance factors of soil nailed walls and slopes through First Order Second Moment also known as First Order Reliability Method (FORM).

II. LRFD AND SOIL NAILING

Soil nailing is an in-situ earth reinforcement technique. Due to copious gains such as prompt construction, no difficulty in application, less environmental impact etc, slope engineers prefer soil nailing as a feasible substitute to the other earth retaining systems. Soil Nail is frequently used in Malaysia particularly in cut slope. The fundamental conception of soil nailing is to underpin and strengthen the cut slope by installing closely-spaced steel bars, called ‘nails’, into a slope as construction takings from ‘top-down’. The spacing of soil nail normally used is in a range of 1 to 2 m at centre to centre and install using grid type or diamond type and the diameter of nail is 100mm. Several types of lengths have been used such as 6m, 9m and 12 m. This process develops a reinforced section that itself is stable and competent to keep hold of the ground behind it. The reinforcements are passive and build up their reinforcing action through nail-ground interactions.

Soil nailed walls were introduced more than thirty years back to deal with the problems of earth-retaining structures. Various methods were developed for the analysis of such walls like force equilibrium approach and moment equilibrium approach. In conjunction with that, it is also documented that there are uncertainties in many design parameters; thus, it is essential to follow a reliability analysis so that one can approximate the effects of such uncertainties on walls stability[1]. Basically the stability of a soil nail wall is radically prejudiced by the complex attitude of the main components, like, in-situ soil, anchors (i.e. nails) and wall facing. Therefore, to ensure a proper design of soil nail walls, it is compulsory to have perception about its construction sequencing, stabilization mechanisms, function of various failure modes, and pressure of spatial variations of soil parameters on its stability. The past studies also expose the fact that very little work till now is offered on the relevance of reliability theory of soil nailed walls. It is worth mentioning that, excluding the studies of [1] and [2] that no significant work has been found in context of reliability analysis of soil nail walls. Referring to LRFD methodology with respect to soil nailed walls, [3] has given the brief background about its emergence and modifications by taking FHWA 1998 manual and AASHTO LRFD Specifications.

Load Resistance Factor Design or limit state design equation in actual represents the condition of an adequate design of the system. Mathematically the condition can be expressed as:

$$\phi R_n \geq \sum \gamma Q_i \qquad (1)$$

The left side of Equation (1) is the resistance term and represents the nominal (ultimate) resistance, R_n, reduced by the multiplicative resistance factor, φ to counter for uncertainties in resistances. The right side of Equation (1) represents load effects and consists of the sum of load components, Q_i, multiplied by associated load factors, γ. The load factors account for uncertainties in loads comes up from the load type, variability, and predictability connected with a particular limit state

According to [4], the reported definition of limit state is:
“A limit state is a condition beyond which a structural component, such as a foundation or other bridge component, ceases to fulfill the function for which it is designed”.

In the above mentioned definition of a limit state, both the resistance and load are built-in. For example, if sufficiency of bearing strength of a soil under a footing is being probed, more than one load combination might be required for assessment, particularly if the footing is focused to eccentric or inclined loads. When the bearing pressures due to the loads go beyond the bearing strength, a limit state (i.e., a Strength Limit State) is reached and structure collapses. Similarly, if the structure displaces and the load crosses the tolerable deformations/settlements, the Service Limit State is reached.

III. FAILURE MECHANISMS OF SOIL NAIL WALLS

In order to ensure the design adequacy, a soil-nailed system must have the capacity to fulfil the stability criteria. In a broad spectrum, the design should carry the surety of safety, against various failure modes. Failure modes of soil nail walls are mainly classified into three diverse groups as: external failure modes, internal failure modes and facing failure modes (Fig, 1).

A.External Failures

External failure denotes to the expansion of potential failure surfaces fundamentally at the exterior of the soil-nailed ground mass. The failure can be in the shape of sliding, rotation, bearing, or may be of overall/global stability. Global stability and sliding stability are the two major external failure modes of soil nail walls. Global stability consigns to the whole stability of the reinforced soil nail wall mass. In this failure mode, along the slip surface the driving force owed to the self-weight and external loading on the retained mass surpass the resisting force provided by the in-situ soil and the nails. Conversely, sliding stability regards as the ability of the soil nail wall to defy sliding along the base of the retained system in reply to tangential lateral earth pressures at the back of the soil nails. Sliding failure may crop up when additional lateral earth pressures, mobilized by the excavation, go beyond the sliding resistance along the base. From time to time, bearing capacity of soil may also to be tackled when a soil nail wall is unearthed in fine-grained, soft soils. Fact is, the wall facing does not broaden below the bottom of the unearthed portion, and the unbalanced load due to the excavation may trigger the bottom of the excavation to heave and fuel a bearing capacity foundation failure.

B.Internal Failures

Internal failures are the inner failures of the soil-nailed ground mass. Internal failures can happen in the active, passive or in both of the two zones of a soil-nailed system. Active zone internal failure modes include, ground mass failure, bearing failure, structural failure of soil nail and soil nail heads and surface failure. In passive zone, pull out failure between ground-grout or grout reinforcement interface will count [5].

Pullout failure and tensile failure of soil nails contributes most significantly as compared to other internal failures. As it is already reported in the literature [6] that shear and bending strengths of soil nails, has very minute contribution like 10% to the overall stability that’s why this failure mode seems to be less significant or unattended in lieu with other failure modes.

Nail pullout failure is a failure along the soil-grout or soil-nail interface is due to inadequate built-in bond strength and/or deficient nail length. Tensile failure of a soil nail takes place when the nails tensile capacity is not sufficient or maximum tensile axial force in the soil nail is greater than nails tensile capacity.

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C.Facing Failure

Facing flexure failure and facing punching shear failure are the two prominent facing failure modes of the soil nail walls. Facing flexure failure is originally the outcome of the excessive bending further than the facing's flexural capacity, while facing punching failure occurs due to lacking in shear capacity of the facing element in the region of the nail head.

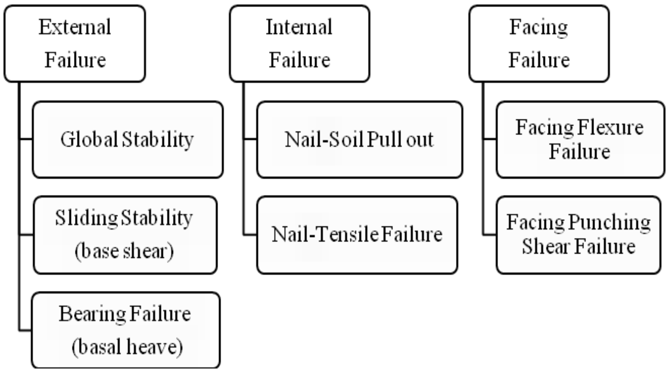

Figure 1 Soil Nailing Failure Modes (Babu and Pratap 2009) (FHWA 1998)

This research takes in to account the case of a failed soil nailed slope of Federal Route 59 Cameron Highland Pahang, Malaysia. The primary question is what are the factors that instigated it? Is it due to design lapses such as wrong data, wrong assumptions, mistake on theory, mistake on correlations and faults in assumptions? An investigation of the slope failure is carried out. The investigation comprises on a review of existing record, failure investigation, causes of the failure and conclusion. It is referring that, factors such as geological, physical and human factor have caused to the failure but which factor dominates more to the failure is still unclear. In this connection, this case history is considered in detail for further explorations.

A 10 m high soil nail wall supporting a vertical cut is considered. Figure 2 illustrates the schematic layout of the soil nail wall considered for the reliability analysis along with the various forces acting on it. The properties of the in-situ soil and other soil nail wall parameters are shown in Table 1. Influence of variability of in-situ soil is studied over a range of coefficients of variation (COV) of soil parameters (cohesion c, angle of internal friction and unit weight of the soil), in accordance with the values reported by past researchers [7, 8].

Description	Symbol	Value
Height of the wall	H	25m
Slope of backfill	β•	0
Surcharge	q_s	15KPa
Soil cohesion	c	5KPa
Friction angle	φ	28 degree
Unit weight of soil	γ	18KN/m3
Diameter of the nail	d	25mm
Length of the nail	L	10m
Drill hole diameter	D_{DH}	100mm
Nail spacing	S_h, S_v	1m*1m
Yield strength of nail	fy	415MPa
Shotcrete facing thickness	h	100mm
Bearing plate length	l	225mm*225mm*225mm

Table 1 Properties of Soil Nail Wall

IV. PERFORMANCE FUNCTION OF SOIL NAIL WALLS

The six failure modes of the soil nail wall discussed previously are considered and a reliability analysis is carried out by taking three statistical means of First Order Reliability Method (FORM), Crude Monte Carlo simulation (MCS) and Importance sampling (IS) technique. These methods vary slightly in their accuracies but not comparable in terms of their efficiencies [9]. In contrast with FORM, Monte Carlo simulation and Importance sampling are basically variance reduction methods used to measure reliability through probability of failure. This is also an indirect parameter on the basis of which reliability of the structure/component can be estimated.

Initial stability analyses are also performed to check that whether the section (nail lengths, nail diameters etc) has to be revised or not. After the stability check performance functions for every single failure mode are used to calculate the reliability index (refer Table 2.). Expressions used to work out the limit state conditions are given below:

a) Global Stability

$$\mathbf{Perf}_{(G)} = \mathbf{R_G} - \mathbf{S_G} \qquad (2)$$

Where

$$\mathbf{R_G} = \mathbf{CL_f} + \left[(\mathbf{W} + \mathbf{Q_T})\cos\theta + \mathbf{T_{eq}}\sin\theta\right]\tan\varphi \qquad (2a)$$

$$\mathbf{S_G} = (\mathbf{W} + \mathbf{Q_T})\sin\theta + \mathbf{T_{eq}}\cos\theta \qquad (2b)$$

b) Sliding Failure

$$\mathbf{Perf}_{(S)} = \mathbf{R_S} - \mathbf{S_S} \qquad (3)$$

Where

$$\mathbf{R_S} = \mathbf{CBL} + [(\mathbf{W} + \mathbf{QT})\tan\varphi] \qquad (3a)$$

$$S_S = PA \qquad (3b)$$

c) Pull out Failure

$\mathbf{Perf}_{(P)} = \mathbf{R_P} - \mathbf{S_P}$ (4)

Where

$R_P = \pi D_{DH} [C + (q_s + \gamma z) \tan \varphi] L_P$ (4a)

$S_P = T_{max} = K_a (q_s + \gamma H) S_h S_v$ (4b)

d) Tensile Failure

$\mathbf{Perf}_{(T)} = \mathbf{R_T} - \mathbf{S_T}$ (5)

Where

$R_T = (0.25\pi d^2 f_y)_z$ (5a)

$S_T = T_{max} = K_a (q_s + \gamma z) S_h S_v$ (5b)

e) Flexural Failure (Facing Failure)

$\mathrm{Perf}_{(FF)} = R_{FF} - L_{FF}$ (6)

Where

$R_{FF} = 0.004 C_F h A_T f_y$ (6a)

$L_{FF} = T_0 = T_{max} [0.6 + 0.2(S_{max} - 1)]$ (6b)

f) Punching Failure

$\mathbf{Perf}_{(FP)} = \mathbf{R_{FP}} - \mathbf{L_{FP}}$ (7)

Where

$R_{FP} = 330\sqrt{f_{ck}}\,(\pi D_c h_c)$ (7a)

$L_{FP} = T_0 = T_{max} [0.6 + 0.2 (S_{max} - 1)]$ (7b)

Table 2 Estimated Results of Soil Nailing Failure Modes

Modes	Factor of Safety	FORM β	MCS (10000000) P_f	IS (646-859) P_f
Global	1.95	4.5	2.9*10^-6	3.1*10^-6
Sliding	2.04	4.3	3.0*10^-6	3.2*10^-6
Pull out	2.42	5.2	1.45*10^-8	1.45*10^-8
Tension	1.90	3.4	3.2*10^-6	3.76*10^-6
Flexural	1.50	3.8	3.44*10^-5	3.87*10^-5
Punching	1.8	3.9	4.41*10^-5	5.01*10^-5

V. SYSTEM RELIABILITY AND ANALYSIS

Performance of engineering system often involves multiple failure modes. For a structural or geotechnical structure with several components, the overall reliability will not depend only on the reliabilities of individual components but also the correlation between the failure modes. Like [10] pinpointed the case of combinatorial reliability involving a foundation near a slope which is supported by the retaining wall. Among many modes of failure, ten were identified in that particular case. However to make the problems accessible theory of series and parallel system has been taken into account.

In geotechnical problems, exact solution is totally impossible to furnish but the concept of upper and lower bounds are one of the suitable ways to approximate the system probability of failure. In this consideration, the system reliability is evaluated utilizing the extension of theory of FORM[11]

In structural design, every component or system needs to be verified that it fulfills the required safety levels. Due to the uncertainties allied with the functioning conditions, design parameters, and materials, this job becomes intricate and tedious. Characteristically these uncertainties are distinguished by using random or nebulous variables, relying on the data/information in hand. Most of the work accessible in the literature about uncertainty analysis is involved with the inference of the safety of a single failure mode based on a particular performance criterion rather than multiple failure modes/criteria. These failure criteria are often linked, because their dependencies are on the same uncertain variables. Previously this type of work has been carried out in connection with slope stability like [12], [13] and [14] but till now for slope strengthening solutions the work seem to be untouched. Therefore, one of the motives of this study is to work on the methodology that can capably deal with multiple forms of uncertainty give rise to multiple failure modes in soil nailed walls/slopes.

In comparison with the methods for system reliability analysis, the theory recommended by Ditlevsen [15] is approached here as it is in actual the extension of the first-order reliability method commonly worked to measure the chances of failure, and it can usually give practically constricted failure probability bounds. Generally when discussing about system reliability, it refers the reliability of the whole unit, it may be a series or a parallel system. Series systems are those systems in which failure of any of the component/element leads to whole system failure. In parallel systems combined failure of the elements/components serves for system failure.

If referring to a series system with number of limit state functions (failure modes), violation of any limit state function would result in system failure. Let E_i represents the event that the ith limit state is exceeded, and $P_{f,sys}$ denotes the probability of system failure. Accurate computation of $P_{f,sys}$ is no doubt tedious. In this connection following bimodal bounds which report for the correlation between pairs of potential failure modes are used to estimate the system reliability.

$$\left\{P(E_1) + \sum_{i=2}^{m} max\left[P(E_i) - \sum_{j=1}^{i-1} P(E_iE_j), 0\right]\right\} \leq P_{f,sys}$$
$$\leq \left\{P(E_1) + \sum_{i=2}^{m}\left\{P(E_i) - max\left[P(E_iE_j)\right]\right\}\right\} \quad (8)$$

Where *j<i* and $P(E_i)$ = the failure probability related to the ith failure mode; and $P(E_iE_j)$ = the probability that the i and jth limit state functions are violated at once.

In Equation (8.), $P(E_i)$ can be examined through array of techniques, such as Point estimate methods [16], FORM [17] [18] Monte Carlo simulation [19], or SORM [19, 20]. To calculate *P(EiEj)*, which is the intersection of two failure events, is normally not simple. In this regard modified version of equation (8) is suggested and scatter into lower and upper bounds of *P(EiEj)*. It can be evaluated as follows:

$$\max\,[a, b] \leq P(E_iE_j) \leq a + b, \ for \rho_{ij} \geq 0 \quad (9a)$$

$$0 \leq P(E_iE_j) \leq min\,[a, b], \ for \rho_{ij} \leq 0 \quad (9b)$$

Where a and b are defined as

$$a = \phi(-\beta_i)\phi\left(-\frac{\beta_j - \rho_{ij}\beta_i}{\sqrt{1-\rho_{ij}^2}}\right) \quad (10a)$$

$$b = \phi(-\beta_j)\phi\left(-\frac{\beta_i - \rho_{ij}\beta_j}{\sqrt{1-\rho_{ij}^2}}\right) \qquad (10b)$$

Where β_i and β_j = reliability indices corresponding to failure modes i and j, respectively and

ρ_{ij} = correlation coefficient between failure modes i and j. Substituting the bounds of derived from Equations 10a and 10b into Equation 8, the upper and lower bounds for $P_{f,sys}$ can be estimated. (Table 3 and Table 8)

Table 3 System Reliability Bounds between Global and Sliding Mode (Uncorrelated Variables)

Global β=3.37	Sliding β=4.67					
Design Points	Design Points	Correlation Matrix				System Probability Bounds
			c	φ	γ	
1.9529	2.108	c	1	0	0	0.000377328
2.5416	0.0296	φ	0	1	0	0.000377337
-1.059	0.9543	γ	0	0	1	-

Table 4 System Reliability Bounds between Global and Sliding Mode Correlated Variables (Cohesion and Angle of Friction)

Global β=2.81	Sliding β=4.48					
Design Points	Design Points		Correlation Matrix			System Probability Bounds
			c	φ	γ	
2.28	-2.475	c	1	0.5	0	0.002477
2.414	-4.475	φ	0.5	1	0	0.002477
-0.739	0.0801	γ	0	0	1	-

Table 5 System Reliability Bounds between Global and Sliding Mode Correlated Variables unit weight and Angle of Friction

Global β=3.83	Sliding β=4.74					
Design Points	Design Points		Correlation Matrix			System Probability Bounds
			c	φ	γ	
2.89	-0.577	c	1	0	0	6.51402E-05
2.22	-4.711	φ	0	1	0.5	6.51402E-05
0.0824	-2.246	γ	0	0.5	1	-

Table 6 : System Reliability Bounds between Global and Sliding Mode Correlated Variables unit weight and Cohesion

Global β=3.69	Sliding β=4.68					
Design Points	Design Points		Correlation Matrix			System m Probability Bounds
			c	φ	γ	
1.283	-0.453	c	1	0	0.5	0.000113561
3.294	-4.663	φ	0	1	0	0.000113561
-0.274	-0.128	φ	0.5	0	1	-

Table 7 System Reliability Bounds between Five Failure Mode, Unit weight and Angle of Friction (Uncorrelated)

Failure Modes	Reliability Index	Correlation Matrix			System Probability Bounds
			φ	γ	
Global	3.37	φ	1	0	0.087375
Sliding	4.67	γ	0	1	0.090416
Tensile	2.82	-	-	-	-
Pull out	2.56	-	-	-	-
Punching	1.34	-	-	-	-

Table 8 System Reliability Bounds between Five Failure Mode, Unit weight and Angle of Friction (Correlated)

Failure Modes	Reliability Index	Correlation Matrix			System Probability Bounds
			φ	γ	
Global	3.83	φ	1	0.5	0.009417
Sliding	4.74	γ	0.5	1	0.009449
Tensile	4.58	-	-	-	-
Pull out	3.04	-	-	-	-
Punching	2.35	-	-	-	-

VI. CONSLUSION

Noteworthy investigations have been conducted in previous years to develop load and resistance factor design (LRFD) methods for geotechnical applications. It is also suggested to carry out reliability based resistance factors to get a range of target probabilities of failure. Results of probabilistic analysis demonstrate that LRFD methods can be used for overall stability evaluations without unnecessary complications. Soil nailing and piled walls are most often used in Malaysia to counter slope instabilities but the practices needs some improvements in this regard. . These improvements can easily be fulfilled by incorporating reliability based design method. The ability of the LRFD method with the probabilistic approach can be extended to variety of geotechnical applications, accounting for the uncertainty of parameters associated to both the loads and the resistance.

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