

# A Survey on Pilot Symbol Assisted Channel Estimation in OFDM Systems\*

Ramji Verma

Electronics and Telecommunication Engineering  
Institute of Engineering and Technology (IET)  
Indore (M.P), India

Mrs. Sneha Moghe

Electronics and Telecommunication Engineering  
Institute of Engineering and Technology (IET)  
Indore (M.P), India

**Abstract:-**On account of frequency and time selective nature of fading channels, channel estimation is needed in Orthogonal Frequency Division Multiplexing (OFDM) systems and is investigated using pilot arrangement. Channel estimation based on comb type pilot arrangement is taken into account using different algorithms for estimating channel at pilot frequencies and interpolating the channel. Least Square(LS) and Minimum mean square(MMSE) error based estimators are used for estimating the channel at pilot frequencies and channel interpolation is done using linear interpolation, second order interpolation, spline cubic interpolation. Furthermore, comparison of various schemes is performed by measuring bit error rate against signal to noise ratio (SNR) with 16 QAM, QPSK as modulation techniques and multipath Rayleigh fading channel as channel model.

**Keywords:-**Least Square (LS); LMMSE; MMSE; QAM; QPSK; Singular Value Decomposition (SVD)

## I. INTRODUCTION

Due to increasing and bandwidth demanding applications high data rate transmission is required over mobile or wireless channels. To fulfill this requirement Orthogonal Frequency Division Multiplexing has been introduced which provides high data rate and mitigates effectively inter symbol interference (ISI) resulting from the delay spread of wireless channels. OFDM is adopted in many wireless standards such as digital audio broadcasting (DAB), Digital video broadcasting (DVB-T), the IEEE 802.11 a local area network and the IEEE 802.16a metropolitan area network (MAN) standard. In this paper, we propose channel estimation based on Least Square (LS) and Minimum mean square estimators in OFDM systems. Since the radio channel is frequency selective and time varying a dynamic estimation of channel is needed to be done for wideband mobile communication systems[1]. There are two types of channel estimation based on insertion of pilot tones into the subcarriers of OFDM symbol. The first one is block type channel estimation in which pilot tones are inserted into all of the subcarriers of OFDM symbol and is developed under the assumption of slow fading channel. The estimation of the channel for this block type pilot arrangement can be based on Least Square (LS) or Minimum Mean square (MMSE). It is observed that MMSE estimate improves the signal to noise ratio by 15 db for the same mean square error over LS estimate [2]. The major drawback of MMSE is complexity, therefore a low rank approximation is applied to linear MMSE which uses the frequency correlation of the channel to eliminate this drawback. The second one is comb type channel estimation which is developed under the assumption of fast fading channels. Here, the radio channel changes very rapidly even from one OFDM block to the subsequent one. Thus, the channel estimation in the present block cannot be used as the

channel response of the next block. Thus it is adopted to estimate the channel transfer function in each block. The channel estimation at pilot frequencies can be performed by Least Square (LS), MMSE or Least Mean Square (LMS). MMSE is shown to be more robust to Additive White Gaussian noise (AWGN) and inter carrier interference (ICI) than LS. Therefore it performs much better than LS. The computational complexity of MMSE is further reduced by employing simplified linear minimum mean squared (LMMSE) estimator with singular value decomposition (SVD)[3]. To interpolate the channel in comb type channel estimation linear interpolation, second order interpolation, spline cubic interpolation are studied. In this paper we compared the performances of all the above schemes by considering 16 QAM, QPSK (Quadrature phase shift keying) with Rayleigh fading channel as channel model. The organization of paper is as follows, In section II, pilot based OFDM system is described. In section III, channel estimation based on block type pilot arrangement is studied. Section IV discusses the comb type channel estimation at pilot frequencies. Section V deals with different interpolation techniques. In section VI simulation results are presented which shows BER against SNR. Section VII concludes the paper. The block diagram of pilot assisted channel estimation is shown in Fig.1. A signal mapper uses the binary information and modulates this information in accordance with modulation techniques QAM, QPSK). Pilots can be either inserted into all the subcarriers or regularly between OFDM blocks. IDFT takes the inverse fourier transform of data sequence of length  $N\{X(K)\}$  and converts into time domain signal  $\{x(n)\}$ . Because channel introduces inter symbol interference guard insertion is done, which is larger than the expected delay spread. This guard time also eliminates inter carrier interference (ICI) by including cyclically extended part of OFDM symbol. After passing data sequences through parallel to serial converter (P/S) the signal  $x_f(n)$  is finally passed through time varying frequency selective channel in addition with additive white Gaussian noise(AWGN). At the receiver the received signal is passed through A/D and low pass filter in order to transform signal into discrete domain, after which the guard deletion is done and  $y(n)$  is sent to DFT block. After DFT operation the channel estimation is done at the pilot sub carriers by extracting the pilot signals. Following this, channel estimation at the data sub carriers is done in channel estimation block and the estimated channel  $H_e(K)$  is used to estimate the data at the respective sub carriers by:  
$$X_e = \frac{Y(K)}{H_e(K)} \quad k=0,1,\dots,N-1 \quad (1)$$
  
The original data is obtained back using "signal demapper" block.

## II. SYSTEM DESCRIPTION

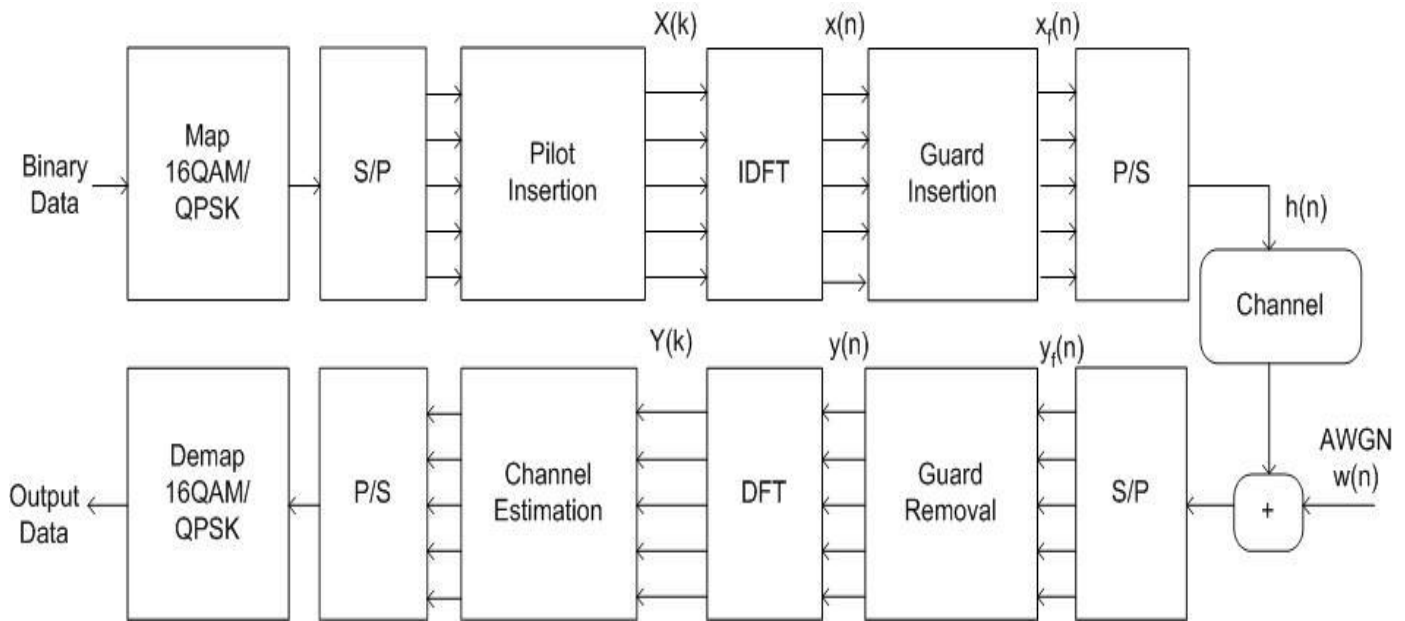


Fig.1 Baseband OFDM System

## III. CHANNEL ESTIMATION BASED ON BLOCK TYPE PILOT ARRANGEMENT

In block type pilot symbol assisted channel estimation, pilot symbols are sent periodically, where all sub channels are used as pilots. THE estimation of the channel is done by using either Least Square(LS) or Minimum Mean Square(MMSE)[2],[3].The MMSE performs better than LS in terms of signal to noise ratio gain. When the channel coefficient h is Gaussian and uncorrelated with channel noise which is Additive White Gaussian Noise (AWGN), then MMSE channel estimate h is given by[3]:

$$H_{MMSE} = F R_{hY} R_{YY}^{-1} Y \quad (2)$$

Where  $R_{hY}$  represents cross-covariance or correlation matrix between time domain channel coefficient h and received vector Y and  $R_{YY}$  indicates auto-covariance matrix of Y.

The channel estimate using Least Square(LS) estimator, which is based on minimizing  $(Y - XFh)^H (Y - XFh)$  is given by,

$$H_{LS} = X^{-1} Y, \quad (3)$$

In case of slow fading channel, decision feedback equalizer can be used to update channel estimate inside the block at each sub-carrier. Decision feedback equalizer can be explained as follows:

- The previous symbol is used to find the response at the  $k^{th}$  sub-carrier which is in turn used to find the estimated transmitted symbol  $\{X_e(k)\}$ .

$$X_e(k) = \frac{Y(k)}{H_e(k)} \quad k=0,1,\dots,N-1 \quad (4)$$

- The obtained estimated symbol  $X_e(k)$  is mapped to binary data using “signal demapper” and then binary data is again mapped to get  $\{X_{pure}(k)\}$ .
- The estimated channel is updated by:

$$H_e(k) = \frac{Y(k)}{X_{pure}(k)} \quad k=0,1,\dots,N-1 \quad (5)$$

Since the decision taken by the decision feedback is assumed correct, this will result in complete loss of estimated parameters in case of fast fading channel.

## IV. CHANNEL ESTIMATION AT PILOT SUBCARRIERS AT COMB TYPE PILOT ARRANGEMENT

In case of comb type pilot arrangement, the number of pilot symbols  $N_p$  are uniformly inserted into symbols  $X(K)$  in accordance with the following equation:

$$X(k) = X(mL+l) = \begin{cases} xp(m), & l = 0 \\ \text{inf. data} & l = 1, \dots, L-1 \end{cases} \quad (6)$$

Where  $L \triangleq$  frequency interval of inserted pilot  $= \frac{N}{N_p}$ ,  $x_p(m)$  is the  $m^{th}$  pilot carrier value.  $\{H_p(k) \quad k=0,1,\dots,N_p\}$  is defined as frequency response of the channel at pilot frequencies. The Least Square (LS) estimate of the channel response at the pilot subcarriers is as follows:

$$H_e(K) = Y_p / X_p \quad K=0,1,\dots,N_p-1 \quad (7)$$

Where  $Y_p(k)$  and  $X_p(K)$  are the output and inputs at the  $k^{th}$  pilot subcarrier respectively. The channel estimate based on Least Square(LS) at pilot subcarriers  $H_p$  is sensitive to Gaussian noise and inter-carrier interference(ICI),therefore an alternative to this is taken into account which is Minimum Mean Square Estimate(MMSE) while compromising complexity. Because MMSE requires matrix inversion at each iteration, the simplified Linear MMSE (LMMSE) is proposed in order to reduce complexity[5]. Here, inversion is only needed to be calculated only once. To reduce further complexity an optimal low rank estimator by using singular value decomposition is proposed [4].

## V. INTERPOLATION TECHNIQUES IN COMB TYPE PILOT ARRANGEMENT

Since the comb type pilot arrangement is designed under the assumption of fast fading channel, therefore an efficient interpolation technique is necessary to estimate the channel at data carriers by making use of channel response at pilot subcarriers.

### A. Linear interpolation method

Linear interpolation technique makes use of the two successive pilot subcarriers in order to determine the channel response at the data subcarriers between these pilot subcarriers [6]. By making use of digital filtering such as Farrow-structure [7] linear channel interpolation can be implemented. The estimated channel response at the data subcarrier  $k$ ,  $mL < k < (m+1)L$ , by using linear interpolation method is given by:

$$H_c(k) = H_c(mL + l) \quad 0 \leq l < L$$

$$= (H_p(m+1) - H_p(m)) \frac{l}{L} + H_p(m) \quad (8)$$

### B. Second order interpolation method

Higher order polynomial interpolation is shown to fit channel response better than linear interpolation. A linear time-invariant FIR filter can be used to implement second order interpolation [8]. Thus, we take into account second order interpolation method and the channel estimated by this method is given by:

$$\hat{H}(K) = \hat{H}(mL + l)$$

$$= c_1 \hat{H}_p(m-1) + c_0 \hat{H}_p(m) + c_{-1} \hat{H}_p(m+1) \quad (9)$$

Where  $c_1 = \alpha(\alpha-1)/2$ ,  
 $c_0 = -(\alpha-1)(\alpha+1)$ ,  $\alpha = l/N$   
 $c_{-1} = \alpha(\alpha+1)/2$

TABLE I. SIMULATION PARAMETERS

Parameters	Specifications
FFT Size	1024
Number of sub carriers(N)	128
Guard interval	256
Guard Type	Cyclic prefix
Channel Bandwidth	1MHz
Modulation model	16 QAM or QPSK
Channel Model	Rayleigh Fading
Doppler frequency shift	40 Hz

### C. Spline cubic interpolation

The spline cubic interpolation (spline function in MATLAB) produces a smooth and continuous polynomial fitted to given data points.

$$\hat{H}(K) = \hat{H}(mL + l)$$

$$= \alpha_1 \hat{H}_p(m+1) + \alpha_0 \hat{H}_p(m) + L \alpha_1 \hat{H}'_p(m+1) - L \alpha_0 \hat{H}'_p(m) \quad (10)$$

$$m = 0, 1, \dots, N_p - 1$$

$\hat{H}'_p(m)$  is the first order derivative of  $\hat{H}_p(m)$  and  
 $\alpha_1 = 3(L-1)^2/L^2 - 2(L-1)^3/L^3$   
 $\alpha_0 = 3L^2/L^2 - 2L^3/L^3$

## VI. SIMULATION

### A. Description of Simulation

#### 1) System parameters

Different system parameters are depicted in Table 1. We have assumed exact synchronization as our objective is to

calculate channel estimation performance. Furthermore, we have chosen cyclic prefix interval much greater than the maximum delay spread in order to eliminate inter-symbol interference. Simulations are carried out for Bit error rate (BER) for different values of signal-to-noise ratio.

#### 2) Channel model

We have modeled our channel as Rayleigh fading channel. It is a reasonable model when there are many objects in the environment that scatter the radio signal before it arrives at the receiver. The envelope of the channel response is Rayleigh distributed. The Rayleigh channel model is as follows-

$$h(t) = \alpha_n(t) e^{-j\theta_n(t)} \quad (11)$$

where  $\alpha_n(t)$  is the attenuation and  $\theta_n(t)$  is the phase of the  $n^{\text{th}}$  path.

#### 3) Channel estimation based on block type pilot arrangement

Here we considered two modulation techniques QPSK and QAM. Pilots are inserted at an interval of 5 used in the simulation. Comparison of different estimators (LS, LMMSE, LMMSE(SVD) rank  $p=5$ , LMMSE(SVD) rank  $p=16$ , LMMSE(SVD) rank  $p=20$ ) is done for Bit error rate against different signal to noise ratio (SNR).

#### 4) Channel estimation in comb type pilot arrangement

Here we considered Comb-LS Linear, Comb-LS Second order and Comb-LS Spline cubic interpolation methods and compared their performances by having a plot between BER versus SNR. The basic method by which estimation of channel at pilot frequencies is performed is Least square (LS).

### B. Simulation Results

The legends "Block-LS, Block-LMMSE, Block-LMMSE (SVD) rank  $p=5$ , Block-LMMSE (SVD) rank  $p=16$ , Block LMMSE (SVD) rank  $p=20$ " shows the different estimators for block type pilot arrangement. The channel estimation at pilot sub carriers is performed using LS, LMMSE and LMMSE (SVD) estimators. Figures 2 and 3 compares the Bit error rate against different signal to noise ratio with Doppler frequency 40 Hz using QAM 16 and QPSK as modulation techniques under the assumption of Rayleigh fading channel. The results shows that QPSK performs better than QAM 16 because it has low bit error rate for the same SNR for QAM 16 and this comparison is valid for all estimators. We also noticed that LMMSE (SVD) with rank  $p=5, 16$  and  $20$  have nearly same bit error rate (BER), but obviously have better performance than LS estimate in case of block type pilot arrangement. Figures 4 and 5 are plotted for comb type pilot symbol assisted channel estimation. Legends "Comb-LS linear, Comb-LS second order, Comb-LS Spline" denotes the different interpolation method with LS estimate. The channel estimation is performed using LS estimate. Here we compared the performance of 16 QAM and QPSK using bit error rate against signal to noise ratio with Doppler frequency 40 Hz. The result shows that spline cubic interpolation gives significantly low bit error rate than second order and linear interpolation techniques.

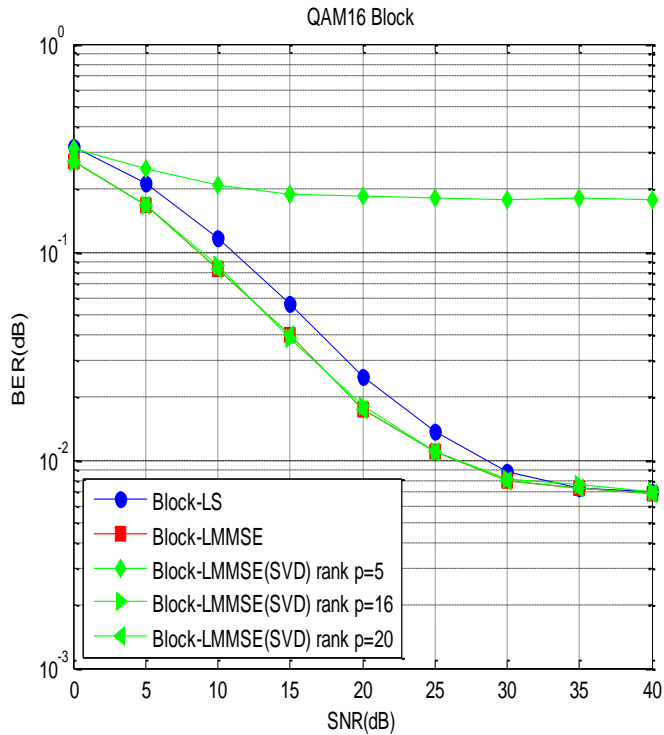


Fig. 2. Plot of BER against SNR for QAM 16 Block (Doppler freq.40Hz)

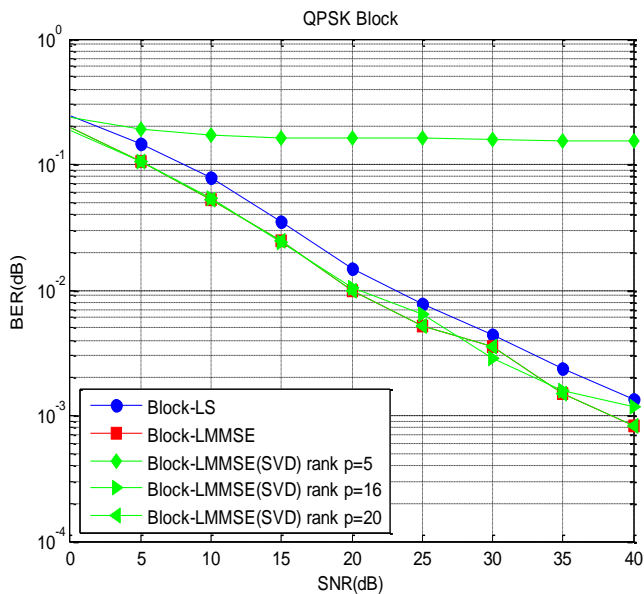


Fig. 3. Plot of BER against SNR for QPSK Block (Doppler freq. 40 Hz)

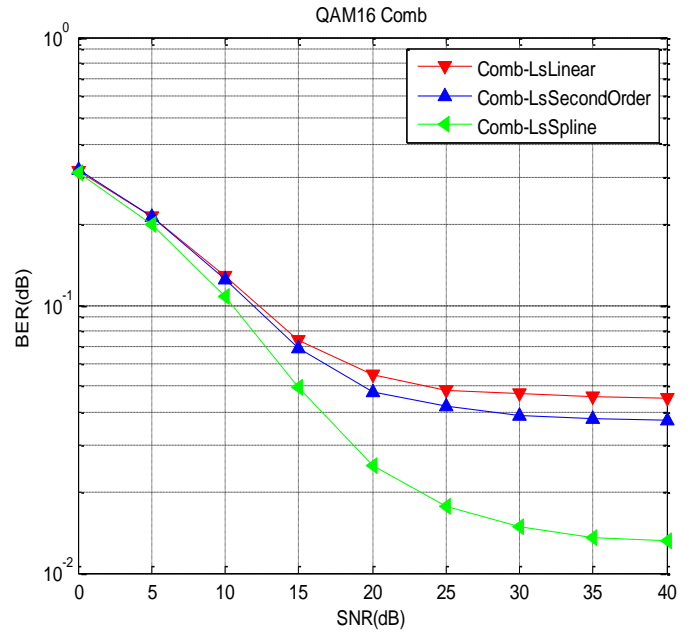


Fig. 4. Plot of BER against SNR for QAM 16 Comb (Doppler freq. 40Hz)

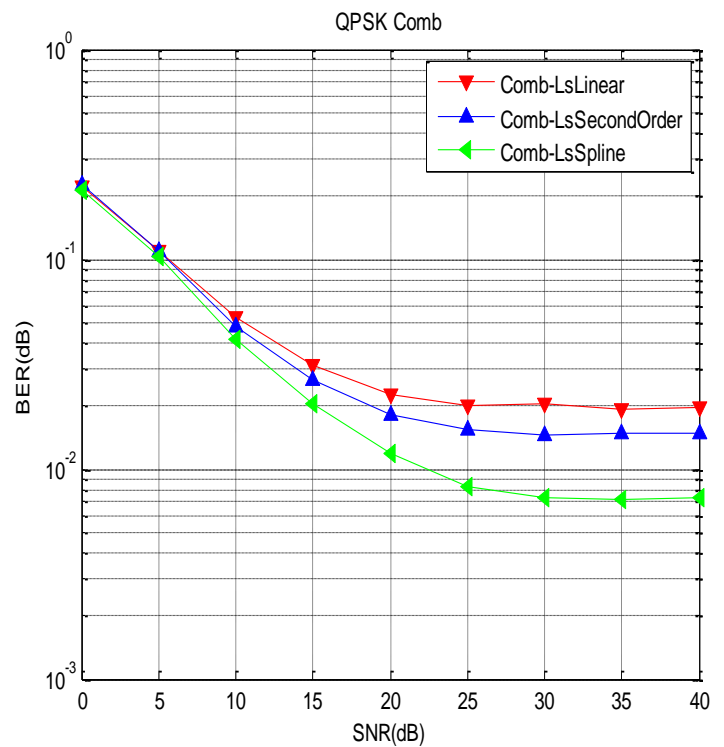


Fig. 5. Plot of BER against SNR for QPSK Comb (Doppler freq. 40Hz)

By comparing figures 4 and 5 we observe spline cubic interpolation is best. Among modulation techniques QPSK performs better than 16 QAM. For the same Doppler frequency shift spline cubic interpolation for QPSK gives lower bit error rate than spline cubic interpolation for 16 QAM.



## VII. CONCLUSION

In this paper, we fully reviewed channel estimation based on block type and comb type pilot arrangement. In addition to this we compared different modulation techniques (16 QAM, QPSK) under the assumption of Rayleigh fading channel. Different interpolation methods are investigated on the basis of plots between Bit error rate (BER) against SNR with Doppler frequency shift 40Hz, we observe spline cubic interpolation performs better than second order interpolation method for comb type channel estimation. This was expected since the comb type pilot arrangement allows the tracking of fast fading channel and spline cubic interpolation does the interpolation such that the mean square error between the interpolated points and their ideal value is minimized.

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