International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 1 Issue 3, May - 2012

Adaptive Filter Analysis for System Identification Using Various Adaptive Algorithms

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Abstract— This paper includes the analysis of various adaptive algorithms such as LMS, NLMS, Leaky LMS, Sign-Sign, Sign-error and RLS for system identification. The problem of obtaining a model of system from input and output measurements is called the system identification problem. Using adaptive filter we can find the mathematical model of unknown system based on the input and output measurement. And analyze different parameter of algorithm such as order of filter, step size, leakage factor, normalized step size and forgetting factor. It has been found that RLS faster than other, but for practical consideration LMS is better. Complexity of LMS is less as compare to RLS because of less floating point operation. As the order increases magnitude response of adaptive filter is nearly equal to the response of unknown system and mean square error also reduced.

Index Terms— Convergence speed, Least mean square error (LMS), Mean Square error, Normalized LMS, System Identification

1. INTRODUCTION

Digital signal processing systems are attractive due to their low cost, reliability, accuracy, small physical sizes, and flexibility. Coefficients of adaptive filter are continuously and automatically adapt to given signal in order to get desired response and improve the performance.

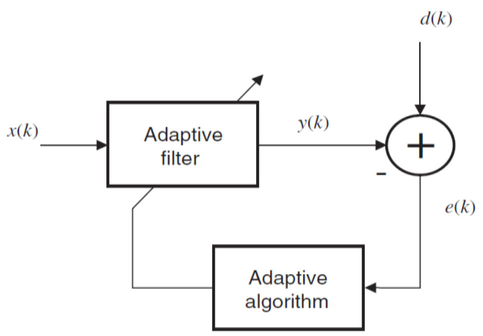


Figure 1. adaptive filter configuration

Figure 1 shows the basic adaptive filter configuration, where x(k) is the input signal, y(k) is filter output, d(k) is the desired signal, e(k) is the error signal.The main objective of adaptive filter is to minimize the error signal. Here FIR filters structure and different Algorithmic methods are used to represents complete adaptive filter specification. There are three main specifications are required for designing adaptive filter, i.e. algorithm, filter structure, application. There are numbers of structures, but widely used FIR filter structure because of its stability. Adaptive filters have been successfully applied in such diverse fields as communications, radar, sonar, seismology and biomedical engineering. Although these applications are indeed quite different in nature, nevertheless, they have one basic common feature: an input vector and desired response are used to compute an estimation error, which is in turn used to control the values of a set of adjustable filter coefficients. However, essential difference between the various applications of adaptive filtering arises in the manner in which the desired response is extracted.

2. ALGORITHMS

The algorithm is the procedure used to adjust the adaptive filter coefficients in order to minimize a prescribed criterion i.e. error signal. Most reported developments and applications use the FIR filter with the LMS algorithm because it relatively simple to design and implement. Many adaptive algorithms can be viewed as approximations of the wiener filter. As shown in figure 1, the adaptive algorithm uses the error signal

$$e(k) = d(k) - y(k) \tag{1}$$

to update the filter coefficients in order to minimize a predetermined criterion. The most widely mean square-error (MSE) criterion is defined as

$$\xi = E[e^2(k)] \tag{2}$$

Most widely used algorithm is LMS (Least Mean Square). Because it is relatively simple to design and implement. There are some set of LMS-type algorithms obtained by the modification of the LMS algorithm [5]. The motivation for each is practical consideration such as faster convergence, simplicity of implementation, or robustness of operation. Mean square error behavior, convergence and steady state analysis of different adaptive algorithms are analyzed in [2]-[4]. The LMS algorithm requires only 2L multiplications and additions and is the most efficient adaptive algorithm in terms of computation and storage requirements. The complexity is much lower than that of other adaptive algorithms such as kalman and recursive least square algorithms.

2.1 LMS Algorithm

The LMS algorithm is a method to estimate gradient vector with instantaneous value. It changes the filter tap weights so that e (k) is minimized in the mean-square sense. The conventional LMS algorithm is a stochastic implementation of the steepest descent algorithm.

$$e(k) = \mathbf{d}(k) - w(k)\, X(k) \tag{3}$$

Coefficient updating equation is

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\, x(k)\, e(k), \tag{4}$$

Where μ is an appropriate step size to be chosen as $0 < \mu < 0.2$ for the convergence of the algorithm. The larger step sizes make the coefficients to fluctuate wildly and eventually become unstable.[6]

The most important members of simplified LMS algorithms are:

2.2 Normalized LMS (NLMS) Algorithm

The normalized LMS algorithm is expressed as

$$\mathbf{w}(k+1) = \mathbf{w}(k) + 2\mu(k)\, e(k)\, \mathbf{x}(k). \tag{5}$$

$$\mu(k) = \alpha / (m+1)\, p_x(k). \tag{6}$$

Where μ (k) is the time varying step size normalized by L= (m+1) and the power of the signal x (k). Where $0 < \alpha < 1$. [3]-[4].

2.3 Leaky LMS Algorithm

Insufficient spectral excitation of the algorithm of LMS algorithm may result in divergence of the adaptive algorithms. Divergence may avoided by using leaky mechanism during the coefficient adaptation process. The leaky LMS algorithm is expressed as

$$\mathbf{w}(k+1) = v\, \mathbf{w}(k) + \mu\, e(k)\, \mathbf{x}(k). \tag{7}$$

Where v is leaky factor with range $0 \ll v < 1$.

2.4 Signed LMS algorithm

This algorithm is obtained from conventional LMS recursion by replacing e(k) by its sign. This leads to the following recursion:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\, \mathbf{x}(k)\, \mathrm{sgn}\{e(k)\} \tag{8}$$

2.5 Signed-Regressor Algorithm (SRLMS)

The signed regressor algorithm is obtained from the conventional LMS recursion by replacing the tap-input vector x (k) with the vector sgn{x(k)}.Consider a signed regressor LMS based adaptive filter that processes an input signal x(k) and generates the output y(k) as per the following:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\, \mathrm{sgn}\{\mathbf{x}(k)\} e(k) \tag{9}$$

2.6 Sign – Sign Algorithm (SSLMS)

This can be obtained by combining signed-regressor and sign recursions, resulting in the following recursion:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu\, \mathrm{sgn}\{x(n)\}\, \mathrm{sgn}\{e(n)\}, \tag{10}$$

2.7 Recursive Least square (RLS)

The RLS method typically converges much faster than the LMS method, but at cost of most computational effort per iteration. Derivation of these results can be found in references books [7]-[9]. Unlike the LMS method, which asymptotically approaches the optimal weight vector using a gradient based search, the RLS method attempts to find the optimal weight at each iteration. The expression for RLS method is

$$\mathbf{w}(k) = \mathbf{R}^{-1}(k)\mathbf{p}(k) \tag{11}$$

the design parameter associated with the RLS method are the forgetting factor $0 < \gamma \leq 1$, the regularization parameter, δ >0, and the transversal filter order, m ≥ 0.the required filter order depends on the application.

3. ADAPTIVE FILTER APPLICATION: SYSTEM IDENTIFICATION

Mathematical models of physical phenomena can be effectively apply analysis and design techniques to practical problems. In many instances, a mathematical model can be developed using underlying physical principles and understanding of the component of the system and how they are interconnected. But in some cases, this approach is less effective, because the physical system or phenomenon is too complex and is not well understood. In these cases, we have to design this mathematical model based on the measurement of the input and output. Typically, we assume that the unknown system can be modelled as a linear time system. The problem of obtaining a model of system from input and output measurements is called the system identification problem.[9]

Adaptive filter are highly effective for performing system identification using the configuration shown in figure 2.

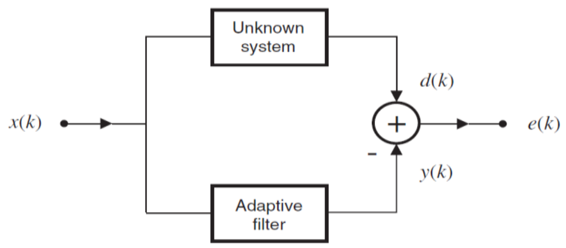


Figure 2 System identification

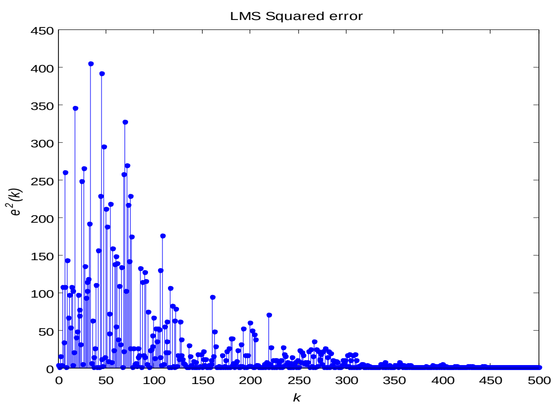
To illustrate the entire algorithm, consider the system identification problem shown in figure 2. Let the system to be identified has the following transfer function.

$$H(z)=\frac{2-3z^{-1}-z^{-2}+4z^{-4}+5z^{-5}-8z^{-6}}{1-1.6z^{-1}+1.7z^{-2}-1.436z^{-3}+0.6184z^{-4}-0.1134z^{-5}-0.0648z^{-6}}$$

Here input x(k) consists of N=1000 samples of white noise uniformly distributed over [-1,1]. Effectiveness of adaptive filter can be assess by comparing the magnitude response of the system, H(z), with the magnitude response of the adaptive filter, w(z), using final steady state weight, w(N-1). Note that this is true in spite of the fact that H(z) is a IIR filter with six poles and six zeros, while the steady state adaptive filter is an FIR filter with different specifications.[10]

4. SIMULATION RESULT

This section presents the results of simulation using MATLAB to investigate the performance behaviours of various adaptive algorithms. The principle means of the comparison is the steady state error of the algorithms which depends on the parameters such as step size, filter length and the number of iterations and identifies the unknown system. Here system is identified using different adaptive algorithms such as LMS, NLMS, Leaky LMS, sign data LMS, sign error LMS, sign sign LMS and RLS. All simulations plots are average over 500 independent runs and filter order m =50.



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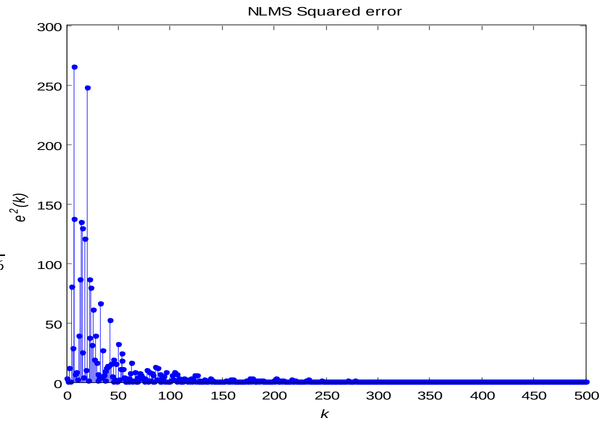

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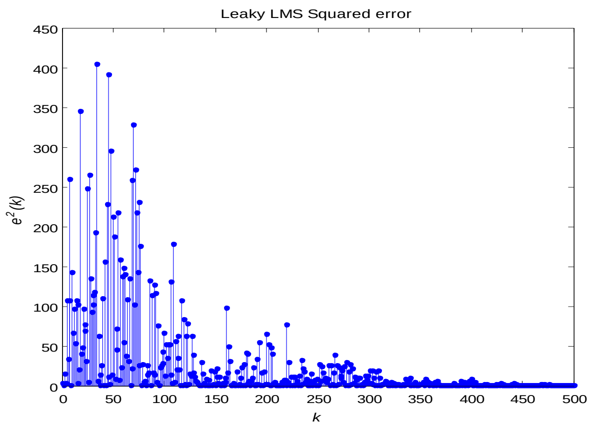
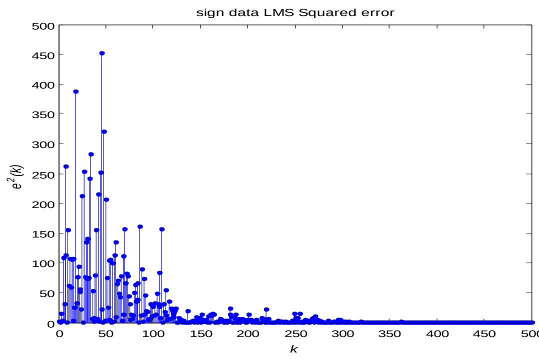


(c)

Figure 3 Plots of MSE using (a) LMS Method (b) NLMS Method (c) Leaky LMS Method using μ=0.01.(continue)

From simulation result shown in figure 3 we have seen that NLMS converge faster than LMS, Leaky LMS have same as LMS but it has excess MSE higher than the LMS. The equation of sign-sign LMS algorithm requires no multiplication. Sign sign LMS method and sign error LMS method is not useful for DSP filter applications. This simplified LMS is designed for a VLSI or ASIC implementation to save multiplications. It is used in the adaptive differential pulse code modulation for speech compression. However, when this algorithm is implemented on DSP processor with a pipeline architecture and parallel hardware multipliers, the throughput is slower than the standard LMS algorithm because the determination of signs can break the instruction pipeline and therefore severely reduce the execution speed.



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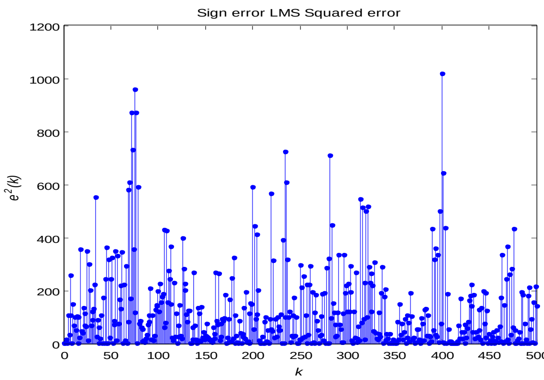

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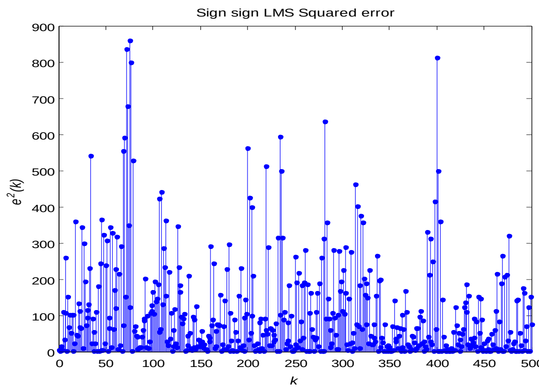

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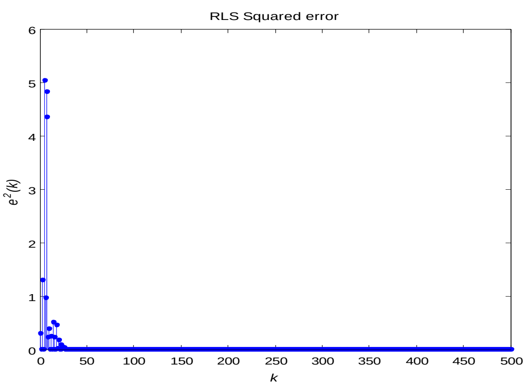
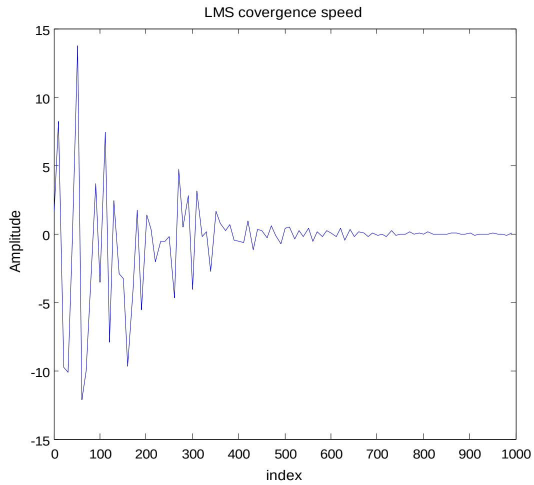


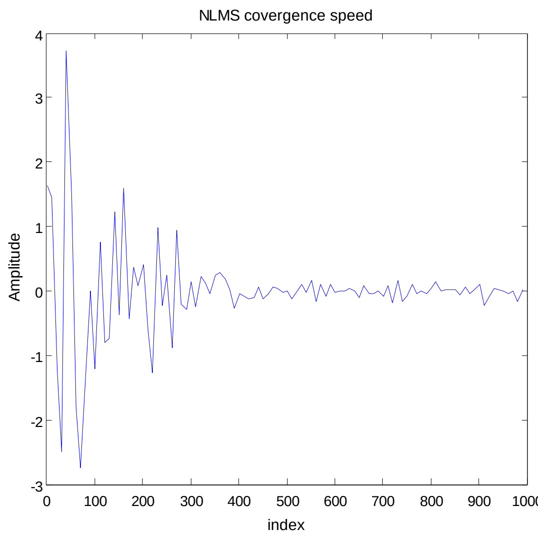
(g)

Figure 3 Figure 3 Plots of MSE using (d) Sign data LMS Method (e) sign error LMS Method (f) Sign Sign LMS Method (g) RLS Method using μ=0.01. (Continue)

Figure 4 shows the plots of convergence speed using different adaptive algorithms. we can see from that RLS method converge faster as compare to other method and sign sign and sign error take to much time and samples to convert into minimum MSE. The results in [1] show that the performance of the signed data LMS algorithm is superior than conventional LMS algorithm, the performance of signed LMS and sign-sign LMS based realizations are comparable to that of the LMS based filtering techniques in terms of signal to noise ratio and computational complexity.

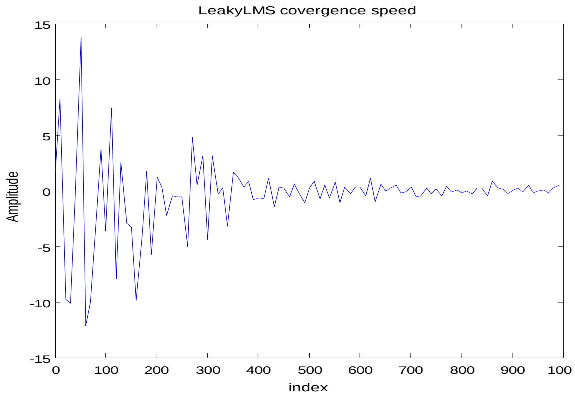


(a)



(b)

Figure 4 Plots of convergence speed using (a) LMS Method (b) NLMS Method using μ=0.01.(continue)



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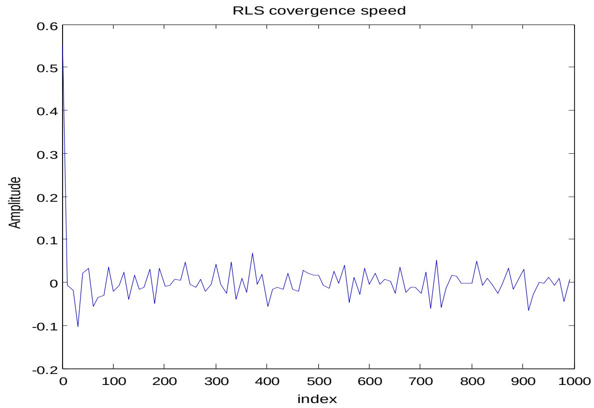

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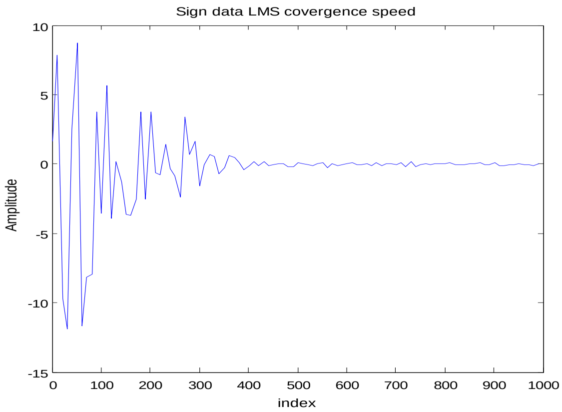

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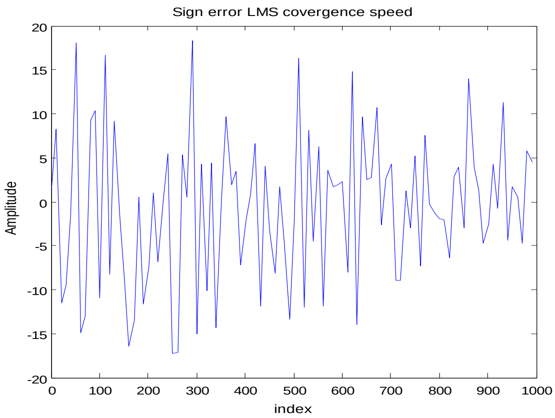

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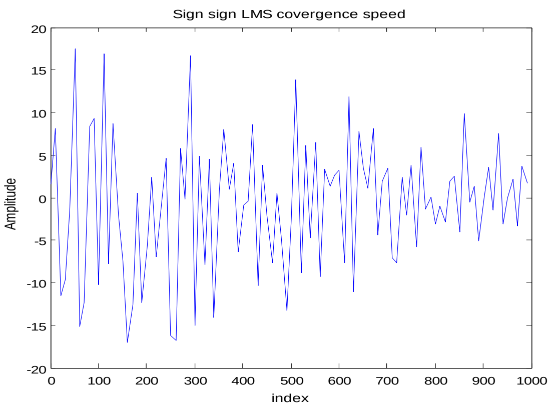

(f)

Figure 4 Plots of convergence speed using (c) Leaky LMS Method (d) sign data LMS Method (e) sign error LMS Method (f) sign sign LMS Method (g) RLS Method using μ=0.01.(continue)

Table 1

Method	**MSE**	**C**	**MSE**	**C**
	μ = 0.01		μ =0.004	
LMS	0.0870	450	0.1967	900
NLMS	0.0170	400	0.0170	400
Leaky LMS	0.0896	600	0.2076	1000
Sign data LMS	0.0630	400	0.1257	700
Sign error LMS	0.6216	2300	0.8309	4000
Sign sign LMS	0.4732	1500	0.7469	3000
RLS	1.4443e-004	30	1.4443e-004	30

Table 1 shows the relation between the MSE and convergence speed for different algorithms using two different values of μ. It shows that for small value of μ MSE is high and convergence time is also high.

$$M = \mu\,(L+1)\,P(x) \qquad (12)$$

Where M gives miss adjustment factor, P(x) gives the power of input signal and L indicates the filter length.

Table 2

μ	M
0.01	0.17
0.004	0.068

Table 2 shows the relation between excess MSE and step size. Where M indicates the miss adjustment factor. if the step size is higher M is also higher. It means that after converge in to minimum MSE there are excess MSE is also presents due to noisy gradient estimation. And it may not be zero at minimum MSE. So there is always tradeoff between the convergence speed and steady state accuracy.

5. CONCLUSION

We have studied and analyzed different adaptive algorithms for system identification. LMS algorithm is useful for practical implementation.RLS method is faster than the LMS methods but require larger number of floating point operation. For LMS m(50) Flops is required and for RLS $3m^2$ (7500) Flops are required. Normalized LMS method, leaky LMS method, sign data, sign error and sign sign LMS are the modified version of LMS method, which are used according to requirement of application. Sign error and sign sign LMS method have larger MSE and take too much time to converge. There is always the tradeoff between convergence speed and steady state accuracy.

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