

An AHP Approach for Determining the Weightage of Reaction Turbine Parameters Influencing its Performance

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Abstract— Cavitation and its negative consequences have a significant effect on the overall performance of a hydro power plant. Various causes which provoke the formation of different types of cavitations have a significant effect on the performance of a hydro power plant. In this present paper an attempt has been made to find the most significant cause and the relative weightage of the causes which influences the performance of a hydropower plant. A Fuzzy scale has been introduced to rate different parameters. The rating has been done on the basis of information provided in different literatures and published research papers. The weightage of different parameters is obtained with AHP method. The waterfall model of AHP has generated the weightage of the causes which significantly affects the performance of a hydro power plant. A Francis Turbine has been considered for study.

Key words: Cavitation; AHP; FUZZY; Francis turbine

1. INTRODUCTION

Cavitation is formation of vapor bubbles in the liquid flowing through any Hydraulic Turbine. Cavitation occurs when the static pressure of the liquid falls below its vapor pressure. In turbines cavitation occurs mainly near the fast moving blades and in the exit region [1]. Knapp et al. [2] defined cavitation as the phenomenon which takes place at constant temperature when a liquid reaches a state at which vapor cavities are formed and grow due to dynamic-pressure reductions to the vapor pressure of the liquid. The emission of large amplitude shock-waves are generated in a very short time of about several nanoseconds due to the violent process of cavity collapse takes place and as demonstrated by Avellan and Farhat [3]. Travelling bubbles, attached cavities or cavitating vortices are the different forms of cavities which can form in a flowing liquid described by Hammit [4] and Arndt [5].

Cavitation is defined as the formation of the vapor phase in a liquid. The initial formation of bubbles (inception) to large-scale, attached cavities (supercavitation) implies the term cavitation. The formation of individual bubbles and subsequent development of attached cavities, bubble clouds, etc., is directly related to reductions in pressure to some critical value, which in turn

is associated with dynamical effects, in a flowing liquid [6].

Cavitation causes hazardous effects on hydraulic turbines such as erosion, vibration, machine efficiency loss and noise depending on the various causes of occurrence such as higher or lower head than the machine design, partial or high load, velocity component of flow discharge and the plant cavitation number. This is why it is necessary to know what brings about the creation of steam bubbles in the liquid flow of hydraulic turbines and the ways to avoid harmful cavitation conditions. The paper presents the cavitation types and the most important causes and effects on the performance of reaction turbine based on their weightage using fuzzy scale and AHP method.

The earliest studies on the theory of a cavitating flow with free boundaries and supercavitation were published in the book [7] followed by [8]. A large number of exact solutions of plane problems derivations can be easily obtained with the help of these books as it includes well-developed theory of conformal mappings of functions of a complex variable. Another venue combining the existing exact solutions with approximated and heuristic models was explored in the work [9] that refined the applied calculation techniques based on the principle of cavity expansion independence, theory of pulsations and stability of elongated axisymmetric cavities, etc [10], [11].

APPENDIX NOMENCLATURE

P_B	bubble pressure (Pa)
R_B	bubble radius (m).
R_o	bubble maximum radius (m)
P_v	vapor pressure (Pa)
P_∞	infinite domain pressure (Pa)
γ	Surface tension (N/m)
ν	viscosity (m ² /s)
ρ	density (Kg/m ³)
τ	Rayleigh time (s)
σ_p	Thoma coefficient
GM	geometric mean
CI	consistency index

RI	random index
LE	Leading Edge
TB	Travelling Bubble
IB	Inter Blade Vortex
DT	Draft Tube Swirl
H	Head
L	Load
V	Velocity Component Flow Discharge
C	Plant Cavitation Number

1.1. Effects Of Cavitation In Turbine

Cavitation can affect the performance of turbomachinery resulting in decreased efficiency of hydro turbines [12], [13], [14]. Noise and vibration occur in many applications. In addition to the deleterious effects of reduced performance, noise and vibration, there is the possibility of cavitation damage [6],[12],[13],[15], [16].

They cause the erosion noise, mechanical vibrations, and modification of flow field of the hydraulic machine [16]. According to the fact that cavitation is connected with efficiency change, increase of noise level and increase of vibrations, the numerous researches are focused on various experimental methods which indirectly via the above mentioned effects estimate the cavitation phenomena in water turbines [17],[18].

2. DIFFERENT FORMS OF CAVITIES IN LIQUID FLOW

2.1. Vortex Cavitation

At low pressures, flow regions with concentrated vortices can develop cavitation in their central cores. The solid surface becomes potentially erosive if the tips of the vapor filled vortices are in contact with it since the final collapse of the whole cavity takes place on them. If Von Karman vortex-shedding occurs at the trailing edge of a hydrofoil this type of cavitation can develop when pressure is low enough. As a result, lift fluctuations are provoked synchronized with the shedding frequency [19].

2.2 Attached Cavities

In the flow macro-cavities are developed on a solid wall that takes the form of cavitation. Cavitation grows from the leading edge on the suction side of a hydrofoil with a positive angle of incidence. This is a very common and complex type of cavitation that can present different regimes depending on the hydrodynamic conditions. Sheet cavitation is one of the regimes, which is characterized by thin stable cavities with smooth and transparent interfaces. At their rear part, the cavity closure presents a slight and weak pulsation due to the shedding of small cavitation vortices so that it represents a low risk of erosion [19].

Cloud cavitation is another regime, shows a strong unsteadiness and a pulsating behavior that provokes significant oscillations of the cavity length. The cavity interface is wavy and turbulent. Large U-shaped transient cavities and clouds of cavities are shed away downstream of the cavity closure that collapse violently on the solid surface [20]. Consequently, this is a very aggressive form of cavitation with a high erosive power. When this type of

cavitation occurs in turbomachinery it can induce abnormal dynamic behavior and cause serious erosion [19].

2.3 Travelling Bubble

In low pressure regions of the flow bubbles usually appear around a body from micron-sized nuclei. They implode when they find an adverse pressure gradient while travelling with the flow. Air content of the liquid influences these bubbles strongly. Nevertheless, their erosive power is considered to be relatively weak [19]. Based on the assumption that remains spherical in an infinite liquid an isolated bubble can be modeled [21]. In this case, the generalized Rayleigh-Plesset equation is a valid approximation of the bubble growth and it can be solved to find the radius of the bubble, $R_B(t)$ provided that the bubble pressure, $P_B(t)$; and the infinite domain pressure, $P_\infty(t)$; are known:

$$\frac{P_B(t) - P_\infty(t)}{\rho} = R_B \frac{d^2 R_B}{dt^2} + \frac{3}{2} \left(\frac{dR_B}{dt} \right)^2 + \frac{4\nu}{R_B} \frac{dR_B}{dt} + \frac{2\gamma}{\rho R_B}$$

Assuming that the bubble reaches a maximum radius, R_0 ; from which the implosion or collapsing process starts, the Rayleigh time or collapse time τ ; until $R_B = 0$ is reached is given by

$$\tau = 0.915 R_0 \sqrt{\frac{\rho}{P_\infty - P_0}}$$

2.4 Leading Edge Cavitation

On the suction side of the runner blades or on the pressure side [14] it takes the form of an attached cavity in (fig 1) due to operation at a higher head than the machine design head when the incidence angle of the inlet flow is positive and largely deviated from the design value or at a lower head than the machine design head when the incidence angle is negative [19]. This type of cavitation is a very aggressive and deeply erodes the blades also provoke pressure fluctuations [1] when it becomes unstable. This type of cavitation is not very sensitive to the value of the Thoma number and it can lead to a severe erosion of the blades [14].



Fig1. Shows Leading edge cavitation Picture courtesy of Saini [22]

2.5 Travelling bubble cavitation

It is attached to the blade suction side near the mid-chord next to the trailing edge which takes the form of separated bubbles (fig 2). When the machine operates in overload condition with the highest flow rate these travelling bubbles appear due to a low plant cavitation number σ_p and they grow with load reaching their maximum [19]. This type of cavitation is a severe and noisy type which significantly reduces the machine efficiency and if the bubbles collapse on the blade it that provoke erosion [1]. This type of cavitation is very sensitive to the content of cavitation nuclei and to the value of the Thoma number [14], [23].



Fig2.Shows travelling bubble cavitation Picture courtesy of Grindoz [24]

2.6 Draft tube swirl

It is a cavitation vortex-core flow that it formed just below the runner cone in the centre of the draft tube fig 3. Its volume depends on σ_p and it appears at partial load and at overload due to the residual circumferential velocity component of the flow discharged from the runner [19]. This type of cavitation provokes large bursts of pressure pulses in the draft tube causing strong vibrations on the turbine and even on the powerhouse [1].



Fig 3.Shows draft tube swirl Picture courtesy of Saini [22]

2.7 Inter-blade vortex cavitation

This is formed by secondary vortices located in the channels between blades that arise due to the flow separation provoked by the incidence variation from the hub to the band (fig 4). These vortices appear at partial load operation and yield a high broadband noise level. They can also appear and cavitate at extremely high-head operation ranges because the σ_p is relatively low [19]. It cause strong vibrations if becomes unstable and if their tip is in touch with the runner surface they can result in erosion.

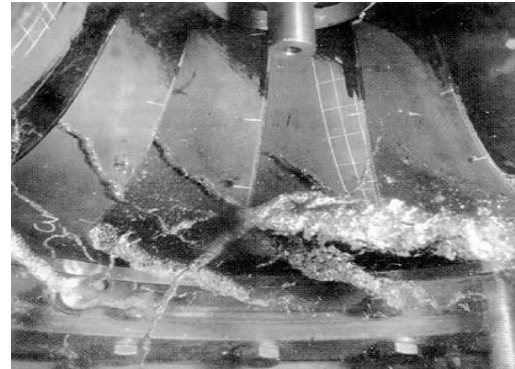


Fig4.Shows Inter-blade vortex cavitation Picture courtesy of Avellan [20]

3. FUZZY LOGIC

Fuzzy logic concept has been introduced by Lotfi Zadeh. It is relatively young theory; the areas of applications are process control, management and decision making, operations research, economics. The major advantage of this theory is that linguistic terms are used in description of problems, rather than in terms of relationships between precise numerical values is.

A linguistic variable are not expressed in numbers but words or sentences in a natural or artificial language, i.e., in terms of linguistic (Zadeh, 1975) [25]. The concept of a linguistic variable is very useful in dealing with situations, which are too complex or not well defined to be reasonably described in conventional quantitative expressions (Zimmermann, 1991)[26]. For example, 'weight' is a linguistic variable whose values are 'very low', 'low', 'medium', 'high', 'very high', etc. Fuzzy numbers can also represent these linguistic values.

Fuzzy logic aims to model human thinking and reasoning. The key advantage of the fuzzy methods is how they reflect the human mind in its remarkable ability to store and process information that is imprecise, uncertain and resistant to classification [27].

Fuzzy logic theory is a uniquely useful tool due to its capability of handling the inherent fuzziness or imprecision of real-world information in a scientific fashion [28] and becomes even more powerful when applied in conjunction with analytical modeling and stochastic simulation.

In the decision-making process fuzzy logic has been with analytic hierarchy process to form a model for risk assessment. Those risk assessment models are widely applied to multiple fields such as floor water invasion in coal mines [29], oil and gas offshore wells [30], electronic

engineering [31], [32], information technology projects [33], green initiatives in the fashion industry [34], food supply chains [35].

The linguistic fuzzy scale used in this paper is given below:

3.1 Analytic Hierarchy Process

Multi Criteria decision making method, Analytic Hierarchy Process (AHP) was developed by Saaty [36], [37], [38] uses a process of pair wise comparison to determine the relative importance of alternatives in decision making. It converts individual preferences into ratio scale weights that can be combined into a linear additive weight for each alternative.

The steps of fuzzy AHP decision making are given as follows

- a) Find out the relative importance of different criteria with respect to the objective. For this a pair-wise comparison matrix using a scale of relative importance has to construct. Using the fundamental scale of the AHP the judgments are entered. An attribute compared with it is always assigned the value 1 so the main diagonal entries of the pair-wise comparison matrix are all 1. The numbers 6, 7, 9, and 11 correspond to the verbal judgments ‘slightly more effective’, ‘effective’, ‘more effective’, and ‘extremely more effective’ (with 1, 2, 3, and 4 for compromise between the previous values). Assuming M criteria, the pair-wise comparison of attribute i with attribute j yields a square matrix A1 where r_{ij} denotes the comparative importance of attribute i with respect to attribute j. In the matrix, $r_{ij} = 1$ when $i = j$ and $r_{ji} = 1 / r_{ij}$

$$AI = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & \dots & M \end{matrix} & \begin{matrix} \text{Criteria} \\ \left[\begin{matrix} r_{11} & r_{12} & r_{13} & \dots & r_{1M} \\ r_{21} & r_{22} & r_{23} & \dots & r_{2M} \\ r_{31} & r_{32} & r_{33} & \dots & r_{3M} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ r_{M1} & r_{M2} & r_{M3} & \dots & r_{MM} \end{matrix} \right] \end{matrix} & \begin{matrix} \left[\begin{matrix} 1 \\ 2 \\ 3 \\ \dots \\ \dots \\ M \end{matrix} \right] \end{matrix} \end{matrix} \quad (1)$$

- b) Find the relative normalized weight (W_j) of each at-tribute by i) calculating the geometric mean of i^{th} row and ii) normalizing the geometric means of rows in the comparison matrix. This can be represented as

$$GM_i = \left(\prod_{j=1}^M r_{ij} \right)^{1/M} \quad (2)$$

$$W_j = GM_i / \sum_{i=1}^M GM_i \quad (3)$$

The geometric mean method of AHP is used in the present work to find out the relative normalized weights of

the attributes because of its simplicity and easiness to find out the maximum Eigen value and to reduce the inconsistency in judgments.

- c) Calculate matrix A3 and A4 such that $A3 = A1 \times A2$ and $A4 = A3 / A2$, where $A2 = [W_1, W_2, \dots, W_M]^T$. Each element of A4 is obtained by dividing each element of A3 by the corresponding element of A2.

Extremely more effective	More effective	Effective	Slightly more effective	Equal weightage	Slightly less effective	Less effective	Very less effective	Extremely less effective
11	9	7	6	5	4	3	2	1

- d) Find out the maximum eigen value λ_{max} (i.e. the average of matrix A4).

- e) Calculate the consistency index $CI = (\lambda_{max} - M) / (M - 1)$. The smaller the value of CI, the smaller is the deviation from the consistency.

- f) Obtain the random index (RI) for the number of attributes used in decision making [39].

- g) Calculate the consistency ratio $CR = CI/RI$. Usually, a CR of 0.1 or less is considered as acceptable and it reflects an informed judgment that could be attributed to the knowledge of the analyst about the problem under study.

4. FORMULATIONS OF COMPARISON MATRIX AND WEIGHTED MATRIX

4.1 Pair wise comparisons of negative effects on the basis of turbine performance

	Vibration	Erosion	Efficiency	Noise
Vibration	1	2	1/3	4
Erosion	1/2	1	1/4	5
Efficiency	3	4	1	6
Noise	1/4	1/5	1/6	1

Table1. Shows the pair-wise comparisons determine the weightage of effects on turbine performance

Weightage of Effects on the basis of turbine performance

Vibration	Erosion	Efficiency	Noise
0.2343	0.1679	0.5404	0.0574

Table2. Shows the weightage of negative effects w.r.t turbine performance

4.2 Weightage of cavitation types depending on their negative effects on performance of turbine

In terms of erosion, pair-wise comparisons determine the weightage of different types of cavitation

	LE	TB	IB	DT
LE	1	9	7	11
TB	3	1	4	6
IB	4	6	1	7
DT	2	4	3	1

Table3. Shows the pair-wise comparisons determine the weightage of different types of cavitation in terms of Erosion.

Weightage of cavitation type depending on erosion effect

Leading edge	Travelling bubble	Inter blade vortex	Draft tube swirl
0.36906	0.21031	0.26094	0.15969

Table4. Shows the weightage of different types of cavitation w.r.t erosion

In terms of vibration, pair-wise comparisons determine the weightage of different types of cavitation:

	LE	TB	IB	DT
LE	1	4	2	3
TB	6	1	3	2
IB	11	7	1	6
DT	9	11	4	1

Table5. Shows the pair-wise comparisons determine the weightage of different types of cavitation in terms of Vibration.

Weightage of cavitation type depending on vibration effect

Leading edge	Travelling bubble	Inter blade vortex	Draft tube swirl
0.16105	0.17824	0.33729	0.32405

Table6. Shows the weightage of different types of cavitation w.r.t vibration

In terms of efficiency, pair-wise comparisons determine the weightage of different types of cavitation:

	LE	TB	IB	DT
LE	1	3	4	6
TB	9	1	11	7
IB	6	2	1	7
DT	4	3	3	1

Table7. Shows the pair-wise comparisons determine the weightage of different types of cavitation in terms of Efficiency.

Weightage of cavitation type depending on efficiency reduction

Leading edge	Travelling bubble	Inter blade vortex	Draft tube swirl
0.21203	0.38341	0.22359	0.18097

Table8. Shows the weightage of different types of cavitation w.r.t efficiency

In terms of noise, pair-wise comparisons determine the weightage of different types of cavitation:

	LE	TB	IB	DT
LE	1	2	3	4
TB	11	1	6	7
IB	9	4	1	4
DT	7	3	6	1

Table9. Shows the pair-wise comparisons determine the weightage of different types of cavitation in terms of Noise.

Weightage of cavitation type depending on noise

Leading edge	Travelling bubble	Inter blade vortex	Draft tube swirl
0.16251	0.33695	0.25421	0.24532

Table10. Shows the weightage of different types of cavitation w.r.t noise

4.3 Weightage of the causes of cavitation in different types of turbine

In terms of leading edge cavitation, pair-wise comparisons determine the weightage of causes:

	H	L	V	C
H	1	11	9	6
L	2	1	6	4
V	3	4	1	4
C	4	7	9	1

Table11. Shows the pair-wise comparisons determine the weightage of causes in terms of LE

Weightage of causes of Leading edge cavitation are

Head	Load	Velocity component of flow discharge	Plant cavitation number
0.34645	0.18732	0.18675	0.27947

Table12. Shows the weightage of cause w.r.t LE

In terms of travelling bubble cavitation, pair-wise comparisons determine the weightage of causes:

	H	L	V	C
H	1	3	4	2
L	9	1	6	4
V	3	2	1	5
C	11	7	6	1

Table13. Shows the pair-wise comparisons determine the weightage of causes in terms of TB

Weightage of causes of travelling bubble cavitation are

Head	Load	Velocity component of flow discharge	Plant cavitation number
0.16511	0.28595	0.19713	0.35181

Table14. Shows the weightage of cause w.r.t TB

In terms of inter blade vortex cavitation, pair-wise comparisons determine the weightage of causes:

	H	L	V	C
H	1	6	9	11
L	4	1	7	9
V	3	2	1	6
C	2	3	4	1

Table15. Shows the pair-wise comparisons determine the weightage of causes in terms of IB

Weightage of causes of inter blade vortex cavitation are

Head	Load	Velocity component of flow discharge	Plant cavitation number
0.36115	0.29193	0.18411	0.16281

Table16.Shows the weightage of cause w.r.t IB

In terms of draft tube swirl cavitation, pair-wise comparisons determine the weightage of causes:

	H	L	V	C
H	1	3	2	4
L	9	1	6	7
V	7	5	1	9
C	6	3	2	1

Table17. Shows the pair-wise comparisons determine the weightage of causes in terms of DT

Weightage of causes of draft tube swirl cavitation are

Head	Load	Velocity component of flow discharge	Plant cavitation number
0.16595	0.32905	0.31423	0.19077

Table18.Shows the weightage of cause w.r.t DT

5. RESULTS

Now for the final weightage of cavitation types from the effects

Vibration	erosion	efficiency	noise		
LE	0.1611	0.3691	0.2120	0.1625	0.2343
TB	0.1782	0.2103	0.3834	0.3370	0.1679
IB	0.3373	0.2610	0.2236	0.2542	0.5404
DT	0.3241	0.1596	0.1810	0.2463	0.0574

Table19. Shows the final comparison matrix between the negative effects and the different types of cavitation

0.2236	LE
0.3035	TB
0.2582	IB
0.2146	DT

Table20. Shows the result weightage matrix of the different types of cavitation

The final weightage of causes on turbine performance from cavitation types:

	LE	TB	IB	DT	
H	0.3465	0.1651	0.3612	0.1660	× [0.2236 LE 0.3035 TB 0.2582 IB 0.2146 DT
L	0.1873	0.2860	0.2920	0.3290	
V	0.1868	0.1971	0.1840	0.3142	
C	0.2795	0.3518	0.1628	0.1908	

Table21. Shows the final comparison matrix of causes and different types of cavitation

0.2564	H
0.2746	L
0.2165	V
0.2522	C

Table22. Shows the weightage of the final result matrix of causes

6. CONCLUSIONS

From literatures different negative consequences of cavitation which are dominant for the declined performance of a hydro power plant are identified and rated on the fuzzy scale to build the AHP model. The AHP model has given the weightage of the negative effects of cavitation which are predominant on the overall performance of a hydro power plant. In the next stage the weightage of different types of cavitation is determined on the basis of their contribution towards causing a particular type of negative effect. Finally the weightage of different causes of cavitation are determined separately for each type of cavitation. These different weightage matrices have given the weightage of the causes of cavitation on the basis of their ultimate effect on hydro power plant performance. It has been seen that Load and Head are two most significant causes affecting the overall performance of a hydro power plant. The weightage of different causes may help in taking strategic decision for the establishment of a hydro power plant and at the same time at dynamic scenario during the operation of the turbine it may help in regulating the operational parameters for maximizing the performance of the plant and turbine.

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