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An Alternative Formulation for Determining Stiffness of Members with Bolted Connections

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***Abstract*— In the present work a simple yet reliable mathematical formulation proposed for determining stiffness of members with bolted connection by subtracting the stiffness of solid cone from the stiffness of internal cylindrical portion (bolted portion). Three dimensional stress distributions of the member are considered in the present formulation. Mathematical validations of the results are done using formulations in the literature. It is found that, the present work gives satisfactory result for member with the ratio of total thickness of the clamped members to diameter of the bolt are approximately up to seven.**

***Keywords: Clamped Zone, Stiffness, Pressure Distribution, Contact Load*.**

I. INTRODUCTION

The bolted joint is a mechanical connection, which is commonly used in automobiles, machine components, pressure vessels, civil structures, etc. Bolted joints find extensive use in the industry due to ease of disassembly without destroying the joined components. This flexibility makes them appropriate fasteners for joints which need frequent disassembly and inspection. As fasteners, bolted joints play a critical role in the safe design of mechanical systems. Failure of a joint can cause catastrophic failure of the system and ultimate economic and human loss. There are several theoretical solutions suggested for determining the stiffness of members with bolted connection by Sneddon [1], Fernlund [2], Nelson [3], Greenwood [4], and Lardner [5], etc. Bradley et al. [6] used a three-dimensional photo-elastic analysis to guess the interface pressure distribution between the members. Gould and Mikic [7] and Tang and Deng [8] have used finite element analysis (FEA) to find the pressure distribution between the members. They also noted that there was a radius at which flat and smooth members become separated. The computations were performed for models of steel plates with various thicknesses. In their studies, the bolts were replaced by uniformly distributed axisymmetric loads on the connected parts of the bolted joint. Osman et al. [9] discussed a design method for calculating an optimal bolt diameter required for a specific fatigue loading situation. He has suggested that a hollow cylinder, whose outside diameter is 1.5 times the bolt diameter, can be used to determine the area under compression and, thus, the member stiffness. Edwards and McKee [10] and Bickford [11] cited suggestion of the Association of German Engineers to determine the member stiffness using an equivalent cylindrical area dependent on the size of the joint. Ito et al. [12] have used ultrasonic techniques to determine the pressure distribution between the members of bolted joints for various surface topographies, materials, and thicknesses of the members. They suggested the use of a pressure-cone method developed by Rotscher [13] for stiffness calculation with variable cone angles that are generally larger than the cone angles thus far theoretically calculated by other authors. Researchers provided many analytical formulae for evaluation of the member stiffness by assuming the clamped zone in the joint to be of a particular shape. Motosh [14] provided the most realistic technique by allowing the stress in the members to vary in both axial and radial directions. Motosh [14] assumed that stress in any plane perpendicular to the axis is maximum at the hole diameter and decreases continuously to zero at the boundary of either a conical or a spherical envelope. The compressive stress in the members is described by a fourth order polynomial depending upon radial coordinate, axial coordinate and dispersion angle. The stiffness is then computed using a series of tedious integrations. Shigley and Mitchell [15] assumed the clamp zone to be in the shape of double conical frusta symmetric about the joint mid plane with semi cone angle and proposed a formula for member stiffness evaluation. Later, Shigley and Mischke [15] modified the model and proposed a new formula by keeping the semi cone angle as a variable and proposed as the best value of semi cone angle and assuming the washer diameter to be equal to 1.5 times the diameter of the bolt. The proposed best practical value of semi cone angle is 30º. Though the conical clamp model of the bolted joint is widely accepted in the industry, arbitrariness in the choice of the pressure cone angle introduces error in the model. Manring [16] performed sensitivity analysis of conical clamp model proposed by Shigley and Mischke [15] and the results show that the calculations are quite sensitive to the value of cone angle and more conservative designs can be obtained by using lower cone angle values. Apart from the analytical techniques, many researchers used finite element method and experiments to calculate the stiffness of the connected parts of a bolted joint. Maruyama performed axisymmetric finite element analysis of specific joint geometry including bolt and nut deflection and, demonstrating reasonable agreement between experiments and finite element analyses. Lehnhoff [17] performed extensive analysis using finite elements considering various bolt sizes, geometries and different

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member materials in the same joint and presented results in the form of graphs of dimensionless stiffness corresponding to the member thickness ratio. The main objective of the present analysis is to use of a smaller cone angle in the conical clamp model improve the agreement between basic theory and FEA. Most of the methods presented here take different forms. This difference is mainly due to the assumptions made during the model development. The differences caused by various assumptions need higher safety factors for reliable design. The present work proposes a new analytical model for calculating the bolted-joint stiffness.

II. MATERIAL USED

The whole joint parts are considered to be made of mild steel. The chemical and mechanical properties are listed in the below:

Table-1: Chemical Properties of the mild steel used:

Characteristics	Value (%)
Carbon	0.16-0.18
Silicon	0.4 (maximum)
Manganese	0.7-0.9
Sulphur	0.04 (maximum)
Phosphorous	0.04

Table-2: Mechanical Properties of the mild steel used:

Characteristics	Value
Modulus of elasticity (E)	210000 N/mm2
Shear modulus (G)	81000 N/mm2
Poisson's ratio (μ)	0.3

III METHODOLOGY

The stiffness of bolt and member are needed to calculate the resultant forces in bolts and members. Calculation of bolt stiffness is easy and is given by the ratio of tensile force induced in the bolt to the deflection produced. There may be more than two members included in the grip of the fastener, which all together acts like compression springs in series.

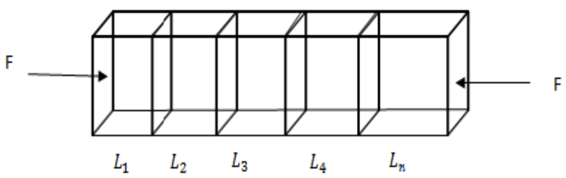

Fig. 1. Several elastic blocks in compression

Due to the three dimensional stress distribution, determination of the member stiffness is not so easy. Computation effort needed to find out the effective cross sectional area of the member, which is actually influencing the member stiffness, is also high. The compression spreads out between the bolt head and the nut and hence the area is not uniform. So it requires better understanding of the forces, stresses and deformations of the member. Moreover the dimension i.e., geometry of the member will affect the stiffness and also other parameters like washer dimensions, clearance between bolt and bolt hole play a significant role in stiffness of the member. The stress distribution within the material under the bolt has a complex geometry. This problem has been studied in a number of investigations where, an accurate computation of the distribution of the stresses volume is found to be quite complicated. The compressive stress in the material is highest directly under the bolt and falls off as laterally from the bolt centre line. At some lateral distance from this centre line, the compressive stress at the joint interface reaches to zero, and beyond that point the joint tends to separate since it cannot sustain a tensile stress. The first step for building the analysis is to guess the pressure distribution through the member as a cone with an envelope angle (α).

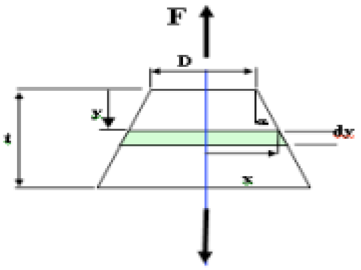

Fig. 2. Schematic diagram Stiffness derivation

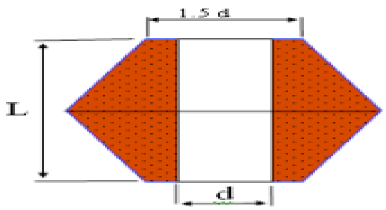

Fig. 3. Identical members

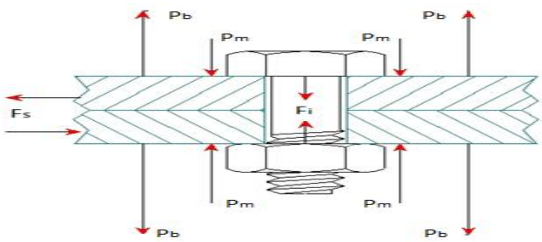

Fig. 4. Schematic diagram of joint force distribution

In the present case, first stiffness of the solid cone is to be found out, and subsequently, stiffness of the cylindrical portion having diameter same as the diameter of hole is to be subtracted from it. Stiffness of the solid cone can be calculated with the help of Fig.2 and considering the strain along the thickness of plates which is in Y direction. If contraction of plate along Y is δ then using elastic theory, the deflection could be formulated as a function of external force (P), modulus of elasticity (E), joint diameter (D), thickness (t) and dispersion angle (α) as,

$$\delta / y = P / (A \times E) \quad \text{or} \quad d\delta = (P \times dy) / (A \times E) \qquad (1)$$

Effective clamped area $(A) = \pi \times x^2 = \pi \times (D/2 + y \tan \alpha)^2$

Integrating (1) with respect to dy from limit 0 to t we get deflection function as

$$\delta = (Pt) / (\pi \times E \times (D/2) \times (D/2 + t \times \tan \alpha)) \qquad (2)$$

From (2) the stiffness of the solid cone (K) is found to be as,

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$$P/\delta = (\pi \times E \times D/2t) \times [D/2 + t \times \tan\alpha] = K \qquad (3)$$

Finally the stiffness of the member is obtained by subtracting the stiffness of cylindrical hole portion i.e., the stiffness of bolt portion from (3)

$$K\,(\text{member}) = K - K(\text{cylinder}) = \pi \times E \times (D/2t) \times [D/2 + t \times \tan\alpha]$$

$$-\ (\pi \times E \times (d_h)^2) / (4t) \qquad (4)$$

IV RESULTS AND DISCUSSION

Fig. 5 to Fig. 11 demonstrates the comparison of the variation of member stiffness with L/d (L is the grip length & d= bolt diameter) ratio using existing formulas and proposed formula and found to be comparable. Different bolt diameter and bolt grip were chosen. The clearly demonstrate the agreement of results, and thereby, establish the validity of the proposed methodology. In standard bolts, increasing bolt diameter leads to larger bolt heads, which in turn, increases the contact area under the head that ultimately increasing the member stiffness. Furthermore, the member deflection increases with increasing member grip length under the application of constant external load. Thus, the stiffness is decreasing with increasing L/d ratio. The maximum percentage of error in the present study is found to vary from 12% to 16%. Consider the difficulty of controlling the contact frictions between the bolt head and member part and also between the joint members assembly, this range of error is acceptable

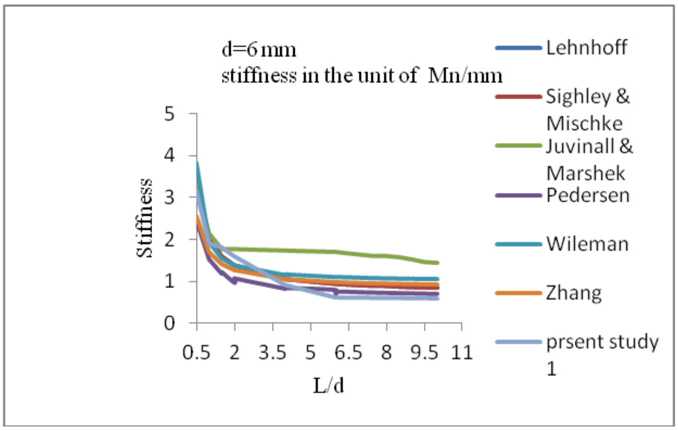

Fig. 5. Variation of stiffness of members for bolt grade 6M

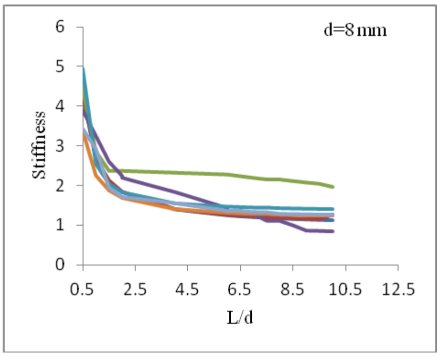

Fig. 6. Variation of stiffness of members for bolt grade 8M

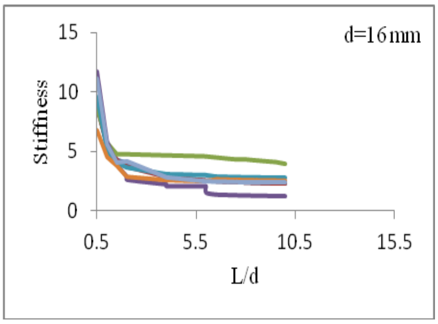

Fig. 7. Variation of stiffness of members for bolt grade 16M

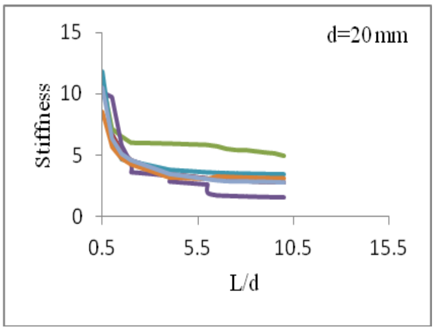

Fig. 8. Variation of stiffness of members for bolt grade 20M

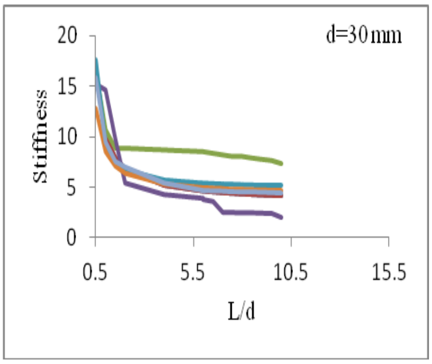

Fig. 9. Variation of stiffness of members for bolt grade 30M

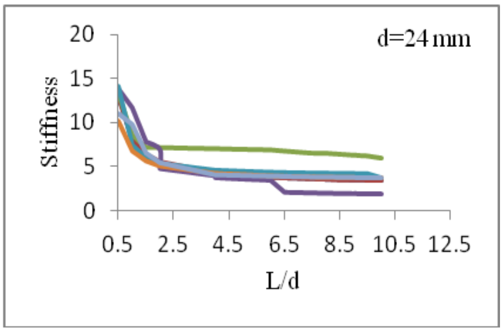

Fig. 10. Variation of stiffness of members for bolt grade 24M

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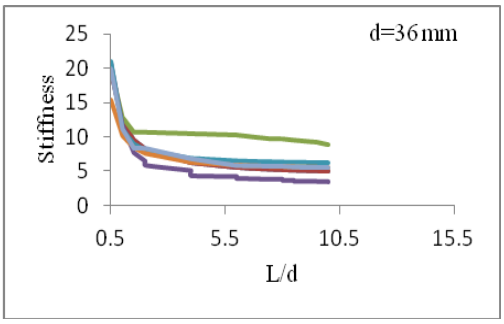

Fig. 11. Variation of stiffness of members for bolt grade 36M

V CONCLUSION

The proposed formulation for stiffness of the members with bolted connections is a simplified expression that gives results with acceptable range of error. It is noted out from the results that the present formula gives quite comparable result with L/d ratio approximately up to 7. Generally, for a particular diameter of bolt, L/d ratio is kept below 7. The validated data and the verified results indicate that the proposed stiffness formulation is considerably robust and reliable.

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