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An Analytical Study of Linear Gravity Modulated Rayleigh-Benard Convection in a Rotating Boussinesq Stokes Suspension with Weak Electric Field

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***Abstract* - The effect of Coriolis force and gravity modulation of small amplitude on a weak electrically conducting Boussinesq- Stokes suspension is investigated by using regular perturbation method to arrive at an expression for the correction Rayleigh number. The Venezian approach is adopted in arriving at the critical Rayleigh and wave number for small amplitudes of gravity modulation. The effect of role of Couple stress parameter, Taylor number and Hartmann number on the onset of convection is studied. The system is most stable with respect to gravity modulation.**

Keywords: Rayleigh-Benard Convection, Boussineq-Stokes suspension, Gravity modulation, Coriolis Force.

INTRODUCTION

The last century saw a thorough understanding of the principles of fluid mechanics and knowledge of how to apply them to many practical problems. Aeronautical, biomedical, civil, marine and mechanical engineers as well as astrophysicists, geophysicists, space researchers, meteorologists, physical oceanographers, physicists and mathematicians have used this knowledge to tackle a multitude of complex flow phenomena. An important class of fluid differs from that of Newtonian fluids, in that the relationship between the shear stress and flow field is more complicated. Such fluids are non-Newtonian.

Couple stress is the consequence of assuming that mechanical action of one part of a body on another across a surface is equivalent to a force and moment distribution. Couple stress fluid theory developed by Stokes[1], is one among the polar fluid theories which considers couple stresses in addition to the classical Cauchy stress. It is the simplest generalization of the classical theory of fluids which allows for polar effects such as the presence of couple stresses and body couples. This fluid theory is discussed in detail by Stokes[2] in his treatise “Theories of fluids with microstructure” wherein he also presented a list of problems discussed by researchers with reference to this theory. The first paper on Rayleigh-Benard situation in Boussinesq-Stokes Suspension is by Siddheshwar and Pranesh[3]. They investigated the effect of Raleigh–Benard situation in Boussinesq–Stokes suspensions using both linear and non-linear stability analyses. Some of the problems of recent interest can also be seen in Neduvinamani *et al*[4].

Convection in porous media has recently been studied since it has many practical, especially biological significance. Bhadauria B. S, Sherani and Md. Aalam[5] investigated the effect of gravity modulation on the onset of Darcy convection in a rotating porous medium. It was concluded that gravity modulation delays the onset of convection as does rotation. T. Sivakumar and S. Saravanan[6] investigated the effect of Gravity Modulation on the Onset of Convection in a Horizontal Anisotropic Porous Layer. A linear stability theory was used to investigate the effect of gravity modulation on the onset of convection in a homogeneous anisotropic porous layer heated from above. The Brinkman model with anisotropic permeability was considered. Free-convection flow past an infinite vertical porous plate with periodic suction and gravity modulation was investigated by S. Baljinder[7]. The double-diffusive convection in a horizontal layer of nanofluid under rotation in a porous medium was studied by Rana *et al.*[8] using the Darcy model. For the case of stationary convection, it was observed that rotation and solute gradient have a stabilizing effect on the system.

Buoyancy-driven convection in microgravity resulting from gravity fluctuations has gained considerable attention owing to the possibility of conducting research in the low gravity environment of space, and because of interest in the fundamental effects of gravity modulation on fluid systems. The effect of such forces on fluid motion is known as Gravity Modulation or g-jitter induced flow and it comes from crew motions, mechanical vibrations (pump, motors, and excitations of natural frequencies of space craft structure), atmospheric drag, solar drag, earth's gravity gradient and other sources.

An extensive overview of the above and other sources of unsteady gravitational accelerations, derived from estimates based on measurements in the space lab engineering model, have been documented by Gresho and Sani[9]. The fluctuating accelerations act on density gradients in the fluid caused by heat and/or mass transfer between the fluid and boundaries, producing convective

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motions. These motions may increase heat transfer significantly. Previous studies of buoyancy-induced fluid motion and heat transfer resulting from gravity modulation under microgravity have focused on a few basic fluid systems, as well as some specific applications. If an imposed modulation can destabilize another stable state, then there can be a major enhancement of heat, mass and momentum transport. Consequently, this has led to research into the possibility of processing materials in space where the low-level background gravitational acceleration can eliminate buoyancy driven convection. However, research has shown that time-dependent accelerations or g-jitter of substantial amplitude resulting from orbital maneuvers and inherent mechanical vibrations may alone induce buoyant convection. It is also of interest to understand how vibration might be used to control convective instabilities.

MATHEMATICAL FORMULATION

Equation of continuity:

$$\nabla.\vec{q} = 0 \tag{1}$$

Equation of conservation of linear momentum:

$$\rho_0\left[\frac{1}{\varepsilon_1}\frac{\partial\vec{q}}{\partial t} + \frac{1}{\varepsilon_1{}^2}(\vec{q}.\nabla)\vec{q} + \frac{2}{\varepsilon_1}(\vec{\Omega}\times\vec{q})\right] = -\nabla p + \rho\vec{g} + \frac{\mu'}{\kappa}\nabla^2\vec{q} - \frac{\mu_{eff}}{\kappa}\vec{q} - \mu_m^2 H_0^2\sigma\vec{q} \tag{2}$$

Equation of conservation of energy:

$$\gamma\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \kappa\nabla^2 T \tag{3}$$

Equation of state:

$$\rho = \rho_0\left[1-\alpha(T-T_0)\right] \tag{4}$$

where, $\vec{q}$ is the velocity, p pressure, $\vec{H}$ magnetic field, B magnetic induction, T temperature, ρ density , $\vec{g}$ acceleration due to gravity, χ thermal conductivity, α coefficient of thermal expansion, ρ_0 reference density, T_0 reference temperature, μ_m magnetic permeability, μ dynamic viscosity, μ' couple stress viscosity, μ_{eff} effective viscosity, σ electrical conductivity, γ porosity and t time. The above equations are solved for free-free isothermal boundary conditions. The gravity modulation is considered as $\vec{g}(t) = g_0(1+\varepsilon cos\Omega t)\hat{k}$, where ε is the small amplitude and Ω is the frequency.

Initially we assume that the fluid is at rest and is described by

$$\vec{q} = \vec{q}_b = (0,0,0), \rho = \rho_b(z), T = T_b(z), p = p_b(z), \vec{\Omega} = \vec{\Omega}_b = \Omega_0\hat{k} \tag{5}$$

When these quantities are substituted in the governing equations we get the following set of equations:

$$\frac{dp_b}{dz} + \rho_b g(1+\varepsilon\cos\Omega t) = 0 \tag{6}$$

$$\frac{\partial^2 T_b}{\partial z^2} = 0 \tag{7}$$

$$\rho = \rho_b[1-\alpha(T_b - T_0)] \tag{8}$$

Solving eqn (3) using the boundary condition $T_b = T_0$ at z=0 and $T_b = T_1$ at z=d we get, $T_b = \frac{-\Delta T}{d}z + T_0$

(9)

where, $\Delta T = (T_0 - T_1)$

Linear Stability Analysis

Let the basic state be disturbed by an infinitesimal thermal perturbation. We now have,

$$\vec{q} = \vec{q}_b + \vec{q}', \rho = \rho_b + \rho', T = T_b + T', p = p_b + p', \vec{\Omega} = \vec{\Omega}_b + \vec{\Omega}' \tag{10}$$

The prime indicates that the quantities are infinitesimal perturbations.

Substituting these into governing equations and using the basic state equations, we get linearized equations governing the infinitesimal perturbations in the form:

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$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial(\nabla^2 w')}{\partial t}+\frac{2}{\varepsilon}\Omega_0\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y)}\right)\right]=\rho_0\alpha g(1+\varepsilon\cos\Omega t)\nabla_1^2 T$$

$$+\frac{\mu'}{\kappa}\nabla^4 w'-\frac{\mu_{eff}}{\kappa}\nabla^2 w'-\mu_m^2 H_0^2\sigma\nabla^2 w' \quad (11)$$

$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial V}{\partial t}+\frac{2}{\varepsilon}\Omega_0\frac{\partial w}{\partial z}\right]=+\frac{\mu'}{\kappa}\nabla^2 V-\frac{\mu_{eff}}{\kappa}V-\mu_m^2 H_0^2\sigma V \quad (12)$$

$$\gamma\frac{\partial T'}{\partial t}+(\vec{q}.\nabla)T_b=\kappa\nabla^2 T' \quad (13)$$

The perturbation equations are non-dimensionalized using the following definitions:

$$(x^*,y^*,z^*)=\left(\frac{x}{d},\frac{y}{d},\frac{z}{d}\right),t^*=\frac{t}{d^2/\chi},\quad w^*=\frac{w}{\chi/d},\quad T^*=\frac{T}{\Delta T},\quad \nabla^*=\nabla d,\ V^*=\frac{V}{\chi/d^2} \quad (14)$$

Eqn (14) is used to get a set of dimensionless equations given below:

$$M_1\left(\partial/\partial t-\nabla^2\right)T=w\ , \quad (15)$$

$$\frac{1}{pr}\frac{\partial(\nabla^2 w)}{\partial t}+Ta^{1/2}\frac{\partial V}{\partial z}=R(1+\varepsilon\cos\Omega t)\nabla_1^2 T+\frac{C}{D_a}\nabla^4 w-\frac{1}{D_a}\nabla^2 w'-M^2\nabla^2 w' \quad (16)$$

$$\frac{1}{pr}\frac{\partial V}{\partial t}-Ta^{1/2}\frac{\partial w}{\partial z}=\frac{C}{D_a}\nabla^2 V-\frac{1}{D_a}V-M^2 V \quad (17)$$

Where the non-dimensional parameters $\Pr=\frac{\mu}{\rho_0\chi}$, $T_a=\left(\frac{2\Omega_0\rho_0 d^2}{\mu}\right)^2$, $R=\frac{\rho_0\alpha g d^3\Delta T}{\mu\chi}$, $C=\frac{\mu'}{d^2\mu}$, $M=\sqrt{\frac{\mu_m^2 H_0^2 d^2\sigma}{\mu}}$, $M_1=\frac{\gamma}{\varepsilon}$, $D_a=\frac{\kappa}{d^2}$ are Prandtl number, Taylor number, Darcy-Rayleigh number, Coupling stress parameter, Hartmann number, Darcy number respectively.

On simplification,

$$\left[\left(\frac{1}{pr}\frac{\partial}{\partial t}\nabla^2-\frac{C}{D_a}\nabla^4+\frac{1}{D_a}\nabla^2+M^2\nabla^2\right)\left(\frac{1}{pr}\frac{\partial}{\partial t}-\frac{C}{D_a}\nabla^2+\frac{1}{D_a}+M^2\right)+TaD^2\right]\left(M_1\frac{\partial}{\partial t}-\nabla^2\right)T=$$

$$\left(\frac{1}{pr}\frac{\partial}{\partial t}-\frac{C}{D_a}\nabla^2+\frac{1}{D_a}+M^2\right)R(1+\varepsilon\cos\Omega t)\nabla_1^2 T \quad (18)$$

Each of T_n is required to satisfy the boundary conditions. The marginally stable solution of the problem is the general solution of the above equation i.e. $T_0=\sin(\pi z)e^{i(lx+my)}$,

corresponding to the lowest mode of convection with the corresponding eigen value.

$$R_0=\frac{(k^2X_1^2+\pi^2 Ta)k^2}{a^2 X_1} \quad (19)$$

Where $X_1=\left(\frac{C}{D}k^2+\frac{1}{D_a}+M^2\right)$ (20)

The consequent equations when solved gives the correction Rayleigh number, R_{2c}

$$R_{2c}=\frac{a^2R_0^2}{2\Pr\left|(L(\Omega,n)^2\right|}\left[\frac{-\Omega^2 Y_1}{X_1\Pr}+\frac{\Omega^2 Y_1}{X_1}+\frac{\Omega Y_2 X_n}{X_1}-X_n Y_1\Pr\right] \quad (21)$$

$$L(\Omega,n)=Y_1+iY_2 \quad (22)$$

$$Y_1 = \frac{2\Omega^2 M_1 k_n^2 X_n}{pr} - X_n^2 k_n^4 - \frac{\Omega^2 k_n^4}{pr^2} - \pi^2 Ta k_n^2 + aR_0 X_n \tag{23}$$

$$Y_2 = \Omega M_1 k_n^2 X_n^2 - \frac{\Omega^3 M_1 k_n^2}{pr^2} + \frac{2\Omega k_n^4 X_n}{pr} + \pi^2 Ta\Omega M_1 - \frac{\Omega a^2 R_0}{pr} \tag{24}$$

RESULT AND DISCUSSIONS

The value of R obtained by this procedure is the eigenvalue corresponding to the eigen function W that, though oscillating, remains bounded in time. Since R is a function of the horizontal wavenumber **a** and the amplitude of perturbationδ, we have $R(a,\delta) = R_0(a) + \delta^2 R_2(a)$. It was shown by Venezian[10] that the critical value is determined by $O(\delta^2)$, by evaluating R_0 and R_2 at $a = a_0$. It is only when one wishes to evaluate R_4 that a_2 must be taken into account where $a = a_2$ minimizes R_2.To evaluate the critical value of R_2 one has to substitute $a = a_0$ in R_2, where a_0 is the value at which R_0 given by equation is minimum.

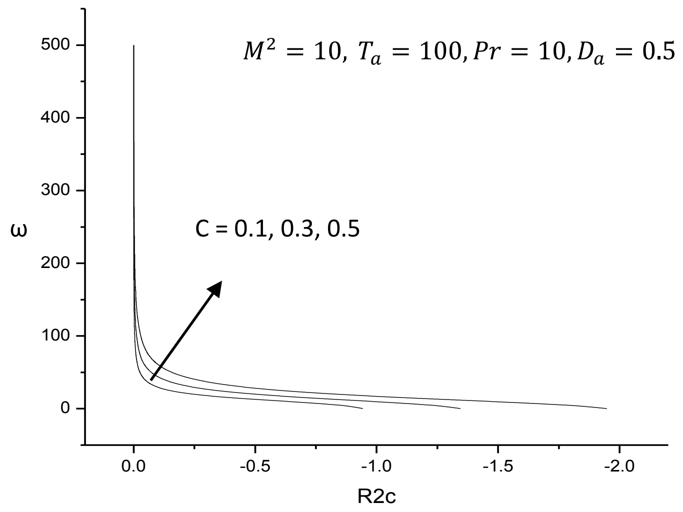

Fig 1: Plot of correction Rayleigh number R_{2c} versus ω for different values of C

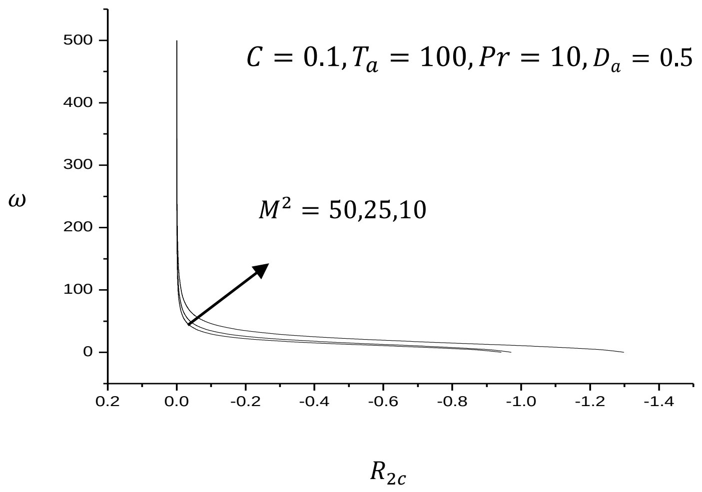

Fig 2: Plot of correction Rayleigh number R_{2c} versus ω for different values of M^2

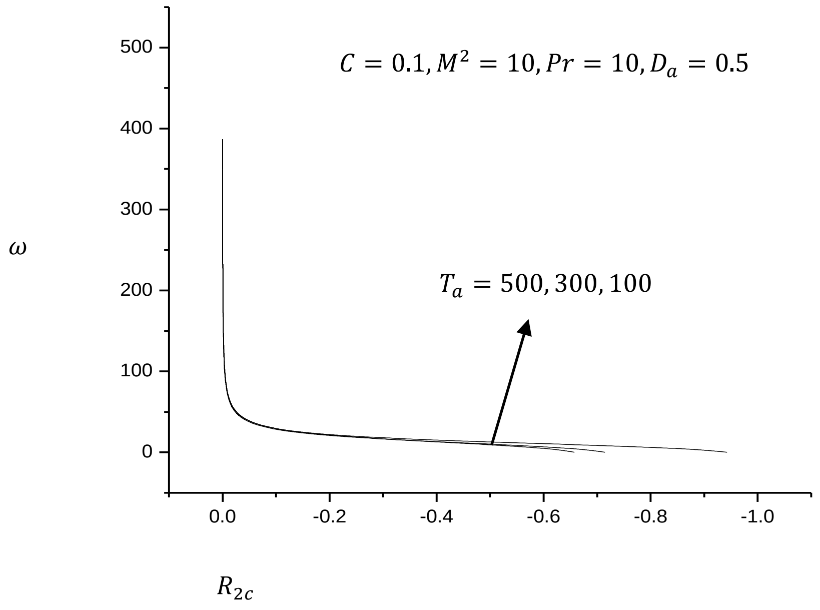

Fig 3: Plot of correction Rayleigh number R_{2c} versus ω for different values of T_a

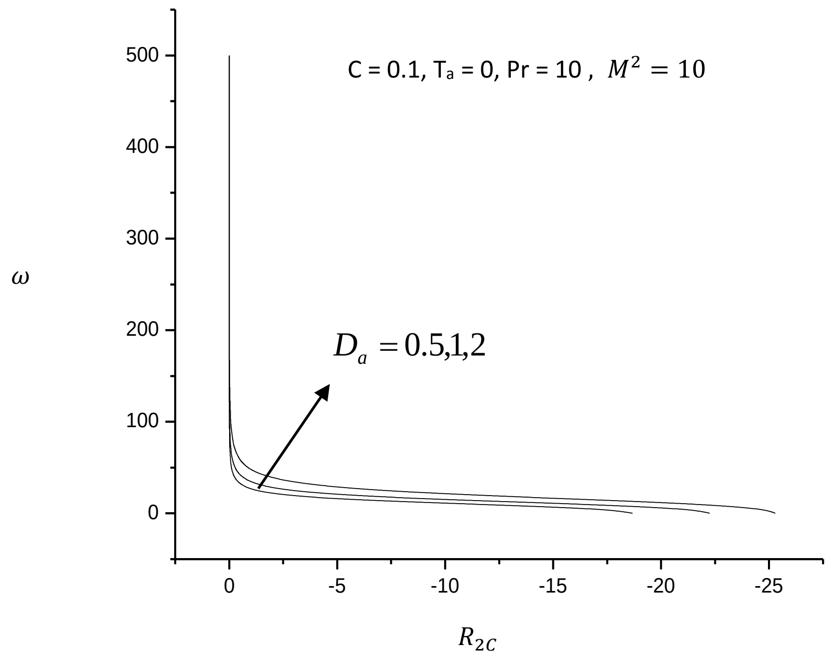

Fig 4: Plot of correction Rayleigh number R_{2c} versus ω for different values of D_a

We make an analytical study of the effect of Coriolis force and gravity modulation on the onset of convection in a weak electrically conducting couple stress fluid in a porous medium. Double diffusive convection in porous medium is studied under Coriolis force, gravity modulation and the inhibition of convection by suspended particles. It should be noted that gravity modulation affects the entire bulk of fluid between the boundary plates.

The analysis presented in this dissertation is based on the assumption that the amplitude of the gravity modulating is small. The validity of the results obtained here depends on the value of the modulating frequency ω. When $\omega < 1$, the period of modulation is large. The gravity modulation affects the entire volume of the fluid, resulting in the growth of the disturbance. On the other hand, the effect of modulation disappears for the large frequency. This is due to the fact that the buoyancy force takes a mean value leading to equilibrium state of the unmodulated case. In view of this, we choose only moderate value of ω in our study. It must be noted here that because of the presence of suspended particles in the fluid and according to Einstein's relation for viscosity, the value of Prandtl number is taken higher than those of clean fluid.

CONCLUSION:

Figure 1 is the plot of correction Rayleigh number R_{2c} versus ω for different values of C. We see that as C increases the value of R_{2c} becomes more and more negative. C is the indicative of the concentration of the suspended particles. Therefore, the couple stress parameter stabilizes the system.

Figure 2 is the plot of correction Rayleigh number R_{2c} versus ω for different values of M^2. From the figure we observe that with increase in M^2, R_{2c} becomes more negative. Magnetic field induces viscosity into the fluid and the magnetic lines are distorted. These magnetic lines hinder the growth of disturbances.

Figure 3 is the plot of correction Rayleigh number R_{2c} versus ω for different values of T_a. We note that as T_a increases, R_{2c} increases, that is it becomes less negative. This implies that rotation causes convection to delay.

Figures 4 is a plot of correction Rayleigh number R_{2c} versus ω for different values of D_a. From the graph it is clear that the Darcy number causes a delay in convection thus stabilizing the system.

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