An Intuitionistic Fuzzy Shortest Path Problem Based On Level λ - LR Type Representation

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Abstract

The shortest path problem is an important, classical network optimization problem which has a wide range of application in various fields. This paper proposes to find the shortest path in an intuitionistic fuzzy weighted graph with nodes remaining crisp and links remaining crisp but the edge weight will be an Intuitionistic fuzzy number. An existing algorithm is proposed for finding the shortest path length. Finally illustrative numerical examples are given to demonstrate the proposed approach

Key words- *Acyclic Digraph,* π_2 *Membership function, Weighted Average Index, Total integral index* (I_T) α *cut interval for LR Trapezoidal Intuitionistic fuzzy number, Convex Index for Intuitionistic fuzzy number, Index Ranking of fuzzy numbers.*

1. Introduction

In network problems the weight of the edge in a shortest path problem is supposed to be real numbers, but in most real world problems was the length of the network which represents the time or cost. Fuzzy set theory proposed by Zadeh L.A [1], The fuzzy shortest path problem was introduced by Dubois and Prade [2]. Klin C.M discussed about the fuzzy shortest path [4]. Hyung LK, Song YS, [7] Lee KM., discussed the Similarity Measure between Fuzzy Sets and elements Yao and Lin [10]developed two types of fuzzy shortest path network problem where the first type of fuzzy shortest path problem uses triangular fuzzy numbers and the second type uses level. $(1-\beta, 1-\alpha)$ interval valued fuzzy numbers. The main result emerging from their study was that the shortest path in the fuzzy sense corresponds to the actual path in the network, and the fuzzy shortest path problem is an extension of the crisp case. Chuang T.N., Kung J.Y.[13] proposed a new algorithm for the discrete fuzzy shortest path problem in a network. L. Sujatha and S. Elizabeth [20,21] found the fuzzy shortest path problem based on Index ranking. In this paper they defined acceptability index, convex index and total integral index using this they proposed many algorithms and found the shortest distance. P.Jayagowri, Dr.G.Geetharamani,[23] discussed, various approaches to solving network problems using λTLR intuitionistic fuzzy numbers. This paper is organized as follows; In Section 2 provides the preliminary concepts required for analysis and definitions in fuzzy set theory used throughout the paper, and some new indices and α -cut interval for LR Trapezoidal intuitionistic fuzzy numbers are introduced. Section 4 gives proposed algorithms based on the indices defined in section2. Section 5 showcases the proposed algorithm and explains its functions. In section 6 some conclusions are drawn.

Preliminaries

In this section ,some basic definitions relating to our work has given, and introduces new definition for convex index , Total integral index, Signed index of level λ -LR intuitionistic trapezoidal fuzzy number.

Acyclic Digraph

A digraph is a graph each of whose edges are directed. Hence an acyclic digraph is a directed graph without cycle.

Membership function of the L.R trapezoidal fuzzy number

Membership function of the LR Trapezoidal fuzzy number $A=(a,b,c,d)_{LR}$ is

$$\mu_A(x)=\begin{cases} 0 & x \le a-c \\ \frac{x-(a-c)}{c}, & a-c < x < a \\ 1 & a < x \le b \\ \frac{(b+d)-x}{d} & b < x < b+d \\ 0 & x \ge b+d \end{cases}$$

π_2 Membership function

The membership function that has the "π_2"shape has four parameters ;

$$\pi_2(x: lw, lp, rp, rw) = \begin{cases} \frac{lw}{lp+lw-x}, & x < lp \\ 1, & lp \le x \le rp \\ \frac{rw}{x-rp+rw}, & x > rp \end{cases}$$

It is now used in current research to define the shortest path in a fuzzy sense.

Addition operation on π_2 shaped fuzzy numbers

Assuming that both $A=(lw_1, lp_1, rp_1, rw_1)$ and $B=(lw_2, lp_2, rp_2, rw_2)$ are fuzzy numbers, then

$$A+B=(lw_1+lw_2, lp_1+lp_2, rp_1+rp_2, rw_1+rw_2)$$

Minimum Operation on π_2 shaped fuzzy numbers

The two π_2 shaped fuzzy numbers $L_1=(lw_1, lp_1, rp_1, rw_1)$ and $L_2=(lw_2, lp_2, rp_2, rw_2)$ then,

$$L_{min}(L_1, L_2)=\begin{pmatrix} \max(lw_1, lw_2), \min(lp_1, lp_2), \min(rp_1, rp_2), \\ \min(rw_1, rw_2) \end{pmatrix}$$

Weighted Average Index for level λ π_2 Membership Function

Let $L_I=((lw_i, lp_i, rp_i, rw_i;\lambda)$ and $L_{min}=(lw, lp, rp, rw;\lambda)$ be two level λ π_2 Membership function, If $lp \le lp_i, rp \le rp_i$, then the Weighted Average Index between L_i and L_{min} can be calculated as $WAI(L_{min}, L_I) = \frac{\lambda A+\lambda A_i}{2\lambda}, 0<\lambda\le 1$, where

$$A=\frac{lp+rp}{2} \text{ and } A_i=\frac{lp_i+rp_i}{2} \quad i=1 \text{ to } n.$$

In the Weighted Average Index, we have $L_1 < L_2$ if and only if $WAI(L_{min}, L_1) < WAI(L_{min}, L_2)$

$\alpha-$ cut interval for LR trapezoidal fuzzy number

$\alpha-$ cut interval for LR trapezoidal fuzzy number $A=(a,b,c,d)_{LR}$ is

$$A_\alpha=[A_\alpha^L, A_\alpha^U]=[\alpha c+(p-c), (q+s)-\alpha d]$$

$\alpha-$ cut interval for LR type representation of fuzzy interval

$\alpha-$ cut interval for LR type representation of fuzzy interval $A=[a,b]$ is given by

$$A_\alpha=[A_\alpha^L, A_\alpha^U]=[a-\alpha c, \alpha d+b]$$

Convex Index (COI)

Let A be an LR trapezoidal fuzzy number, then $CoI=\lambda(A_\alpha^L)+(1-\lambda)(A_\alpha^U)$ is the $\alpha-$ cut interval of

$A = (a, b, c, d)_{LR}$,if $\mu_A[\lambda(A_\alpha^L) + (1-\lambda)(A_\alpha^U)] \geq \min\{\mu_A(A_\alpha^L), \mu_A(A_\alpha^U)\}$ for all $\alpha, \lambda \varepsilon [0,1]$. If A and B are two LR trapezoidal fuzzy numbers, then in convex Index, we have A < B if and only if CoI(A) < CoI(B)

Total Integral Index

For fuzzy number $A = (a, b, c, d)_{LR}$ the Total integral index $I_T(A)$ can be constructed from the left integral values $I_L(A)$ and the right integral values $I_R(A)$. the total integral value $I_T(A)$ with index of optimism λ where $0 < \lambda \leq 1$ is then defined as, $I_T(A) = \lambda I_L(A) + (1-\lambda) I_R(A)$. -----------(3)The left integral values $I_L(A)$ and the right integral values $I_R(A)$ of a LR trapezoidal fuzzy number can be found a

$$I_L(A) = \left[\left(a - \frac{c}{2}\right) + \left(b - \frac{m}{2}\right) + \left(l - \frac{n}{2}\right)\right]$$

$$= \left[(a + b + l) - \left(\frac{c + m + n}{2}\right)\right]$$

$$I_R(A) = \left[\left(a + \frac{n}{2}\right) + \left(b + \frac{c}{2}\right) + \left(l + \frac{m}{2}\right)\right]$$

$$= \left[(a + b + l) + \left(\frac{n + c + m}{2}\right)\right]$$

The signed distance of level $\lambda - LR$ Type Representation of Fuzzy interval

Let $A = \{\langle p, q, r, s; \lambda\rangle\}_{LR}$ be level $\lambda - LR$ type representation of fuzzy interval. The α – cut of A is $A(\alpha) = [A_L(\alpha\alpha) A_R(\alpha\alpha)]$, $0 \leq \alpha \leq \lambda$ and $\lambda \ \varepsilon \ (01]$

Where $A_L(\alpha) = \frac{\lambda p - \alpha r}{\lambda}$, $A_R(\alpha) = \frac{\lambda q + \alpha s}{\lambda}$ Since the function is continuous over the interval $0 \leq \alpha \leq \lambda$ integration is used for obtaining the mean value of signed distance as follows: d(A.0) = ¼(2(p+q)+(s-r))

The ranking of signed distance of level $\lambda - LR$ type representation of fuzzy interval.

Let $A = \{\langle p_1, q_1, r_1, s_1; \lambda\rangle\}_{LR}$; $B = \{\langle p_2, q_2, r_2, s_2; \lambda\rangle\}_{LR}$ be two level $\lambda - LR$ type representation of fuzzy intervals. For $\lambda \ \varepsilon \ (01]$ the ranking is defined as A $A \leq B$ iff $d(A.0) \leq d(B,0)$

Mean and standard deviation of level λ LR Trapezoidal Fuzzy number

Let $A = \{\langle p_1, q_1, r_1, s_1; \lambda\rangle\}_{LR}$ be a trapezoidal fuzzy number, then the membership function in terms of mean and standard deviation is defined as follows:

$$\mu_A(x) = \begin{cases} \dfrac{\lambda[x - (\mu - \sigma)]}{\sigma}, & \text{if } \mu - \sigma \leq x \leq \mu \\ \dfrac{\lambda[(\mu + \sigma) - x]}{\sigma}, & \text{if } \mu \leq x \leq \mu + \sigma \\ \lambda & \text{if } x = \mu. \end{cases}$$

where $\mu = \frac{2(p+q) + (s-r)}{4}$ and $\sigma = \frac{3(q-r) + 2(s+r)}{4}$

If the i^{th} fuzzy path Length is $L_i = (p_i, q_i, r_i, s_i; \lambda)_{LR}$ and the fuzzy shortest length is $L_{min} = (p, q, r, s; \lambda)_{LR}$ Then the shortest Mean and Standard Deviation Index between L_{Min} and L_i is

$$I(L_{Min}, L_i) = \frac{\lambda(\mu + \sigma) - (\mu_i - \sigma_i)}{\sigma + \sigma_i}$$

Where $\mu = \frac{2(p+q) + (s-r)}{4}$ and $\sigma = \frac{3(q-r) + 2(s+r)}{4}$

Introduces new definition are here.

Minimum Operation on π_2 shaped Intuitionistic fuzzy numbers:

For two π_2 shaped Intuitionistic fuzzy numbers

$$L_1 = \langle (lw_{\mu1}, lp_{\mu1}\ rp_{\mu1}, rw_{\mu1})\ (lw_{\gamma1}, lp_{\gamma1}\ rp_{\gamma1}, rw_{\gamma1}) \rangle,$$

$$L_2 = \langle (lw_{\mu2}, lp_{\mu2}\ rp_{\mu2}, rw_{\mu2})\ (lw_{\gamma2}, lp_{\gamma2}\ rp_{\gamma2}, rw_{\gamma2}) \rangle$$

$$L_{min}(L_1, L_2) = \begin{Bmatrix} \max(lw_{\mu 1}, lw_{\mu 2}), \min(lp_{\mu 1}, lp_{\mu 2}), \\ \min(rp_{\mu 1}, rp_{\mu 2}), \min(rw_{\mu 1}, rw_{\mu 2}) \end{Bmatrix}$$

$$L_{max}(L_1, L_2) = \begin{Bmatrix} \min(lw_{\gamma 1}, lw_{\gamma 2}), \max(lp_{\gamma 1}, lp_{\gamma 2}), \\ \max(rp_{\gamma 1}, rp_{\gamma 2}), \max(rw_{\gamma 1}, rw_{\gamma 2}) \end{Bmatrix}$$

Weighted Average index for level $\lambda\pi_2$ Membership function

Let $L_i = (lw_{\mu i}, lp_{\mu i}, rp_{\mu i}, rw_{\mu i}; \lambda)$ and $L_{min} = (lw_\mu, lp_\mu, rp_\mu, rw_\mu; \lambda)$ be two $\lambda\pi_2$ membership functions.

If $lp \le lp_i$, $rp \le rp_i$ then the weighted average index between L_i and L_{min} can be calculated as

$$WAI(L_{min}, L_i) = \frac{\lambda A + \lambda A_i}{2\lambda} \quad 0 < \lambda \le 1$$

$$\text{Where } A = \frac{lp_\mu + rp_\mu}{2} \text{ and } A_i = \frac{lp_{\mu i} + rp_{\mu i}}{2}$$

$i = 1,2,....n.$

Weighted Average index for level $\lambda\pi_2$ non-membership function

Let $L_i = (lw_{\gamma i}, lp_{\gamma i}, rp_{\gamma i}, rw_{\gamma i}; \lambda)$

and $L_{max} = (lw_\gamma, lp_\gamma, rp_\gamma, rw_\gamma; \lambda)$ be two non membership functions, If $l > lp_i$, $rp > rp_i$ then the weighted average index between L_i and L_{max} can be calculated as

$$WAI(L_{max}, L_i) = \frac{\lambda A + \lambda A_i}{2\lambda} \quad 0 < \lambda \le 1$$

$$\text{Where } A = \frac{lp_\gamma + rp_\gamma}{2} \text{ and } A_i = \frac{lp_{\gamma i} + rp_{\gamma i}}{2} \quad i = 1,2,....n.$$

$\alpha-$ cut interval for LR trapezoidal Intuitionistic fuzzy number

$\alpha-$ cut interval for LR trapezoidal Intuitionistic fuzzy number

$A = \{\langle p_\mu, q_\mu, r_\mu, s_\mu \rangle \langle p_\gamma, q_\gamma, r_\gamma, s_\gamma \rangle\}_{LR}$ is $A_\alpha = [A_\alpha^L, A_\alpha^U]$. The above $\alpha-$cut interval is membership function and nonmembership function as follows $\forall\ \alpha \varepsilon [0,1]$

$A_{\alpha\mu}^L = \alpha c_\mu + (a_\mu - c_\mu)$; $A_{\alpha\mu}^U = (b_\mu + d_\mu) - \alpha d_\mu$ and

$A_{\alpha\gamma}^L = \alpha c_\gamma - (a_\mu - c_\mu)$; $A_{\alpha\gamma}^U = (b_\gamma + d_\gamma) + \alpha d_\gamma$

$\alpha-$ cut interval for LR type representation of Intuitionistic fuzzy interval

$\alpha-$ cut interval for LR type representation of fuzzy interval $A = [a, b]$ is given by $A_\alpha = [A_\alpha^L, A_\alpha^U]$. The above $\alpha-$ representation cut interval is obtained for membership function and nonmember ship function as follows $\forall\ \alpha \varepsilon [0,1]$

$A_{\alpha\mu}^L = b_\mu - \alpha c_\mu$ $A_{\alpha\mu}^U = \alpha d_\mu + b_\mu$

and $A_{\alpha\gamma}^L = b_\gamma + \alpha c_\gamma$ $A_{\alpha\gamma}^U = \alpha d_\gamma - b_\gamma$

Convex Index for Intuitionistic fuzzy number

Let A be an LR trapezoidal Intuitionistic fuzzy number, then

$CoI(A_\mu) = \lambda(A_{\alpha\mu}^L) + (1-\lambda)(A_{\alpha\mu}^U)$; $CoI(A_\gamma) = (1-\lambda)(A_{\alpha\gamma}^L) + \lambda(A_{\alpha\gamma}^U)$,

Where $[A_{\alpha\mu}^L\ A_{\alpha\mu}^U]$ and $[A_{\alpha\gamma}^L\ A_{\alpha\gamma}^U]$ are the $\alpha-$ cut inter al of $A = \{\langle p_\mu, q_\mu, r_\mu, s_\mu \rangle \langle p_\gamma, q_\gamma, r_\gamma, s_\gamma \rangle\}$ If

$$\mu_A[\lambda(A_{\alpha\mu}^L) + (1-\lambda)(A_{\alpha\mu}^U)] \ge \min\{\mu_A(A_{\alpha\mu}^L), \mu_A(A_{\alpha\mu}^U)\};$$

$$\gamma_A[(1-\lambda)(A_{\alpha\gamma}^L) + \lambda(A_{\alpha\gamma}^U)] < \max\{\gamma_A(A_{\alpha\gamma}^L), \gamma_A(A_{\alpha\gamma}^U)\}$$

for all $\alpha, \lambda \varepsilon [0,1]$

Total Integral Index

For an intuitionistic fuzzy number $A = \{\langle p_\mu, q_\mu, r_\mu, s_\mu \rangle \langle p_\gamma, q_\gamma, r_\gamma, s_\gamma \rangle$

The total integral index $I_T(A)$ can be constructed from the left integrals values $I_L(A)$ and the right interval value $I_R(A)$. The total integral value $I_T(A)$ with index of optimism λ where $0 < \lambda \le 1$.

Is then defined for both membership and non membership function.

$$I_T(A_\mu) = \lambda I_L(A_\mu) + (1-\lambda) I_R(A_\mu) \quad ;$$

$$I_T(A_\gamma) = (1-\lambda) I_L(A_\gamma) + \lambda I_R(A_\gamma)$$

$$I_L(A_\mu) = \left\{(a+b+l) - \left(\frac{c+m+n}{2}\right)\right\}$$

$$I_R(A_\mu) = \left\{(a+b+l) + \left(\frac{n+d+m}{2}\right)\right\} \quad l = \frac{a+b}{2},$$

$n = l - a,\ m = b - l$

where $$I_L(A_\gamma) = \left\{(a+b+l) + \left(\frac{c+m+n}{2}\right)\right\} \quad ;$$

$$I_R(A_\gamma) = \left\{(a+b+l) - \left(\frac{n+d+m}{2}\right)\right\}$$

Signed distance of level λ – LR Intuitionistic Fuzzy number(Membership Function)

$$A = \left\{\langle p_\mu, q_\mu, r_\mu, s_\mu\rangle \ \langle p_\gamma, q_\gamma, r_\gamma, s_\gamma\rangle\right\}_{LR} \quad \text{is } A_\alpha = [A_\alpha^L, A_\alpha^U]$$

The mean value of signed distance is

$$d(A,0) = \frac{1}{4}(2(p_\mu + q_\mu) + (s_\mu - r_\mu))$$

Signed distance of level λ – LR Intuitionistic Fuzzy number(Non Membership Function)

$$A = \left\{\langle p_\mu, q_\mu, r_\mu, s_\mu\rangle \ \langle p_\gamma, q_\gamma, r_\gamma, s_\gamma\rangle\right\}_{LR} \quad is\ A_\alpha = [A_\alpha^L, A_\alpha^U]$$

The mean value of signed distance *is*

$$d(A,0) = \frac{1}{4}(2(p_\gamma + q_\gamma) + (r_\gamma - s_\gamma))$$

Mean and standard deviation of level λ LR trapezoidal Intuitionistic Fuzzy Number

$$A = \left\{\langle p_\mu, q_\mu, r_\mu, s_\mu\rangle \ \langle p_\gamma, q_\gamma, r_\gamma, s_\gamma\rangle\right\}_{LR}$$

is intuitionistic trapezoidal fuzzy number

If the i^{th} fuzzy path Length for membership function is $L_{\mu i} = (p_{\mu i}, q_{\mu i}, r_{\mu i}, s_{\mu i}; \lambda)_{LR}$ and the nonmembership function $L_{\gamma i} = (p_{\gamma i}, q_{\gamma i}, r_{\gamma i}, s_{\gamma i}; \lambda)_{LR}$ then fuzzy shortest lengths are $L_{min} = (p_\mu, q_\mu, r_\mu, s_\mu)$ and $L_{max} = (p_\gamma, q_\gamma, r_\gamma, s_\gamma)$ Then the shortest Mean and Standard Deviation Index between L_{Min} and L_i is L_{Max} and L_i,

$$I(L_{Min}, L_i) = \frac{\lambda(\mu+\sigma) - (\mu_i - \sigma_i)}{\sigma + \sigma_i},$$

$$I(L_{Max}, L_i) = \frac{\lambda(\mu+\sigma) - (\mu_i - \sigma_i)}{\sigma + \sigma_i}$$

$$\mu = \frac{2(p+q) + (s-r)}{4} \quad \text{and}$$

$$\sigma = \frac{3(q-p) + 2(s+r)}{4} \quad \text{for membership function and}$$

$$\mu = \frac{2(p+q) + (r-s)}{4} \quad \text{and}$$

$$\sigma = \frac{3(p-q) - 2(s-r)}{4} \quad \text{for Nonmembership Function}$$

Proposed Algorithm

Algorithm for Intuitionistic fuzzy shortest path problem based on convex index

In this section a new algorithm is proposed for finding the Intuitionistic fuzzy shortest path based on convex index.

Step 1: Construct a network G= (V, E) Where V is the set of vertices and E is the set of edges.

Step 2: Form the possible path P_i from source node to destination node and compute the corresponding path length L_i ii=1,2,…m for possible' m' path and set

$$L_i = \left\{\langle p_{\mu i}, q_{\mu i}, r_{\mu i}, s_{\mu i}\rangle \ \langle p_{\gamma i}, q_{\gamma i}, r_{\gamma i}, s_{\gamma i}\rangle\right\}_{LR}$$

Step3: Calculate α cut interval for LR trapezoidal Intuitionistic fuzzy number (or LR Type representation of fuzzy interval) for all possible path length, L_i I =1,2,…m and set

$$L_{i(\alpha\alpha\mu}=\left[L^L_{i(\alpha\alpha\mu}, L^U_{i(\alpha\alpha\mu}\right] \quad i=1,2,...m ;$$

$$L_{i(\alpha\alpha\gamma}=\left[L^L_{i(\alpha\alpha\gamma}, L^U_{i(\alpha\alpha\gamma}\right] \quad i=1,2,...m$$

Step 4: Calculate Convex Index

$$CoI(L_{i\mu})=\lambda\left(L^L_{i(\alpha\alpha\mu}\right)+(1-\lambda)\left(A^U_{i(\alpha\alpha\mu}\right);$$

$$CoI(L_{i\gamma})=(1-\lambda)\left(L^L_{i(\alpha\alpha\gamma}\right)+\lambda\left(L^U_{i(\alpha\alpha\gamma}\right)$$

for all possiblepathlengths

Step 5: Ranking the shortest path with the minimum convex index and the corresponding path length L_i is the Intuitionistic fuzzy shortest path.

Algorithm for Intuitionistic fuzzy shortest path problem based on weighted average index

A new algorithm is proposed for finding the Intuitionistic fuzzy shortest path based on weighted average index

Step1: Construct a network G =(V,E) Where V is the set of vertices and E is the set of edges. Here G is an acyclic digraph.

Step 2: Calculate all possible paths P_i from source node to destination node and compute the corresponding path length L_i i =1,2,…m for possible m path and set

$$L_i=\left\{\left\langle p_{\mu i}, q_{\mu i}, r_{\mu i}, s_{\mu i}\right\rangle \left\langle p_{\gamma i}, q_{\gamma i}, r_{\gamma i}, s_{\gamma\ i}\right\rangle\right\}_{LR}$$

Step3: Calculate the Intuitionistic fuzzy shortest length.

L_{min} using definition and set

$$L_{min}=\left\{\left\langle p_{\mu}, q_{\mu}, r_{\mu}, s_{\mu}\right\rangle\right\};$$ L_{max} using definition

and set $L_{max}=\left\{\left\langle p_{\gamma}, q_{\gamma}, r_{\gamma}, s_{\gamma}\right\rangle\right\}$

Step4: Calculate weighted average index using definition and the rank to the path based on weighted index.

Step5: Rank the shortest path with the highest weighted index.

Algorithm For Intuitionistic Fuzzy Shortest path problem based on signed distance

A new algorithm is proposed for finding the Intuitionistic fuzzy shortest path based on signed distance in this paper introduces a new method Signed distance

Let $L_i=\left\{\left\langle p_{\mu i}, q_{\mu i}, r_{\mu i}, s_{\mu i}\right\rangle \left\langle p_{\gamma i}, q_{\gamma i}, r_{\gamma i}, s_{\gamma\ i}\right\rangle\right\}_{LR}$

and i=1,2,…n where

L_i denotes the level λ -LR Intuitionistic fuzzy path length.

Step 1: Construct a network G=(V,E) Where V is the set of vertices and E is the set of edges. Here G is an acyclic digraph and the arc length takes the level λ - LR intuitionistic fuzzy numbers.

Step 2: Calculate all possible paths P_i from source node to destination node and compute the corresponding path length L_i i =1,2,…m for possible m paths using definition and set and

$$L_i=\left\{\left\langle p_{\mu i}, q_{\mu i}, r_{\mu i}, s_{\mu i}\right\rangle \left\langle p_{\gamma i}, q_{\gamma i}, r_{\gamma i}, s_{\gamma\ i}\right\rangle\right\}_{LR}$$

i = 1 to n and $0 < \lambda \le 1$.

Step 3: calculate signed distance of level λ - LR type representation of fuzzy intervals for each possible path using definition 3.8&3.9

Step 4: The path having the minimum signed distance is identified as the shortest path.

Algorithm for Intuitionistic Fuzzy Shortest path problem based on Mean and standard deviation index

A new algorithm is proposed for finding the Intuitionistic fuzzy shortest path based on Mean and standard deviation index in this paper introduces a new method called 'Mean and Standard Deviation index' which acts as the measurement tool between L_{min} and L_i , L_{max} and L_i

Let $L_i = \left\{ \left\langle p_{\mu i}, q_{\mu i}, r_{\mu i}, s_{\mu i} \right\rangle \left\langle p_{\gamma i}, q_{\gamma i}, r_{\gamma i}, s_{\gamma\ i} \right\rangle \right\}_{LR}$

and i=1,2,…n where L_i denotes the level λ _LR Intuitionistic fuzzy path length.

Step1: Construct a network G =(V,E) Where V is the set of vertices and E is the set of edges. here G is an acyclic digraph.

Step 2: Calculate all possible paths P_i from source node to destination node and compute the corresponding path length L_i ,

i =1,2,…m for possible m path and set $L_i = \left\{ \left\langle p_{\mu i}, q_{\mu i}, r_{\mu i}, s_{\mu i} \right\rangle \left\langle p_{\gamma i}, q_{\gamma i}, r_{\gamma i}, s_{\gamma\ i} \right\rangle \right\}_{LR}$

Step 3: Calculate the Intuitionistc fuzzy shortest length.

L_{min} using definition and set $L_{min} = \left\{ \left\langle p_{\mu}, q_{\mu}, r_{\mu}, s_{\mu} \right\rangle \right\}$;

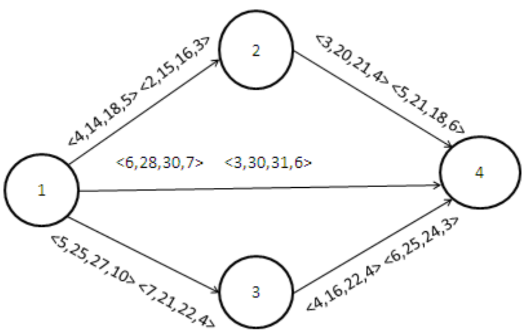
L_{max} using definition and set $L_{max} = \left\langle p_{\gamma}, q_{\gamma}, r_{\gamma}, s_{\gamma} \right\rangle$

Step 4: Calculate Mean and standard deviation index between L_{min}, and L_i, L_{max} and L_i using definition 3.10.

Step 5: The path having the maximum Mean and Standard deviation index is identified as the shortest path.

ILLUSTRATIVE EXAMPLE

Step1: Construct a network with 4 vertices and 5 edges as follows: Fig (1)



Step 2: The possible paths and the corresponding path lengths are as follows

$P_1 : 1 \to 2 \to 4 = L_{\mu 1} = (7,34,39,9;1)$

and $L_{\gamma 1} = (7,36,34,9;1)$

$P_2 : 1 \to 4 = L_{\mu 2} = (6,28,30,7;1)$

and $L_{\gamma 2} = (3,30,31,6;1)$

$P_3 : 1 \to 3 \to 4 = L_{\mu 3} = (9,41,49,14;1)$

and $L_{\gamma 3} = (13,46,46,7;1)$

Step 3:

$L_{min} = \{9,28,30,7;1\}$ and $L_{max} = \{3,46,46,9;1\}$

Step 4: Let $\lambda = 1$ for weighted index, signed distance index.

Table 1 Results of the Network Based on Level Lamda TLR Weighted Index

i	paths	$W_A(L_{min}, L_{\mu i})$	Rank	$W_A(L_{max}, L_{\gamma i})$	Rank
1	$P_1 : 1 \to 2 \to 4$	32.75	2	40.5	2
2	$P_2 : 1 \to 4$	29	1	38.25	1
3	$P_3 : 1 \to 3 \to 4$	37	3	46	3

Table 2 Results of the Network Based on Level Lamda TLR Signed Distance Index

i	paths	$d(L_{\mu i}, 0)$	Rank	$d(L_{\gamma i}, 0)$	Rank
1	$P_1 : 1 \to 2 \to 4$	**13**	**2**	**4**	**2**
2	$P_2 : 1 \to 4$	**11.3**	**1**	**5**	**1**
3	$P_3 : 1 \to 3 \to 4$	**16.3**	**3**	**7.8**	**3**

Let $\lambda = .3$ and $\alpha = .5$ for convex index

Table 3 Results of the Network Based on Convex Index

i	paths	$COI_A(L_{min}, L_{\mu i})$	Rank	$COI_A(L_{max}, L_{\gamma i})$	Rank
1	$P_1: 1 \to 2 \to 4$	5.15	2	7.35	2
2	$P_2: 1 \to 4$	1.15	1	4.85	1
3	$P_3: 1 \to 3 \to 4$	7.85	3	12.45	3

Total integral index for different values of λ

(i) Let $\lambda = .3$ then,

Table 4 Results of the Network Based on Total Integral Index

i	paths	$IT_A(L_{min}, L_{\mu i})$	Rank	$I_{T_A}(L_{max}, L_{\gamma i})$	Rank
1	$P_1: 1 \to 2 \to 4$	64.2	2	80.85	2
2	$P_2: 1 \to 4$	53.4	1	64.85	1
3	$P_3: 1 \to 3 \to 4$	79.0	3	110.15	3

(i) $\lambda = 1$ then,

Table 5 Results of the Network Based on Total Integral Index

i	paths	$I(L_{min}, L_{\mu i})$	Rank	$I(L_{max}, L_{\gamma i})$	Rank
1	$P_1: 1 \to 2 \to 4$	.9967	2	1.070	2
2	$P_2: 1 \to 4$	1.022	1	1.131	1
3	$P_3: 1 \to 3 \to 4$	.9603	3	.9421	3

Step 5: Path $P_2: 1 \to 4$ is the Intuitionistic fuzzy shortest path since it has the highest level λ Trapezoidal LR weighted average index and the corresponding shortest path length is $L_{\mu 2} = (6,28,30,71)_{LR}$ and $L_{\gamma 2} = (3,30,31,61)_{LR}$. The solution obtained for Intuitionistic fuzzy shortest path problem in this paper coincides with the solution of the existing algorithm.

Conclusion

This paper developed three algorithms for solving the shortest path problem on a network with intuitionistic fuzzy arc length. The ranking given to the paths is helpful for the decision-makers as they make decisions in choosing the best of all the possible path alternatives. Verification is also done with the existing methods, which helps to conclude that the algorithms developed in the current paper are an alternative and improved form of previous methods, to get the shortest path in intuitionistic fuzzy environments.

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