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An Inventory Ordering Policy Using Constant Deteriorating Items With Constant Demand

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Abstract

In this paper, a deterministic inventory model for constant deteriorating items with constant demand is developed. optimal solution with respect to the parameter of the system is carried out with the help of numerical example using the software MATHEMATICA 4.1 version.

Keywords: Inventory, Ordering policy, Constant demand, Shortages, Deterioration

Introduction:

The analysis of deteriorating inventory began with Ghare and Schrader (1963), who established the classical no shortage inventory with a constant rate of decay. However it has been empirically observed that failure and life expectancy of many items can be expressed in items of weibull distribution. This empirical observation has prompted researchers to represent the time to deterioration of a product by a weibull distribution. Covert and Philip (1973) extended Ghare and Schrader (1963) model and obtain an economic order quantity model for a variable rate of deterioration buy assuming a two-parameter weibull distribution. Researchers including Philip (1974), Misra (1975) Tadikamalla (1978), Wee (1997, 93), Chakrabarty etal. (1998) and Mukhopadhaya etal (2005), developed economic order quantity models that focused on this type of products. Therefore, a realistic model is the ones that treat the deterioration rate as a time varying function. In general deterioration is defined as decay damage, spoilage, evaporation, obsolesce, pilferage, loss of utility or loss of marginal value of a commodity that results in decrease of usefulness from the original one. The decrease of loss of utility due to decay is usually a function of the on hand inventory. It is reasonable to note that a product may be understood to have life time which ends when utility reaches zero. The continuously decaying/Deterioration of items is classified as age dependent ongoing deterioration and age independent ongoing deterioration. Blood, fish, strawberry are some of the examples of the former while alcohol, gasoline and radioactive chemical and grain products are examples of the latter. Time varying demand patterns are commonly used to reflect sales in different phases of the product life cycle in the market. For example, the demand for inventory items increases over time in the growth phase and decreases in the decline phase. Donaldson (1997) initially developed an inventory model with a linear trend in demand. After that, many research works for example Dave and Patel (1981), Mak (1982), Holier and Mak (1983), Sachan (1984), Goyal (1987), Bahari-Kashani (1989), Aggrawal and Bahari-Kashani (1991), Goswami and Chaudhuri (1992), Xu and Wang (1992), Chung and Ting (1994), Hargia and Benkherouf (1994), Jain and Silver (1994), Hariga (1994,1995) Sahu,Dash and Sukla(2007), have been devoted to incorporating a time-varying demand rate into their models for deteriorating items with or with out shortages under a variety of circumstance.

In this paper, a deterministic inventory model for constant deteriorating items with constant demand is developed. optimal solution with respect to the parameter of the system is carried out with the help of numerical example using the software MATHEMATICA 4.1 version.

Assumptions:

(i) Lead time is negligible,

(ii) Replenishment is instantaneous,

(iii) A deteriorating item is neither repaired nor replaced during a given cycle,

(iv) Shortages are allowed and fully back-logged,

Notations:

T – Length of one cycle,

q – Order quantity,

A – Order Cost per order,

$r(t)$ – Constant demand rate,

$C -$ Unit cost of item,

$I_c -$ Inventory carrying,

$\pi -$ Shortage cost per unit time,

$\theta -$ Constant rate of deterioration, $0 \le \theta \le 1,$

$I(t) -$ Inventory level at time t,

Mathematical Formulation and Solutions :

Let $I(t)$ be the inventory level at any time t $(0 \le t \le T)$. and demand rate is depleting due to the simultaneously occurrence of demand and the deterioration.

The differential equations for the instantaneous state of $I(t)$ over $(0,T)$ are given by

$$\frac{dI(t)}{dt} + \theta I(t) = -R \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{di(t)}{dt} = -R \qquad t_1 \le t \le T \tag{2}$$

The solution of (1) (boundary condition at t=0, $I(t) = I(0)$) is

$$I(t) = I(0)e^{-\theta t} + \frac{R}{\theta}(e^{-\theta t} - 1) \tag{3}$$

The solution of (3) boundary conditions at $t = t_1$ $I(t_1) = 0,$ is

$$I(t) = R(t_1 - t) \qquad t_1 \le t \le T \tag{4}$$

at $t = t_{1,}$ $I(t_1) = 0,$ hence (3) becomes

$$\Rightarrow \frac{R}{\theta}(e^{-\theta t_1} - 1) + I(0)e^{-\theta t_1} = 0$$

$$\Rightarrow t_1 = \frac{1}{\theta}\log\left[1 + \frac{\theta I(0)}{R}\right] \tag{5}$$

Stock loss dueto deterioration in the interval $(0,t)$ is given by

$$\text{D (t)=I (0)-} \int_0^t R dt - I(t) \tag{6}$$

Again (3), gives

$$I(0)=\left(I(t+\frac{R}{\theta}\right)e^{\theta t}-\frac{R}{\theta} \tag{7}$$

Using (7) in (6),we get $D(t)=(e^{\theta t}-1)I(t)+\frac{R}{\theta}(e^{\theta t}-1)-Rt$ (8)

Loss of stock due to deterioration within cycle T is till time t_1.

$$D(t_1)=(e^{\theta t_1}-1)I(t_1)+\frac{R}{\theta}(e^{\theta t_1}-1)-Rt_1$$

Since $I(t_1)=0$,

$D(t_1)=0,$

$$D(t_1)=\frac{R}{\theta}(e^{\theta t_1}-1)-Rt_1 \tag{9}$$

$$B(t_1)=\int_{t_1}^{T}Rdt$$

$$=R(T-t_1)$$

Now $q=D(t_1)+\int_{0}^{T}Rdt$

$$\Rightarrow q=\frac{R}{\theta}(e^{\theta t_1}-1)-Rt_1+RT$$

$$\Rightarrow q=\frac{R}{\theta}(e^{\theta t}-1)+R(T-t_1) \tag{10}$$

and $I(0)=\frac{R}{\theta}(e^{\theta t_1}-1)$ (11)

Thus I (t)= $\frac{R}{\theta}(e^{-\theta t}-1)+\left\{\frac{R}{\theta}(e^{-\theta t}-1)\right\}e^{-\theta t}$ (12)

The total cost per unit time $C(T,t_1)$, is the sum of cost of the units, ordering cost, inventory holding cost and shortage cost

$$C(T,t_1)=\frac{A}{T}+\frac{cq}{T}+\frac{I_c}{T}\int_{0}^{t_1}I(t)dt-\frac{\pi}{t}\int_{t_1}^{T}I(t)dt \tag{13}$$

$$C(T,t_1)=\frac{A}{T}+\frac{C}{T}\left[\frac{R}{\theta}\left(e^{\theta t}-1\right)+R(T-t_1)\right]+\frac{I_c}{T}\left(\frac{R}{\theta^2}\left(1-e^{\theta t_1}-\theta t_1\right)\right)+\frac{\pi R}{T}\frac{\left(t_1-T\right)^2}{2} \qquad (14)$$

Our objective is to minimize the cost function $C(T,t_1)$, which is a differentiable function.

Let $t_1=ZT, \qquad 0<Z<1 \qquad (15)$

By substituting (15) in(14), it can be written as

$$C(T)=\frac{A}{T}+\frac{C}{T}\left[\frac{R}{\theta}(e^{\theta TZ}-1)+R(T-TZ)\right]+\frac{I_c}{T}\left\{\frac{R}{\theta^2}(1-e^{\theta TZ}-\theta TZ)\right\}+\frac{\pi R}{T}\frac{(TZ-T)^2}{2}$$

$$=\frac{A}{T}+\frac{C}{T}\left[\frac{R}{\theta}(e^{\theta TZ}-1)+R(T-TZ)\right]+\frac{I_c}{T}\left[\frac{R}{\theta^2}(1-e^{\theta TZ}-\theta TZ)\right]+\frac{\pi R}{2}(TZ^2+T-2TZ)$$

minimize the cost function $C(T)$, the necessary condition is $\frac{dC(T)}{dT}=0.$

$$\begin{aligned}\frac{dC(T)}{dT}=&\frac{-A}{T^2}-\frac{C}{T^2}\left[\frac{R}{\theta}(e^{\theta TZ}-1)+R(T-TZ)\right]+\tfrac{C}{T}\left[RZe^{\theta TZ}+R(1-Z)\right]\\&-\frac{I_C}{T^2}\left[\frac{R}{\theta^2}(1-e^{\theta TZ}-\theta TZ)\right]+\frac{\pi R}{2}(Z-1)^2\end{aligned} \qquad (17)$$

$$\frac{d^2C}{dT^2}=\frac{2A}{T^3}+\frac{2C}{T^3}\left[\frac{R}{\theta}(e^{\theta TZ}-1)+R(T-TZ)\right]-\frac{2C}{T^2}\left[(RZe^{\theta TZ}+(R(1-Z)\right]$$

$$+\frac{C}{T}(R\theta Z^2e^{\theta ZT})+\frac{2I_c}{T^3}\left[\frac{R}{\theta^2}(1-e^{\theta TZ}-\theta TZ)\right]+\frac{2I_c}{T^2}\left[\frac{Rz}{\theta}(e^{\theta TZ}+1)\right]-\frac{I_c}{T}RZ^2e^{\theta TZ}$$

(18)

The optimal value of T say T^* can be obtained from equation (18) and optimal order quanity q^* can be obtained from (10) $\frac{d^2C}{dT^2}>0$

Numerical Example:

As an illustration to the above-developed model we consider a system with the following parameters:

$A = Rs.200/-$ Per order. $C = Rs.20/-$ Per unit, $I_c = Rs.5/-$ Per unit time, $\pi = Rs.5/-$ Per unit per unit time, $\theta = 0.05$, R=7200. ,we gets the optimal length of ordering cycle T^* is 3.0422, the optimal order quantity q^* is 1.77889 and the optimal total variable cost C^* is 145.342. The optimal solution with respect to the above parameters of the system is carried out with the help of the software MATHEMATICA 4.1version.

CONCLUSION:

Inventory ordering policy using constant deteriorating items with constant demand is presented in this paper, method is efficient for getting accurate results, and no doubt it is a bit lengthy and complicated. While the method is faster and good for mathematical simplication.

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