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An Inverse Thermoelastic Problem Of Thin Finite Rectangular Plate Due To Internal Heat Sources

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ABSTRACT
This paper is concerned with three dimensional inverse thermoelastic problem of thin rectangular plate due to internal heat sources. To determine unknown temperature distribution, displacement and thermal stresses on edge z=h of the thin rectangular plate due to internal heat sources with known third kind boundary and initial condition by applying Marchi-Fasulo and Laplace transform technique. The results are obtained in terms of infinite series and the numerical calculations are carried out by using MATHCAD -7 software and shown graphically.

KEYWORDS:
Three dimensional inverse thermoelastic problem, thin rectangular plate, integral transform.

INTRODUCTION

Tanigawa et al.[1] have studied thermal stress analysis of a rectangular plate and its thermal stress intensity factor for compressive stress field. Khobragade et al.[3,6,7] to determine an inverse unsteady-state thermoelastic problem of thick rectangular plate. Kishor et al.[5]have discuss three dimensional non homogeneous thermoelastic problem of thick rectangular plate due to internal heat generation. Lamba et al.[9] have studied thermoelastic problem of thin rectangular plate due to partially distributed heat supply.

In the present paper to determine the temperature, displacement and thermal stresses on the edge z=h, of thin rectangular plate due to internal heat sources occupying the region D: x ϵ(-a, a), y ϵ (-b, b), z ϵ (-h, h) with known boundary conditions here the finite Marchi-Fasulo and Laplace transforms has been used to find the solution.

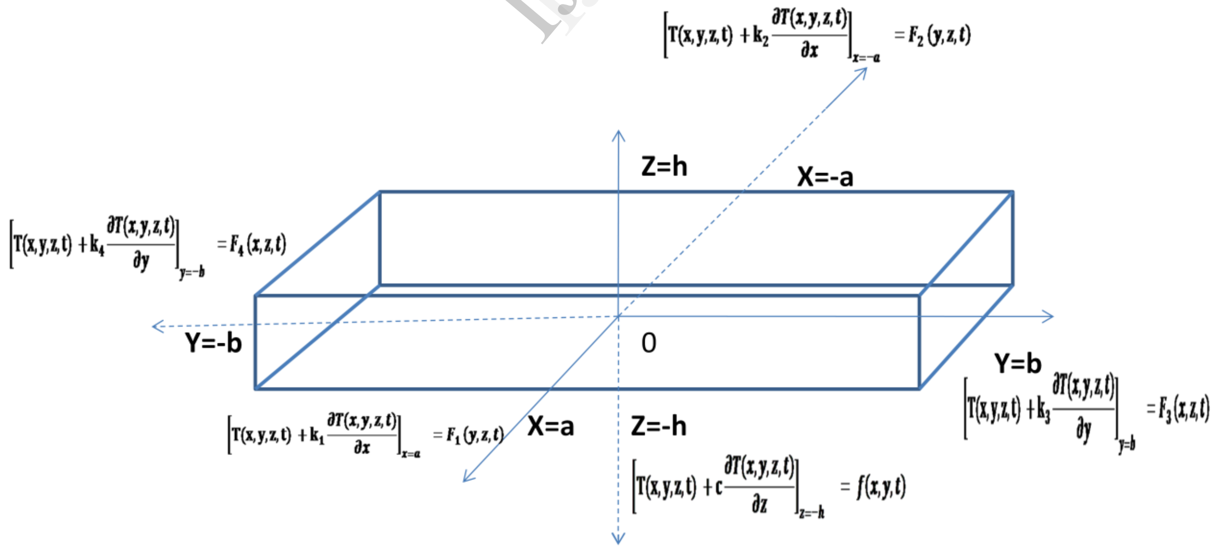

Fig.1 Thin rectangular plate with known third kind boundary condition

2. STATEMENT OF THE PROBLEM

Consider a thin rectangular plate from fig.1 occupying the space D:$-a \le x \le a, -b \le y \le b, -h \le z \le h$.The displacement components u_x, u_y, u_z in the x, y, z direction respectively are in the integral form as in [2]

$$u_x = \int_{-a}^{a} \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - \nu \frac{\partial^2 U}{\partial x^2}\right) + \lambda T\right] dx \tag{2.1}$$

$$u_y = \int_{-b}^{b} \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} - \nu \frac{\partial^2 U}{\partial y^2}\right) + \lambda T\right] dy \tag{2.2}$$

$$u_z = \int_{-h}^{h} \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial z^2}\right) + \lambda T\right] dz \tag{2.3}$$

Where E, ν and λ are the Young modulus, the poisson ratio and the linear coefficient of thermal

expansion of the material of the plate respectively, U(x, y ,z ,t) is the Airy stress function which satisfies the differential equation.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z,t) = -\lambda E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) T(x,y,z,t) \tag{2.4}$$

Here $T(x,y,z,t)$ denotes the temperature of the thin rectangular plate satisfying the following differential equation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\theta(x,y,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2.5}$$

Where k is thermal conductivity and α is the thermal diffusivity of the material of the plate and $\theta(x,y,z,t)$ is heat generated within the rectangular plate for t>0 subject to initial conditions

$$T(x,y,z,0) = 0 \tag{2.6}$$

The boundary conditions

$$\left[\mathrm{T}(x,y,z,t) + k_1 \frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=a} = F_1(y,z,t) \tag{2.7}$$

$$\left[\mathrm{T}(x,y,z,t) + k_2 \frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=-a} = F_2(y,z,t) \tag{2.8}$$

$$\left[\mathrm{T}(x,y,z,t) + k_3 \frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=b} = F_3(x,z,t) \tag{2.9}$$

$$\left[\mathrm{T}(x,y,z,t) + k_4 \frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=-b} = F_4(x,z,t) \tag{2.10}$$

$$\left[\mathrm{T}(x,y,z,t) + c \frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=-h} = f(x,y,t) \tag{2.11}$$

$$[\mathrm{T}(x,y,z,t)]_{z=h} = g(x,y,t) \quad \text{(Unknown)} \tag{2.12}$$

The interior condition

$$\left[\mathrm{T}(x,y,z,t) + c \frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=\xi} = 0 \quad \text{(Known)} \tag{2.13}$$

The components in term of U(x, y, z, t) are given by

$$\sigma_{xx} = \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right) \tag{2.14}$$

$$\sigma_{yy} = \left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2}\right) \tag{2.15}$$

$$\sigma_{zz} = \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) \tag{2.16}$$

The equations (2.1) to (2.16) constitute the mathematical formulation of the problem under consideration.

*3. SOLUTION OF THE PROBLEM

The finite Marchi-Fasulo integral transform of f (z), -h < z < h is defined to be

$$\bar{F} = \int_{-h}^{h} f(z)\, P_n(z) dz \tag{3.1}$$

Then at each point of (-h,h) at which f(z) is continuous. Also the inverse finite Marchi-Fasulo transform is defined as

$$f(z) = \sum_{n=1}^{\infty} \frac{\bar{F}(n)}{\lambda_n} P_n(z) \tag{3.2}$$

Where

$$P_n(z) = Q_n \cos(a_n z) - W_n \sin(a_n z)$$

$$Q_n = a_n(\alpha_1 + \alpha_2)\cos(a_n h) + (\beta_1 - \beta_2)\sin(a_n h)$$

$$W_n = (\beta_1 + \beta_2)\cos(a_n h) + (\alpha_1 - \alpha_2)a_n \sin(a_n h)$$

$$\lambda_n = \int_{-h}^{h} P_n^{\,2}(z)dz$$
$$= h\left[Q_n^{\,2} + W_n^{\,2}\right] + \frac{\sin(2a_n h)}{2a_n}\left[Q_n^{\,2} - W_n^{\,2}\right]$$

The Eigen values a_n are the solutions of the equation

$$[\alpha_1\, a_n \cos(a_n h) + \beta_1 \sin(a_n h)] \times [\beta_2 \cos(a_n h) + \alpha_2\, \mathrm{a}\sin(a_n h)]$$
$$= [\alpha_2\, a_n \cos(a_n h) - \beta_2 \sin(a_n h)] \times [\beta_1 \cos(a_n h) - \alpha_1\, a_n \sin(a_n h)] \qquad (3.3)$$

Where $\alpha_1, \alpha_2, \beta_1, \beta_2$ are constants

By applying the finite March- Fasulo transform two times and Laplace transform to equation (2.5) and their inverses, we obtain

$$\frac{d^2\bar{\bar{T}}^*}{dz^2} - p^2\bar{\bar{T}}^* = \propto \left(\bar{\bar{\emptyset}}^* + \frac{\bar{\bar{\theta}}^*}{k}\right) \qquad (3.4)$$

Where $p^2 = a_m^{\,2} + a_n^{\,2} + \frac{s}{\propto}$

The Eigen values a_m , a_n are the solutions of the equation

$$[\alpha_1\, a_n \cos(a_n a) + \beta_1 \sin(a_n a)] \times [\beta_2 \cos(a_n a) + \alpha_2\, \mathrm{a}\sin(a_n a)]$$
$$= [\alpha_2\, a_n \cos(a_n a) - \beta_2 \sin(a_n a)] \times [\beta_1 \cos(a_n a) - \alpha_1\, a_n \sin(a_n a)] \quad \text{and}$$

$$[\alpha_1\, a_n \cos(a_n b) + \beta_1 \sin(a_n b)] \times [\beta_2 \cos(a_n b) + \alpha_2\, \mathrm{a}\sin(a_n b)]$$
$$= [\alpha_2\, a_n \cos(a_n b) - \beta_2 \sin(a_n b)] \times [\beta_1 \cos(a_n b) - \alpha_1\, a_n \sin(a_n b)]$$

Where $\alpha_1, \alpha_2, \beta_1, \beta_2$ are constants
and

$$-\emptyset = P_m(a)F_2 - P_m(-a)F_1 + P_n(b)F_4 - P_n(-b)F_3$$

The equation (3.4) is a second order differential equation whose solution is in form

$$\bar{\bar{T}}^* = Ae^{pz} + Be^{-pz} + PI \qquad (3.5)$$

where $PI = \frac{\propto(Q^* - \frac{\bar{\Psi}^*}{k})}{D^2 - q^2}$, $D \equiv \frac{d}{dz}$ and A, B are constant. Using boundary conditions we obtain the values of A and B substituting these values (3.5) and then apply inverse of Laplace transform and Marchi- Fasulo integral transform. We obtain

$$T(x, y, z, t) = \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}} \frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}}$$
$$\times \sum_{\mathrm{m}=1}^{\infty} (-1)^{m+1}\, \mathrm{m} \left[\mathrm{Sin}\left[\left(\frac{\mathrm{m}\pi}{h+\xi}\right)(\mathrm{z}+\mathrm{h})\right] - \mathrm{cpCos}\left[\left(\frac{\mathrm{m}\pi}{h+\xi}\right)(\mathrm{z}+\mathrm{h})\right]\right]$$
$$\times \int_0^{\mathrm{t}} \left[- [\mathrm{PI}]_{\mathrm{z}=\xi} - \mathrm{c}\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{\mathrm{z}=\xi}\right] e^{-k\left(a_m^{\,2} + a_n^{\,2} + \left(\frac{\mathrm{m}\pi}{h+\xi}\right)^2\right)(t-t')} dt'$$
$$- \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}} \frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}} \sum_{\mathrm{m}=1}^{\infty} (-1)^{m+1}\, \mathrm{m} \left[\mathrm{Sin}\left[\left(\frac{\mathrm{m}\pi}{h+\xi}\right)(\mathrm{z}-\xi)\right] - \mathrm{cpCos}\left[\left(\frac{\mathrm{m}\pi}{h+\xi}\right)(\mathrm{z}-\xi)\right]\right]$$
$$\times \int_0^{\mathrm{t}} \left[\bar{\bar{f}} - [\mathrm{PI}]_{\mathrm{z}=-\mathrm{h}} - \mathrm{c}\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{\mathrm{z}=-\mathrm{h}}\right] e^{-k\left(a_m^{\,2} + a_n^{\,2} + \left(\frac{\mathrm{m}\pi}{h+\xi}\right)^2\right)(t-t')} dt'$$
$$+ \sum_{m,n=1}^{\infty} \frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}} \frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}} L^{-1}[PI] \int_0^{\mathrm{t}} e^{-k\left(a_m^{\,2} + a_n^{\,2} + \left(\frac{\mathrm{m}\pi}{h+\xi}\right)^2\right)(t-t')} dt' \qquad (3.6)$$

$$T(x, y, z, t) = \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}} \frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}} \sum_{\mathrm{m}=1}^{\infty} (-1)^{m+1}\, \mathrm{m}[\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)] +$$
$$\sum_{m,n=1}^{\infty} \frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}} \frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}} L^{-1}[PI]\psi_3(t) \qquad (3.7)$$

Where

$$\phi_1(z) = \left[\mathrm{Sin}\left[\left(\frac{m\pi}{h+\xi}\right)(z+h)\right] - cp\mathrm{Cos}\left[\left(\frac{m\pi}{h+\xi}\right)(z+h)\right]\right]$$

$$\psi_1(t) = \int_0^t \left[-[PI]_{z=\xi} - c\left[\frac{dPI}{dz}\right]_{z=\xi}\right] e^{-k\left(a_m{}^2+a_n{}^2+\left(\frac{m\pi}{h+\xi}\right)^2\right)(t-t')} dt'$$

$$\phi_2(z) = \left[\mathrm{Sin}\left[\left(\frac{m\pi}{h+\xi}\right)(z-\xi)\right] - cp\mathrm{Cos}\left[\left(\frac{m\pi}{h+\xi}\right)(z-\xi)\right]\right]$$

$$\psi_2(t) = \int_0^t \left[\bar{\bar{f}} - [PI]_{z=-h} - c\left[\frac{dPI}{dz}\right]_{z=-h}\right] e^{-k\left(a_m{}^2+a_n{}^2+\left(\frac{m\pi}{h+\xi}\right)^2\right)(t-t')} dt'$$

$$\psi_3(t) = \int_0^t e^{-k\left(a_m{}^2+a_n{}^2+\left(\frac{m\pi}{h+\xi}\right)^2\right)(t-t')} dt'$$

$$g(x,y,t) = \frac{2k\pi}{(h+\xi)^2}\sum_{m,n=1}^{\infty}\frac{1}{(1-c^2p^2)}\frac{P_m(x)}{\lambda_m}\frac{P_n(y)}{\mu_n}$$

$$\times \sum_{m=1}^{\infty}(-1)^{m+1}\, m\left[\mathrm{Sin}\left[\left(\frac{m\pi}{h+\xi}\right)(h+h)\right] - cp\mathrm{Cos}\left[\left(\frac{m\pi}{h+\xi}\right)(h+h)\right]\right]\times$$

$$\int_0^t \left[-[PI]_{z=\xi} - c\left[\frac{dPI}{dz}\right]_{z=\xi}\right] e^{-k\left(a_m{}^2+a_n{}^2+\left(\frac{m\pi}{h+\xi}\right)^2\right)(t-t')} dt'$$

$$-\frac{2k\pi}{(h+\xi)^2}\sum_{m,n=1}^{\infty}\frac{1}{(1-c^2p^2)}\frac{P_m(x)}{\lambda_m}\frac{P_n(y)}{\mu_n}\sum_{m=1}^{\infty}(-1)^{m+1}\, m\left[\mathrm{Sin}\left[\left(\frac{m\pi}{h+\xi}\right)(h-\xi)\right] - cp\mathrm{Cos}\left[\left(\frac{m\pi}{h+\xi}\right)(h-\xi)\right]\right]$$

$$\times \int_0^t \left[\bar{\bar{f}} - [PI]_{z=-h} - c\left[\frac{dPI}{dz}\right]_{z=-h}\right] e^{-k\left(a_m{}^2+a_n{}^2+\left(\frac{m\pi}{h+\xi}\right)^2\right)(t-t')} dt'$$

$$+\sum_{m,n=1}^{\infty}\frac{P_m(x)}{\lambda_m}\frac{P_n(y)}{\mu_n}L^{-1}[PI]\int_0^t e^{-k\left(a_m{}^2+a_n{}^2+\left(\frac{m\pi}{h+\xi}\right)^2\right)(t-t')} dt' \qquad (3.8)$$

Here $\bar{\bar{f}}$ denote the of Marchi- Fasulo integral transform of $\bar{f}$ and f denote the of Marchi- Fasulo integral transform of f

$\bar{\bar{f}} = \int_{-b}^{b}\bar{f}\,P_n(y)dy$ and $\mu_n = \int_{-b}^{b}P_n{}^2(y)dy$

$P_n(y) = Q_n\cos(a_n b) - W_n\sin(a_n b)$
$Q_n = a_n(\alpha_3+\alpha_4)\cos(a_n b) + (\beta_3 - \beta_4)\sin(a_n b)$
$W_n = (\beta_3 + \beta_4)\cos(a_n b) + (\alpha_4 - \alpha_3)a_n\sin(a_n b)$
The equations (3.6) and (3.7) are the desired solutions of the given problem with $\beta_3 = \beta_4 = 1$
And $\alpha_3 = k_3$, $\alpha_4 = k_4$

4. DETERMINATION OF THE AIRY STRESS FUNCTION

Substituting the value of T(x, y, z ,t)from equation (3.7) in the equation (2.4), one obtains

$$U(x,y,z,t) = -\propto E\frac{2k\pi}{(h+\xi)^2}\sum_{m,n=1}^{\infty}\frac{1}{(1-c^2p^2)}\frac{P_m(x)}{\lambda_m}\frac{P_n(y)}{\mu_n}$$

$$\times\sum_{m=1}^{\infty}(-1)^{m+1}\frac{1}{a_m{}^2+a_n{}^2-\left(\frac{m\pi}{h+\xi}\right)^2}\, m[\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)]$$

$$-\propto E\sum_{m,n=1}^{\infty}\frac{1}{a_m{}^2+a_n{}^2}\frac{P_m(x)}{\lambda_m}\frac{P_n(y)}{\mu_n}L^{-1}[PI]\psi_3 \qquad (4.1)$$

5. DETERMINATION OF THE DISPLACEMENT COMPONENTS

Substituting the value of U(x, y, z ,t)from equation (4.1) in the equation (2.1),(2.2),(2.3) one obtains

$$u_x = -\propto \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P''_n(y)}{\mu_n} \int_{-a}^{a} \frac{P_m(x)}{\lambda_m} dx$$

$$\times \sum_{m=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2+a_n{}^2-\left(\frac{m\pi}{h+\xi}\right)^2} m[\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)]$$

$$- \propto \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2+a_n{}^2} \frac{P''_n(y)}{\mu_n} \int_{-a}^{a} \frac{P_m(x)}{\lambda_m} dx L^{-1}[PI]\, \psi_3(t)$$

$$+ \propto \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P_n(y)}{\mu_n} \int_{-a}^{a} \frac{P_m(x)}{\lambda_m} dx$$

$$\times \sum_{m=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2+a_n{}^2-\left(\frac{m\pi}{h+\xi}\right)^2} m[\phi''_1(z)\psi_1(t) - \phi''_2(z)\psi_2(t)]$$

$$+ \propto \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2+a_n{}^2} \frac{P_n(y)}{\mu_n} \int_{-a}^{a} \frac{P_m(x)}{\lambda_m} dx \frac{\partial^2 [\, L^{-1}\{PI\}]}{\partial z^2} \psi_3(t)$$

$$+ \propto \upsilon \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P_n(y)}{\mu_n} \int_{-a}^{a} \frac{P''_m(x)}{\lambda_m} dx \times$$

$$\sum_{m=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2+a_n{}^2-\left(\frac{m\pi}{h+\xi}\right)^2} m[\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)]$$

$$+ \propto \upsilon \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2+a_n{}^2} \frac{P_n(y)}{\mu_n} \int_{-a}^{a} \frac{P''_m(x)}{\lambda_m} dx L^{-1}[PI]\, \psi_3(t)$$

$$+ \lambda \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P_n(y)}{\mu_n} \int_{-a}^{a} \frac{P_m(x)}{\lambda_m} dx \sum_{m=1}^{\infty} (-1)^{m+1} m[\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)]$$

$$+ \lambda \sum_{m,n=1}^{\infty} \frac{P_n(y)}{\mu_n} \int_{-a}^{a} \frac{P_m(x)}{\lambda_m} dx L^{-1}[PI]\, \psi_3(t) \qquad (5.1)$$

$$u_y = \propto \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P_m(x)}{\lambda_m} \int_{-b}^{b} \frac{P_n(y)}{\mu_n} dy$$

$$\times \sum_{m=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2+a_n{}^2-\left(\frac{m\pi}{h+\xi}\right)^2} m[\phi''_1(z)\psi_1(t) - \phi''_2(z)\psi_2(t)]$$

$$+ \propto \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2+a_n{}^2} \frac{P_m(x)}{\lambda_m} \int_{-b}^{b} \frac{P_n(y)}{\mu_n} dy \frac{\partial^2 [\, L^{-1}\{PI\}]}{\partial z^2} \psi_3(t)$$

$$- \propto \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P''_m(x)}{\lambda_m} \int_{-b}^{b} \frac{P_n(y)}{\mu_n} dy \times$$

$$\sum_{m=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2+a_n{}^2-\left(\frac{m\pi}{h+\xi}\right)^2} m[\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)]$$

$$- \propto \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2+a_n{}^2} \frac{P''_m(x)}{\lambda_m} \int_{-b}^{b} \frac{P_n(y)}{\mu_n} dy L^{-1}[PI]\psi_3(t)$$

$$+ \propto \nu \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P_m(x)}{\lambda_m} \int_{-b}^{b} \frac{P''_n(y)}{\mu_n} dy$$

$$\times \sum_{m=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2+a_n{}^2-\left(\frac{m\pi}{h+\xi}\right)^2} m[\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)]$$

$$+ \propto \nu \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2+a_n{}^2} \frac{P_m(x)}{\lambda_m} \int_{-b}^{b} \frac{P''_n(y)}{\mu_n} dy\, L^{-1}[PI]\psi_3(t)$$

$$+ \lambda \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P_m(x)}{\lambda_m} \int_{-b}^{b} \frac{P_n(y)}{\mu_n} dy \sum_{m=1}^{\infty} (-1)^{m+1} m[\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)]$$

$$+ \lambda \sum_{m,n=1}^{\infty} \frac{P_m(x)}{\lambda_m} \int_{-b}^{b} \frac{P_n(y)}{\mu_n} dy\, L^{-1}[PI]\psi_3(t) \qquad (5.2)$$

$$u_z = -\propto \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P''_m(x)}{\lambda_m} \frac{P_n(y)}{\mu_n} \times$$

$$\sum_{m=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2 + a_n{}^2 - \left(\frac{m\pi}{h+\xi}\right)^2} m \left[\int_{-h}^{h} \phi_1(z)\psi_1(t)dz - \int_{-h}^{h} \phi_2(z)\psi_2(t)dz\right]$$

$$- \propto \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2 + a_n{}^2} \frac{P''_m(x)}{\lambda_m} \frac{P_n(y)}{\mu_n} \int_0^h L^{-1}[PI]dz\, \psi_3(t)$$

$$-\propto \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P_m(x)}{\lambda_m} \frac{P''_n(y)}{\mu_n}$$

$$\times \sum_{m=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2 + a_n{}^2 - \left(\frac{m\pi}{h+\xi}\right)^2} m \left[\int_{-h}^{h} \phi_1(z)\psi_1(t)dz - \int_{-h}^{h} \phi_2(z)\psi_2(t)dz\right]$$

$$- \propto \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2 + a_n{}^2} \frac{P_m(x)}{\lambda_m} \frac{P''_n(y)}{\mu_n} \int_0^h L^{-1}[PI]dz\, \psi_3(t)$$

$$+ \propto \nu \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P_m(x)}{\lambda_m} \frac{P_n(y)}{\mu_n}$$

$$\times \sum_{m=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2 + a_n{}^2 - \left(\frac{m\pi}{h+\xi}\right)^2} m \left[\int_{-h}^{h} \phi''_1(z)\psi_1(t)\, dz - \int_{-h}^{h} \phi''_2(z)\psi_2(t)dz\right]$$

$$+\propto \nu \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2 + a_n{}^2} \frac{P_m(x)}{\lambda_m} \frac{P_n(y)}{\mu_n} \int_0^h L^{-1}[PI]dz\, \psi_3(t)$$

$$+\lambda \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P_m(x)}{\lambda_m} \frac{P_n(y)}{\mu_n} \sum_{m=1}^{\infty} (-1)^{m+1} m \left[\int_{-h}^{h} \phi_1(z)\psi_1(t)dz - \int_{-h}^{h} \phi_2(z)\psi_2(t)dz\right]$$

$$+ \lambda \sum_{m,n=1}^{\infty} \frac{P_m(x)}{\lambda_m} \frac{P_n(y)}{\mu_n} L^{-1}[PI]\psi_3(t) \qquad (5.3)$$

Where

$$\phi''_1(z) = \left(\frac{m\pi}{h+\xi}\right)^2 \left(3c - \left(\frac{m\pi}{h+\xi}\right)^2\right) \left[\mathrm{Sin}\left[\left(\frac{m\pi}{h+\xi}\right)(z+h)\right] - c\left(1 - \left(\frac{m\pi}{h+\xi}\right)^3\right) p\mathrm{Cos}\left[\left(\frac{m\pi}{h+\xi}\right)(z+h)\right]\right]$$

$$\phi''_2(z) = \left(\frac{m\pi}{h+\xi}\right)^2 \left(3c - \left(\frac{m\pi}{h+\xi}\right)^2\right) \left[\mathrm{Sin}\left[\left(\frac{m\pi}{h+\xi}\right)(z-\xi)\right] - c\left(1 - \left(\frac{m\pi}{h+\xi}\right)^3\right) p\mathrm{Cos}\left[\left(\frac{m\pi}{h+\xi}\right)(z-\xi)\right]\right]$$

6. DETERMINATION OF STRESS FUNCTIONS

Using (3.9) in (3.14), (3.15) and (3.16) the stress functions are obtained as

$$\sigma_{xx} = -\propto E \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P_m(x)}{\lambda_m} \frac{P''_n(y)}{\mu_n}$$

$$\times \sum_{m=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2 + a_n{}^2 - \left(\frac{m\pi}{h+\xi}\right)^2} m[\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)]$$

$$- \propto E \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2 + a_n{}^2} \frac{P_m(x)}{\lambda_m} \frac{P''_n(y)}{\mu_n} L^{-1}[PI]\psi_3(t)$$

$$-\propto E \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{P_m(x)}{\lambda_m} \frac{P_n(y)}{\mu_n}$$

$$\times \sum_{m=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2 + a_n{}^2 - \left(\frac{m\pi}{h+\xi}\right)^2} m[\phi''_1(z)\psi_1(t) - \phi''_2(z)\psi_2(t)]$$

$$-\propto E \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2 + a_n{}^2} \frac{P_m(x)}{\lambda_m} \frac{P_n(y)}{\mu_n} \frac{\partial^2[L^{-1}\{PI\}]}{\partial z^2} \psi_3(t) \qquad (6.1)$$

$$\sigma_{yy} = -\propto E \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}} \frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}}$$

$$\times \sum_{\mathrm{m}=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2 + a_n{}^2 - \left(\frac{\mathrm{m}\pi}{h+\xi}\right)^2} \mathrm{m}[\phi''_1(z)\psi_1(t) - \phi''_2(z)\psi_2(t)]$$

$$-\propto E \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2 + a_n{}^2} \frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}} \frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}} \frac{\partial^2 [\mathrm{L^{-1}\{PI\}}]}{\partial z^2} \psi_3(t)$$

$$-\propto E \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{\mathrm{P''_m(x)}}{\lambda_\mathrm{m}} \frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}} \times$$

$$\sum_{\mathrm{m}=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2 + a_n{}^2 - \left(\frac{\mathrm{m}\pi}{h+\xi}\right)^2} \mathrm{m}[\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)]$$

$$-\propto E \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2 + a_n{}^2} \frac{\mathrm{P''_m(x)}}{\lambda_\mathrm{m}} \frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}} L^{-1}[PI]\psi_3(t) \qquad (6.2)$$

$$\sigma_{zz} = -\propto E \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{\mathrm{P''_m(x)}}{\lambda_\mathrm{m}} \frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}} \times$$

$$\sum_{\mathrm{m}=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2 + a_n{}^2 - \left(\frac{\mathrm{m}\pi}{h+\xi}\right)^2} \mathrm{m}[\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)]$$

$$-\propto E \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2 + a_n{}^2} \frac{\mathrm{P''_m(x)}}{\lambda_\mathrm{m}} \frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}} L^{-1}[PI]\psi_3(t)$$

$$-\propto E \frac{2k\pi}{(h+\xi)^2} \sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)} \frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}} \frac{\mathrm{P''_n(y)}}{\mu_\mathrm{n}}$$

$$\times \sum_{\mathrm{m}=1}^{\infty} (-1)^{m+1} \frac{1}{a_m{}^2 + a_n{}^2 - \left(\frac{\mathrm{m}\pi}{h+\xi}\right)^2} \mathrm{m}[\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)]$$

$$-\propto E \sum_{m,n=1}^{\infty} \frac{1}{a_m{}^2 + a_n{}^2} \frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}} \frac{\mathrm{P''_n(y)}}{\mu_\mathrm{n}} L^{-1}[PI]\psi_3(t) \qquad (6.3)$$

7. SPECIAL CASE

Setting $f(x, y, t) = (1 - e^{-t})(x + a)^2(x - a)^2(y + b)^2(y - b)^2\xi$ (7.1)

$\theta(x, y, z, t) = g(t)\,\delta(z - \mathrm{h})$ and $\phi = 0$, $g(t) = const$ (7.2)

$[\mathrm{PI}]_{z=\xi} = \mathrm{const.}, \left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=\xi} = \mathrm{const.}$

$[\mathrm{PI}]_{z=-\mathrm{h}} = const.\ , \left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=-\mathrm{h}} = \mathrm{const.}$

$L^{-1}[PI] = const$ and $p = \frac{\mathrm{m}\pi}{h+\xi}$

Applying finite Marchi-Fasulo integral transform to (7.1), (7.2) one obtains

$$\bar{\bar{f}}(n, m, t) = [16(k_1 + k_2)(k_3 + k_4)(1 - e^{-t})\xi] \times \left[\frac{(a_m a)cos^2(a_m a) - \cos(a_m a)sin(a_m a)}{a_m{}^2}\right] \times \left[\frac{(a_n b)cos^2(a_n b) - \cos(a_n b)sin(a_n b)}{a_n{}^2}\right]$$

Substitute this values in the equation (3.6),(3.8), 4.1),(5.1),(5.2)(5.3),(6.1),(6.2)and(6.3)one obtains

$$T(x,y,z,t) = \frac{2k\pi}{(h+\xi)^2}\quad [16(k_1+k_2)(k_3+k_4)\ ]\left[\frac{(a_m a)cos^2(a_m a) - \cos(a_m a)sin(a_m a)}{a_m{}^2}\right]\left[\frac{(a_n b)cos^2(a_n b) - \cos(a_n b)sin(a_n b)}{a_n{}^2}\right]\times$$

$$\sum_{m,n=1}^{\infty} \frac{1}{(1-c^2p^2)}\frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}}\frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}} \times \sum_{\mathrm{m}=1}^{\infty}(-1)^{m+1}\,\mathrm{m}\left[\mathrm{Sin}\left[\left(\frac{\mathrm{m}\pi}{h+\xi}\right)(\mathrm{z}+\mathrm{h})\right] - \mathrm{cpCos}\left[\left(\frac{\mathrm{m}\pi}{h+\xi}\right)(\mathrm{z}+\mathrm{h})\right]\right]$$

$$\times\int_0^{\mathrm{t}}\left[-[\mathrm{PI}]_{\mathrm{z}=\xi} - \mathrm{c}\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{\mathrm{z}=\xi}\right] e^{-k\left(a_m{}^2+a_n{}^2+\left(\frac{m\pi}{h+\xi}\right)^2\right)(t-t')}dt'$$

$$-\frac{2k\pi}{(h+\xi)^2}\quad\left[16(k_1+k_2)\binom{k_3+}{k_4}\right]$$

$$\left[\frac{(a_m a)cos^2(a_m a) - \cos(a_m a)sin(a_m a)}{a_m{}^2}\right]\left[\frac{(a_n b)cos^2(a_n b) - \cos(a_n b)sin(a_n b)}{a_n{}^2}\right]\sum_{m,n=1}^{\infty}\frac{1}{(1-c^2p^2)}\frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}}\frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}}$$

$$\sum_{\mathrm{m}=1}^{\infty}(-1)^{m+1}\,\mathrm{m}\left[\mathrm{Sin}\left[\left(\frac{\mathrm{m}\pi}{h+\xi}\right)(\mathrm{z}-\xi)\right] - \mathrm{cpCos}\left[\left(\frac{\mathrm{m}\pi}{h+\xi}\right)(\mathrm{z}-\xi)\right]\right]$$

$$\times\int_0^{\mathrm{t}}\left[\bar{\bar{f}} - [\mathrm{PI}]_{\mathrm{z}=-\mathrm{h}} - \mathrm{c}\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{\mathrm{z}=-\mathrm{h}}\right] e^{-k\left(a_m{}^2+a_n{}^2+\left(\frac{m\pi}{h+\xi}\right)^2\right)(t-t')}dt'$$

$$+\sum_{m,n=1}^{\infty}\frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}}\frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}}L^{-1}[PI]\int_0^{\mathrm{t}} e^{-k\left(a_m{}^2+a_n{}^2+\left(\frac{m\pi}{h+\xi}\right)^2\right)(t-t')}dt' \qquad (7.3)$$

$$g(x,y,t) = \frac{2k\pi}{(h+\xi)^2}[16(k_1+k_2)(k_3+k_4)\ ]\quad\times\left[\frac{(a_m a)cos^2(a_m a) - \cos(a_m a)sin(a_m a)}{a_m{}^2}\right]\times$$

$$\left[\frac{(a_n b)cos^2(a_n b) - \cos(a_n b)sin(a_n b)}{a_n{}^2}\right]\sum_{m,n=1}^{\infty}\frac{1}{\left(1-c^2\left(\frac{m\pi}{h+\xi}\right)^2\right)}\frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}}\frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}}$$

$$\times\sum_{\mathrm{m}=1}^{\infty}(-1)^{m+1}\,\mathrm{m}\left[\mathrm{Sin}\left[\left(\frac{\mathrm{m}\pi}{h+\xi}\right)(\mathrm{h}+\mathrm{h})\right] - \mathrm{c}\left(\frac{\mathrm{m}\pi}{h+\xi}\right)\mathrm{Cos}\left[\left(\frac{\mathrm{m}\pi}{h+\xi}\right)(\mathrm{h}+\mathrm{h})\right]\right]$$

$$\times\int_0^{\mathrm{t}}\left[-[\mathrm{PI}]_{\mathrm{z}=\xi} - \mathrm{c}\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{\mathrm{z}=\xi}\right] e^{-k\left(a_m{}^2+a_n{}^2+\left(\frac{m\pi}{h+\xi}\right)^2\right)(t-t')}dt'$$

$$-\frac{2k\pi}{(h+\xi)^2}[16(k_1+k_2)(k_3+k_4)\ ]\quad\times\left[\frac{(a_m a)cos^2(a_m a) - \cos(a_m a)sin(a_m a)}{a_m{}^2}\right]\times$$

$$\left[\frac{(a_n b)cos^2(a_n b) - \cos(a_n b)sin(a_n b)}{a_n{}^2}\right]\sum_{m,n=1}^{\infty}\frac{1}{\left(1-c^2\left(\frac{m\pi}{h+\xi}\right)^2\right)}\frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}}\frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}}$$

$$\times\sum_{\mathrm{m}=1}^{\infty}(-1)^{m+1}\,\mathrm{m}\left[\mathrm{Sin}\left[\left(\frac{\mathrm{m}\pi}{h+\xi}\right)(\mathrm{h}-\xi)\right] - \mathrm{c}\left(\frac{\mathrm{m}\pi}{h+\xi}\right)\mathrm{Cos}\left[\left(\frac{\mathrm{m}\pi}{h+\xi}\right)(\mathrm{h}-\xi)\right]\right]\quad\times$$

$$\times\int_0^{\mathrm{t}}\left[\bar{\bar{f}} - [\mathrm{PI}]_{\mathrm{z}=-\mathrm{h}} - \mathrm{c}\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{\mathrm{z}=-\mathrm{h}}\right] e^{-k\left(a_m{}^2+a_n{}^2+\left(\frac{m\pi}{h+\xi}\right)^2\right)(t-t')}dt'$$

$$+[16(k_1+k_2)(k_3+k_4)\ ]\quad\times\left[\frac{(a_m a)cos^2(a_m a) - \cos(a_m a)sin(a_m a)}{a_m{}^2}\right]\left[\frac{(a_n b)cos^2(a_n b) - \cos(a_n b)sin(a_n b)}{a_n{}^2}\right]$$

$$+\sum_{m,n=1}^{\infty}\frac{\mathrm{P_m(x)}}{\lambda_\mathrm{m}}\frac{\mathrm{P_n(y)}}{\mu_\mathrm{n}}L^{-1}[PI]\int_0^{\mathrm{t}} e^{-k\left(a_m{}^2+a_n{}^2+\left(\frac{m\pi}{h+\xi}\right)^2\right)(t-t')}dt' \quad (7.4)$$

8. NUMERICAL RESULTS

$\beta_1 = [16(k_1+k_2)(k_3+k_4)\ ]\quad \beta = \left[\frac{2k\pi}{(h+\xi)^2}\right]$, c=1

For Aluminum metal

Modulus elasticity $E = 6.9\times 10^{11}$ Poisson ratio $\upsilon = 0.281$

Thermal Expansion coefficient $\alpha_t = 25.5\times 10^{-6}$ Thermal Diffusivity k=0.86

Thermal Conductivity $\propto = 0.48$ a=2 cm , b=1 cm h=1 ,$\xi = 0.5$

$$\frac{g(x,y,t)}{\beta_1} = \beta\left[\frac{(a_m 2)cos^2(a_m 2) - \cos(a_m 2)sin(a_m 2)}{a_m{}^2}\right]\left[\frac{(a_n)cos^2(a_n) - \cos(a_n)sin(a_n)}{a_n{}^2}\right]$$

$$\sum_{m,n=1}^{\infty}\frac{1}{\left(1-\left(\frac{m\,3.14}{1.5}\right)^2\right)}\frac{\mathrm{P_m}(x)}{\lambda_m}\frac{\mathrm{P_n}(y)}{\mu_n}\sum_{m=1}^{\infty}(-1)^{m+1}\,\mathrm{m}\left[\mathrm{Sin}\left[\left(\frac{m\,3.14}{1.5}\right)(2)\right]-\left(\frac{m\,3.14}{1.5}\right)\mathrm{Cos}\left[\left(\frac{m\,3.14}{1.5}\right)(2)\right]\right]\times$$

$$\int_0^t [\,\mathrm{const.}]e^{-0.86\left(a_m{}^2+a_n{}^2+\left(\frac{m\,3.14}{1.5}\right)^2\right)(t-t')}dt'$$

$$-\beta\left[\frac{(a_m 2)cos^2(a_m 2) - \cos(a_m 2)sin(a_m 2)}{a_m{}^2}\right]\left[\frac{(a_n)cos^2(a_n) - \cos(a_n)sin(a_n)}{a_n{}^2}\right]\sum_{m,n=1}^{\infty}\frac{1}{\left(1-c^2\left(\frac{m\pi}{h+\xi}\right)^2\right)}\frac{\mathrm{P_m}(x)}{\lambda_m}\frac{\mathrm{P_n}(y)}{\mu_n}$$

$$\times\sum_{m=1}^{\infty}(-1)^{m+1}\,\mathrm{m}\left[\mathrm{Sin}\left[\left(\frac{m\,3.14}{1.5}\right)(0.5)\right]-\mathrm{c}\left(\frac{m\pi}{h+\xi}\right)\mathrm{Cos}\left[\left(\frac{m\,3.14}{1.5}\right)(0.5)\right]\right]\times$$

$$\int_0^t \left[\left(1-e^{-t'}\right)(constant)\right]e^{-0.86\left(a_m{}^2+a_n{}^2+\left(\frac{m\,3.14}{1.5}\right)^2\right)(t-t')}dt'$$

$$+\left[\frac{(a_m 2)cos^2(a_m 2) - \cos(a_m 2)sin(a_m 2)}{a_m{}^2}\right]\left[\frac{(a_n)cos^2(a_n) - \cos(a_n)sin(a_n)}{a_n{}^2}\right]\sum_{m,n=1}^{\infty}\frac{\mathrm{P_m}(x)}{\lambda_m}\frac{\mathrm{P_n}(y)}{\mu_n}$$

$$\int_0^t \frac{0.48}{0.86\left(\frac{m\,3.14}{1.5}\right)^2}[const.]\;e^{-0.86\left(a_m{}^2+a_n{}^2+\left(\frac{m\,3.14}{1.5}\right)^2\right)(t-t')}dt' \qquad (8.1)$$

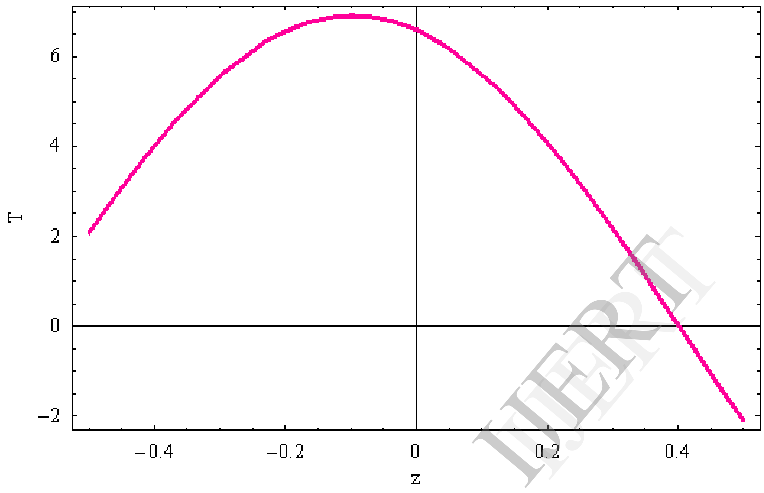

Fig.1 Temperature distribution along z

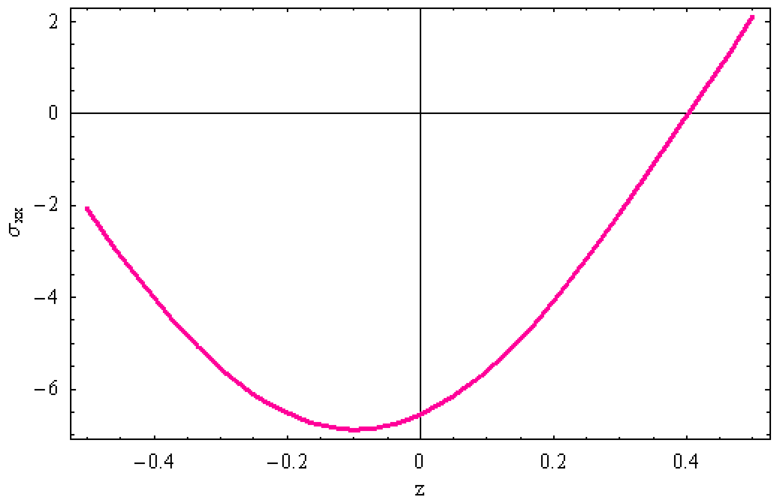

Fig.2 Thermal stresses σ_{xx} along z

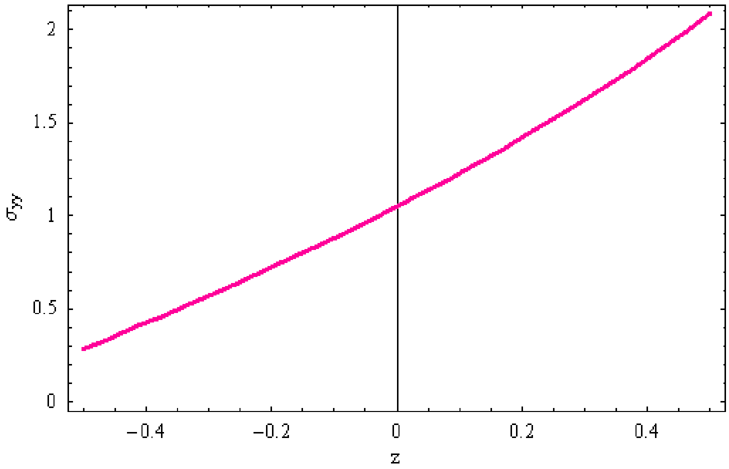

Fig.3 Thermal stresses σ_{yy} along z

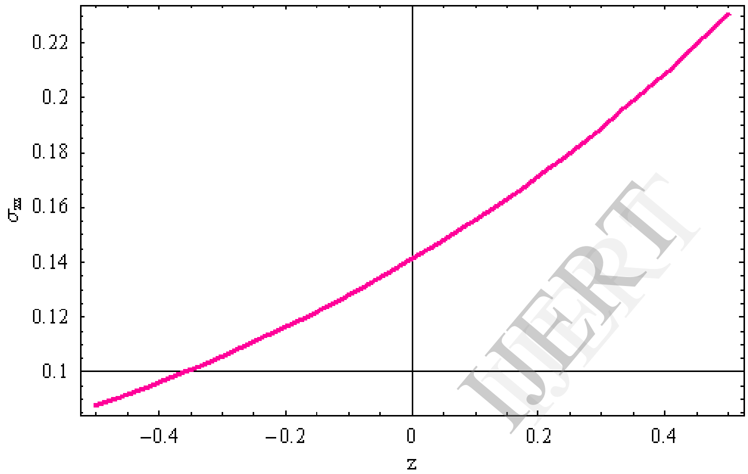

Fig.4 Thermal stresses σ_{zz} along z

9. CONCLUSION

In this Paper, We discussed completely the inverse unsteady-state thermoelastic problem of thin rectangular plate on the edge z=h, where the non homogeneous boundary condition of third kind is varies with position and time on edges x=-a, a, y=-b, b and z=-h with additionally heat sources at the edge z=h of the rectangular plate. The finite Marchi-Fasulo integral and Laplace transform is used to obtain the numerical results the temperature, Displacement and thermal stresses that are obtained can be applied to the design of useful structure or machines in engineering application. Any particular case of special interest can be derived by assigning suitable value of the parameters and function in the expression.

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