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Analylsis of Polynomial Windows for FIR Filters for Better Spectral Response

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Abstract

The analysis of time-domain functions is carried out.FIR filters are designed with windowing techniques which will improve spectral response of filter. In this paper new window functions are proposed for FIR filter design using conventional windows and polynomial windows which improve spectral response of filter than existing windowing techniques in terms of RSA(Relative side lobe attenuation).

Keywords: Polynomial window, Conventional window, RSA.

INTRODUCTION

In signal processing, a window function w(n) also known as an apodization function or tapering function is a mathematical function that is zero-valued outside of some chosen interval. When another function or waveform/data-sequence is multiplied by a window function, the product is also zero-valued outside the interval. Applications of window functions include spectral analysis, filter design, and beam forming.

$$w(n) = w(-n) \neq 0 \;\; for \;\; |n| \leq \frac{[N-1]}{2}$$

$$= 0 \;\; for \;\; |n| > \frac{[N-1]}{2}$$

Polynomial window functions in the time domain, allow desired order of continuity at the boundary of the observation window. Without any loss of generality, we assume the window interval to be [−1, 1]. Let us
assume that $f(t)$ represents the signal of interest and $f(t)$ is the windowed approximation of $f(t)$ over the windowed interval [−1, 1], i.e.,

$$f(t) = w(t)f(t)$$

Where $w(t)$ is the window function with a compact support over [−1, 1].

Section-I describes conventional window functions [3, 4] and in section-II polynomial window functions are discussed and Section-III describes proposed windows.

Section IV discusses the FIR filter and section discusses the spectral response of notch filter with conventional wind polynomial windows.

SECTION-I

Rectangular window:

The rectangular window (sometimes known as the **boxcar** or **Dirichlet window**) is the simplest window, equivalent to replacing all but N values of a data sequence by zeros, making it appear as though the waveform suddenly turns on and off:

$$w(n) = 1.$$

For $-\frac{[N-1]}{2}|n| \leq \frac{[N-1]}{2}$

Triangular window:

is defined by:

$$w(n) = 1 - \frac{2|n|}{[N-1]}$$

$$for\ -\frac{[N-1]}{2}|n| \leq \frac{[N-1]}{2}$$

The end samples are positive (equal to $2/(N + 1)$). This window can be seen as the convolution of two half-sized rectangular windows (for N even), giving it a main lobe width of twice the width of a regular rectangular window. The nearest lobe is −26 dB down from the main lobe.

Hamming window:

The window is optimized to minimize the maximum (nearest) side lobe, giving it a height of about one-fifth that of the Hann window.

$$w(n) = 0.54 + 046cos2\pi n/[N-1]\ for\ -\frac{[N-1]}{2}|n| \leq \frac{[N-1]}{2}$$

$$= 0 \quad otherwis$$

With

$$\alpha = 0.54,\ \beta = 1 - \alpha = 0.46,$$

Hanning window:

The Hann window is defined by:

$$w(n) = 0.5 + 0.5cos2\pi n/[N-1]\ for\ -\frac{[N-1]}{2}|n| \leq \frac{[N-1]}{2}$$

$$= 0 \quad otherwise$$

Blackman window:

defined as:

$$w(n) = 0.42 + \frac{0.5cos2\pi n}{[N-1]} + \frac{0.08cos4\pi n}{[N-1]}:$$

$$for\ -\frac{[N-1]}{2}|n| \leq \frac{[N-1]}{2}$$

$$= 0 \quad otherwise$$

Figure-1: FREQUENCY RESPONSE OF BARTLETT WINDOW:

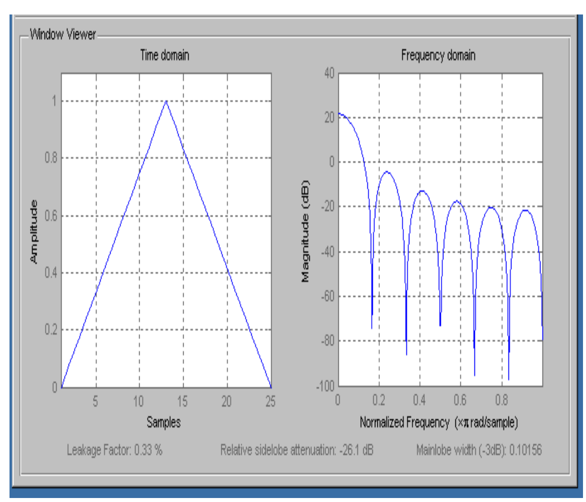

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Figure-2: FREQUENCY RESPONSE OF HAMMING WINDOW:

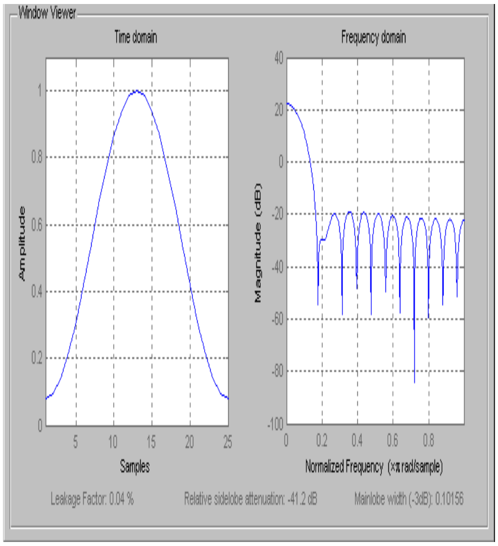

Figure-3: FREQUENCY RESPONSE OF HANNING WINDOW:

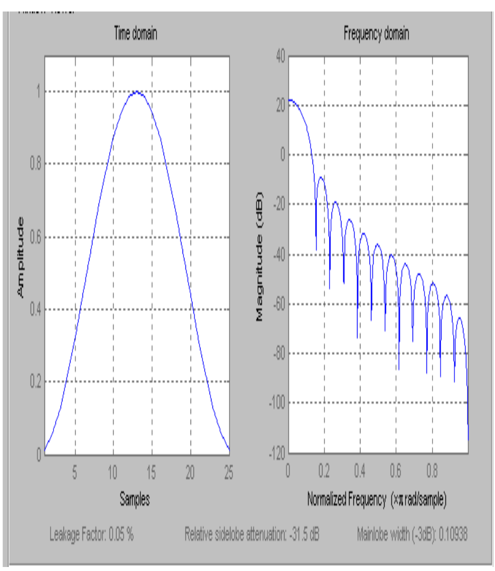

Figure-4: FREQUENCY RESPONSE OF BLACKMAN WINDOW:

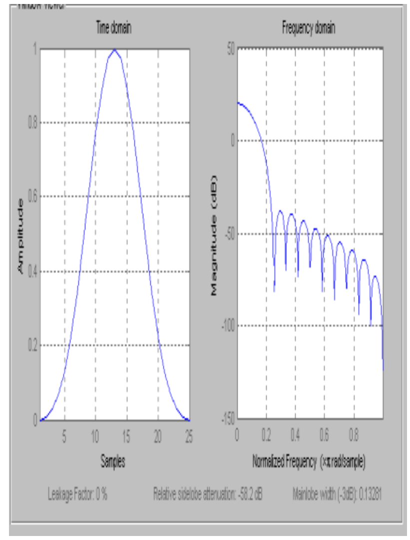

TABLE-1: COMPARASION RELATIVE SIDE LOBE ATTENUATION OF CONVENTIONAL WINDOWS:

Window function	Relativesidelobe attenuation in dB
Bartlett	-26.1
Hamming	-41.2
Hanning	-31.5
Blackmann	-58.2

From TABLE -1 it is observed that relative side lobe attenuation increases from Bartlett to Blackman for the value of N=25.

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SECTION-II

The time domain equation of polynomial [1] window is given as:

$$w_m(t)=1-K_m\sum_{n=0}^{m}A_{m,n}|t|^{2m-n+1}, \quad -1\leq t\leq 1.$$

Where K_m and $A_{m,n}$ are given by:

$$K_m=\frac{(2m+1)!(-1)^m}{(m!)^2} \quad A_{m,n}=\frac{(-1)^n\,{}^mC_n}{2m-n+1}.$$

The frequency domain equation of polynomial window is given as:

$$W_m(\omega)=2K_m\sum_{n=0}^{m}\mathcal{I}_{2m-n-1}A_{m,n}\frac{(2m-n+1)(2m-n)}{\omega^2}, \quad m>0.$$

Where

$$\mathcal{I}_{2m-n+1}=\frac{\sin\omega}{\omega}+\frac{(2m-n+1)\cos\omega}{\omega^2}-\frac{(2m-n+1)(2m-n)}{\omega^2}\mathcal{I}_{2m-n-1}, \quad 2m-n>0.$$

Where 'm' is called order of the polynomial window.

Figure-5: FREQUENCY RESPONSE OF POLYNOMIAL WINDOW FOR m=0:

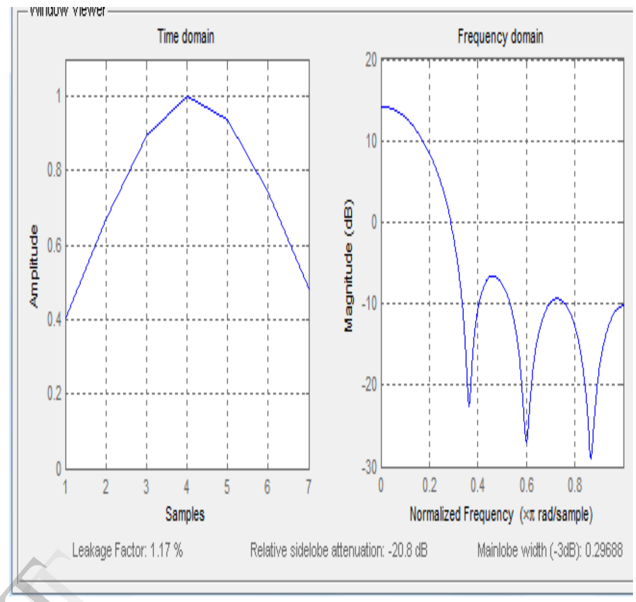


Figure-6: FREQUENCY RESPONSE OF POLYNOMIAL WINDOW FOR m=1:

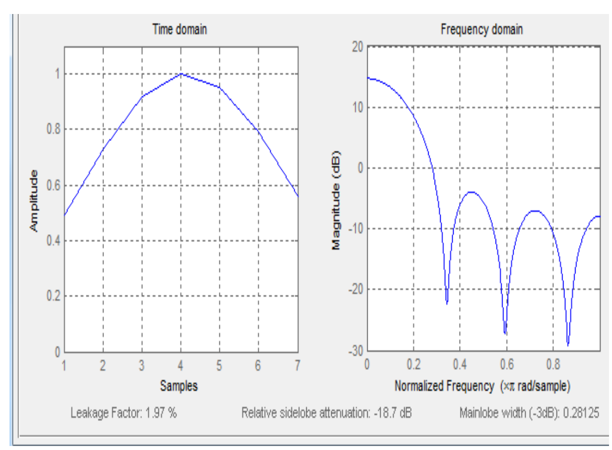


Figure-7: FREQUENCY RESPONSE OF POLYNOMIAL WINDOW FOR m=2:

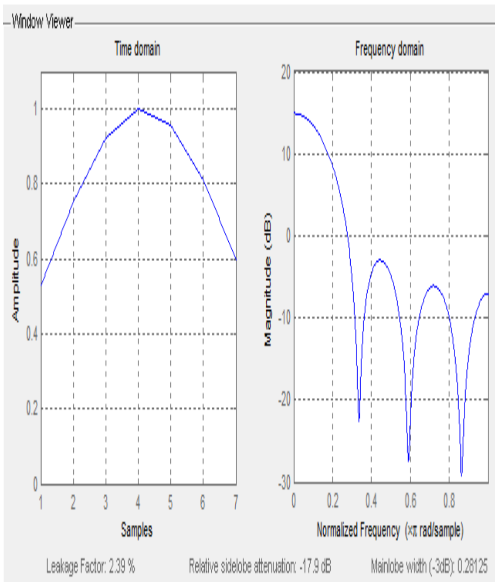


TABLE-2**: COMPARASION OF RELATIVE SIDE LOBE ATTENUATION OF POLYNOMIAL WINDOWS:**

Polynomial window	Relativesidelobe attenuation in dB
For m=0	-20.8
For m=1	-18.7
For m=2	-17.9

SECTION-III

Proposed windowing concept:

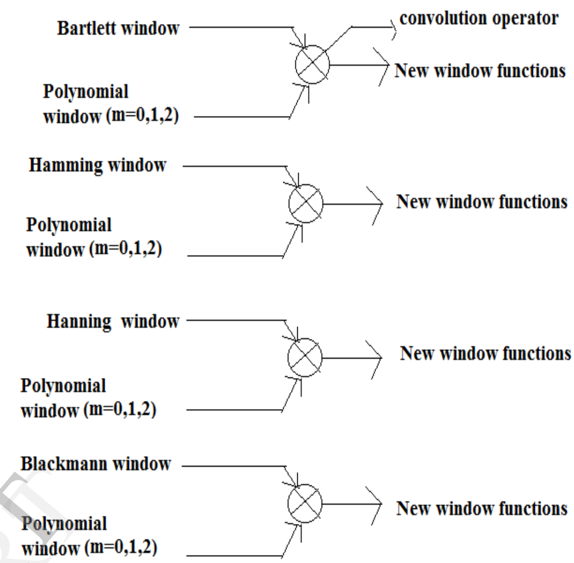


Figure-III: **convolution of conventional and polynomial windows.**

Figure-8: Response of new window1 (convolution of polynomial window for m=0 and Bartlett window):

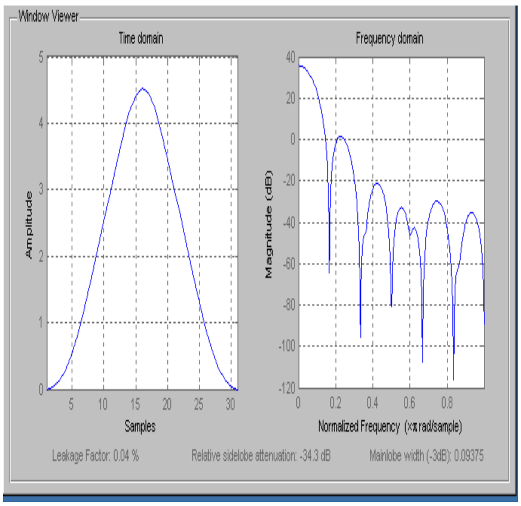


Figure-9: Response of new window2 (convolution of polynomial window for m=1 and Bartlett window):

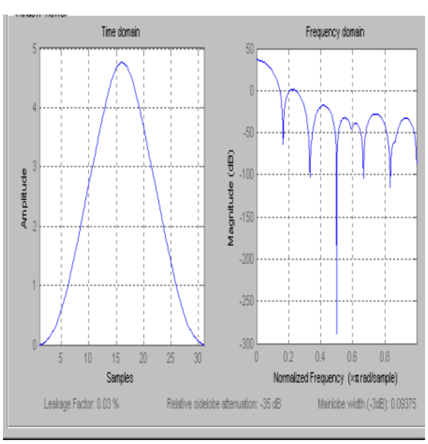


Figure-10: Response of new window3 (convolution of polynomial window for m=2 and Bartlett window):

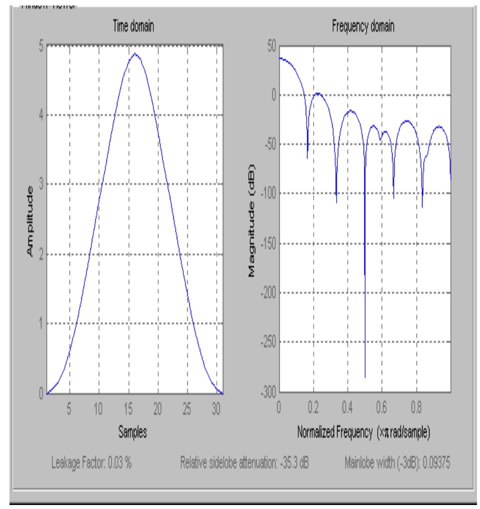
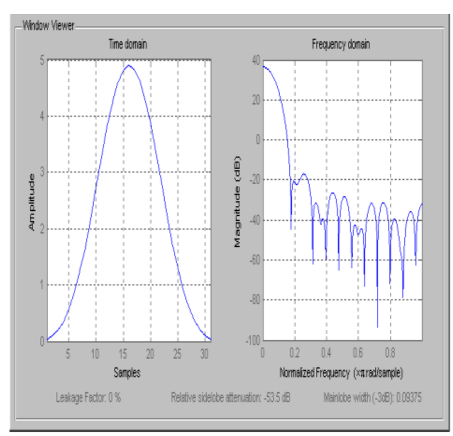


Figure-11: RESPONSE OF NEW WINDOW4 (CONVOLUTION OF POLYNOMIAL WINDOW FOR M=0 AND HAMMIING WINDOW):



igure-12: RESPONSE OF NEW WINDOW5 (CONVOLUTION OF POLYNOMIAL WINDOW FOR M=1 AND HAMMIING WINDOW):

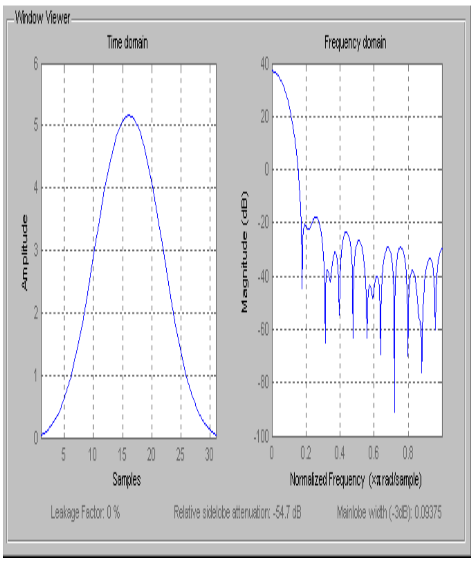


Figure-13: RESPONSE OF NEW WINDOW6 (CONVOLUTION OF POLYNOMIAL WINDOW FOR M=2 AND HAMMIING WINDOW):

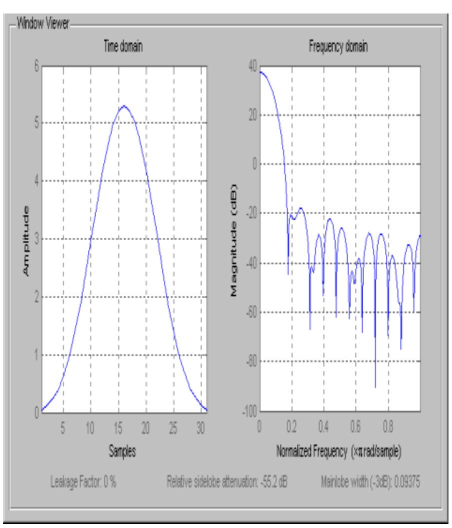


Figure-14: RESPONSE OF NEW WINDOW7 (CONVOLUTION OF POLYNOMIAL WINDOW FOR M=0 AND HANNING WINDOW):

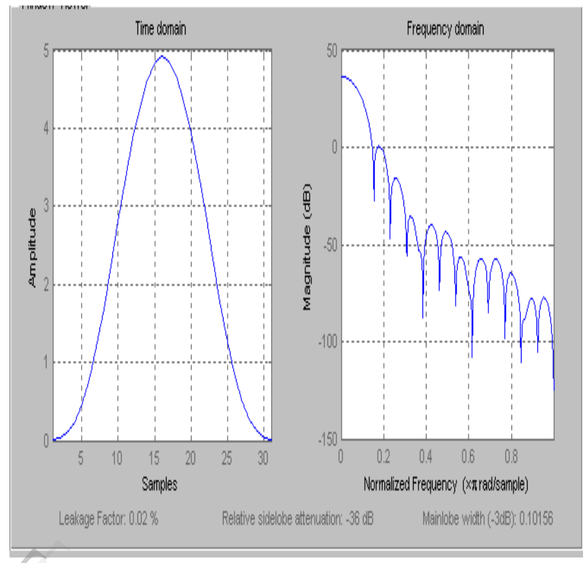


Figure-15: RESPONSE OF NEW WINDOW8 (CONVOLUTION OF POLYNOMIAL WINDOW FOR M=1 AND HANNING WINDOW):

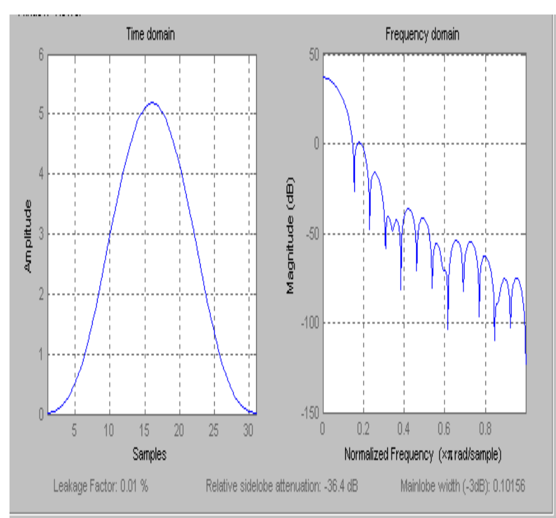


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Figure-16: **RESPONSE OF NEW WINDOW9 (CONVOLUTION OF POLYNOMIAL WINDOW FOR M=2 AND HANNING WINDOW):**

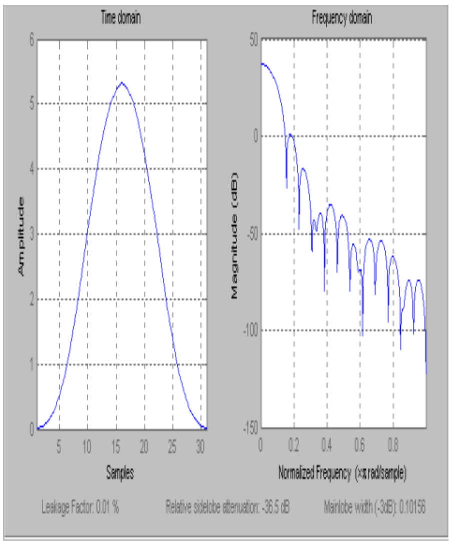

Figure-17: RESPONSE OF NEW WINDOW10 (CONVOLUTION OF POLYNOMIAL WINDOW FOR M=0 AND BLACKMANN WINDOW):

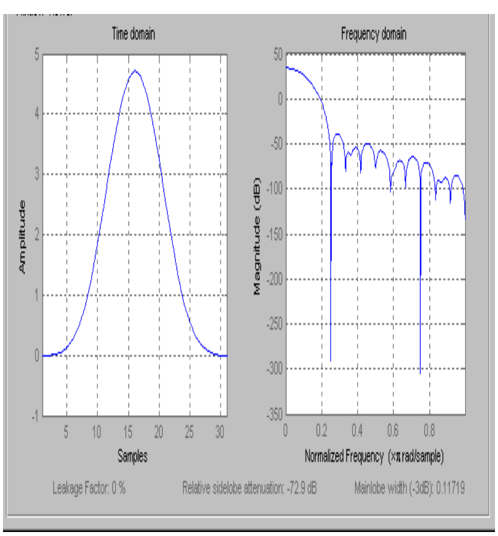

Figure-18: RESPONSE OF NEW WINDOW11 (CONVOLUTION OF POLYNOMIAL WINDOW FOR M=1 AND BLACKMANN WINDOW):

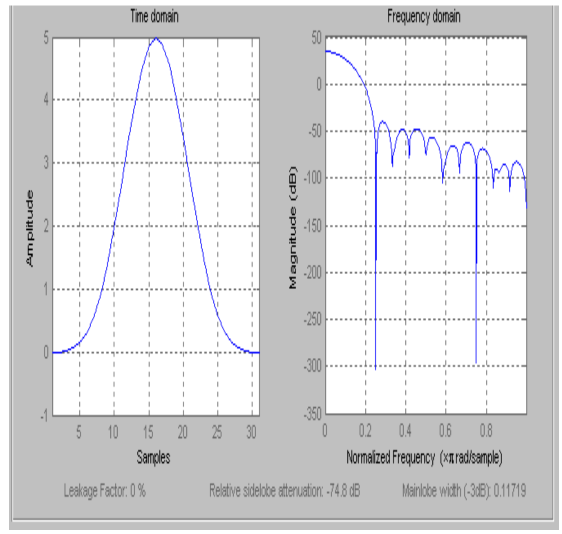

Figure-19: RESPONSE OF NEW WINDOW12 (CONVOLUTION OF POLYNOMIAL WINDOW FOR M=2 AND BLACKMANN WINDOW):

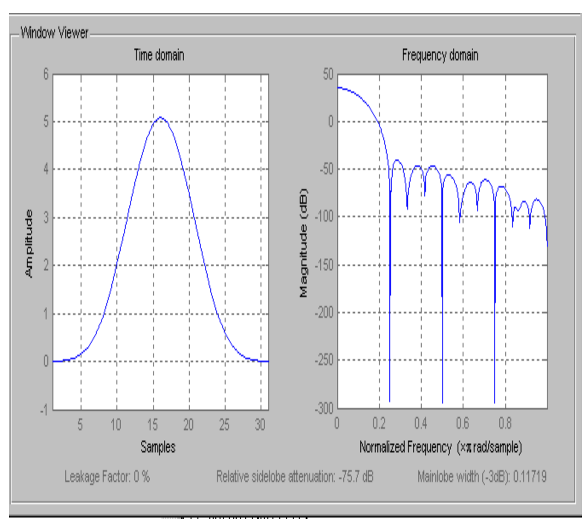

TABLE-3: COMPARASION OF RSA OF CONVENTIONAL AND NEW WINDWOS

CONVENTIONAL WINDOW	RSA IN dB	NEW WINDOW	FOR M=0 RSA dB	FOR M=1 RSA dB	FOR M=2 RSA dB
BARTLETT	-26.1	BARTLETT&POLYNOMIAL	-34.3	-35	-35.3
HAMMING	-41.2	HAMMIG&POLYNOMIAL	-53.5	-54.7	-55.2
HANNING	-31.5	HANNING&POLYNOMIAL	-36	-36.4	-36.5
BLACKMAN	-58.2	BLACKMAN&POLYNOMIAL	-72.9	-74.8	-75.7

SECTION-IV

FIR FITLER DESIGN [2]:

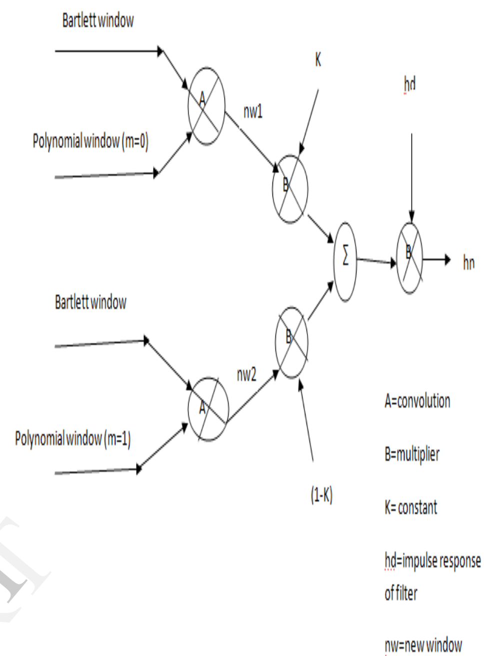

FIGURE-IV-proposed filter design flow

SECTION-V

Spectral responses of FIR notch filter with Bartlett window:

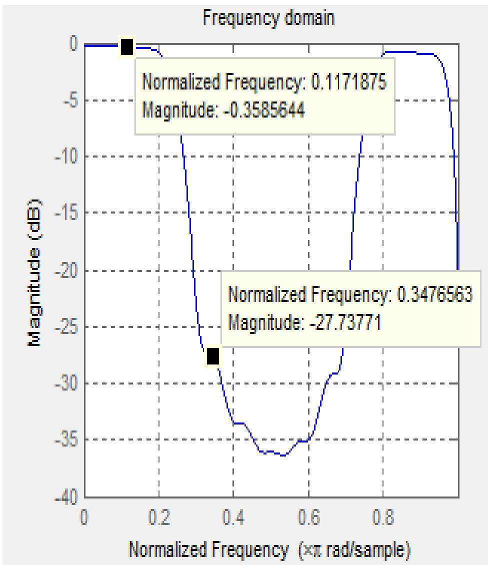

Figure-20 spectral response of notch filter

Spectral response of notch filter with poroposed polynomial windows for K=1.2:

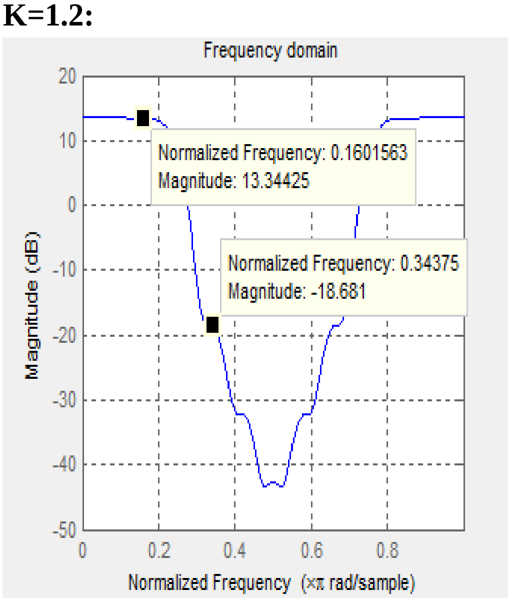

Figure-21 improved spectral response

TABLE-4 comparison of RSA of Bartlett window and with polynomial windows:

Window	RSA(dB)
Bartlett	28.08
K=0	32.71
K=1	31.97
K=1.2	32.02
K=2.5	31.64
K= -0.5	33.14

CONCLULSION

it is clear from the table-3 that for RSA of new window functions is improved by convolving conventional and polynomial windows and useful for better elimination of noise. It is clear from table-4 that spectral response of FIR band reject filter is improved which is observed in terms of RSA. In this paper spectral response of notch filter is analyzed and the proposed concept.

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Muralidhar, P.V. ; Aditya Inst. of Technol. & Manage., Tekkali, India ; Nataraj, D. ; Iokesh Raju, V. ; Naik, S.K.

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