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Analysis And Design Of R.C. Deep Beams By Finite Strip Method And Comparison Of Theoretical Results With Experimental Results

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Abstract

This paper describes analysis and design of deep beams subjected to two points loading with different L/D ratios using Programme in FORTRAN 77 for analysis and codes I.S.456-2000, B.S.8112, ACI 318 and Appendix A of ACI 318 for design purpose, to plot the variation of flexural stress, strains and shear stress in deep beam. The parameter, Shear span of beam was varied during the analysis. Several beams were cast and tested in laboratory.

Key words: *Deep Beam, Finite Strip method, codal provisions, Design.*

1.1. GENERAL INTRODUCTION

Beams with large depths in relation to spans are called deep beams[10]. In IS-456 (2000) Clause 29, a simply supported beam is classified as deep when the ratio of its effective span L to overall depth D is less than 2. Continuous beams are considered as deep when the ratio L/D is less than 2.5. The effective span is defined as the centre-to-centre distance between the supports or 1.15 times the clear span whichever is less[13].

1.2 OBJECTIVE OF STUDIES:

The main objective of this investigation is to conduct an experimental study on strength & behavior of deep beams. The detailed analysis has been carried out using the finite strip method. The study also aimed at testing validity & usefulness of IS 456:2000, B.S.8112, ACI 318-2005 and ACI Appendix A (STM), Draft Eurocode & CEB- FIP code and Canadian code.

The objectives of the experimental investigation can be listed as follows.

1. To observe & explain the deflection, cracking & failure modes of deep beams subjected to two points loading.
2. To compare the flexural steel requirement as per codal provisions with that calculated using the finite strip method.
3. To comment on suitability of finite strip method & codal provisions

2. ANALYSIS OF DEEP BEAM

2.1. FINITE STRIP METHOD

The finite strip approach was first introduced by CHEUNG (1968). For a structure with constant cross section and end boundary conditions that do not change transversely, stress analysis can be performed using finite strips. It is recognized as best method of analysis for simply supported rectangular plate, deep beam and box structure in terms of accuracy and efficiency. Basically, the method is a hybrid procedure which retains advantages of both the orthotropic plate Method and finite element concept.

2.2 THE COMPUTER PROGRAM

2.2.1 Introduction: A computer program has been prepared for the analysis of Deep Beam having simple support. A computer programme is necessary for the solution of Equations. It should be noted that the overall stiffness matrix is symmetrical. Computer programme is developed on the basis of direct stiffness method.

The essential steps in writing a programme are as follows.

1) Presenting input data to computer
2) Evaluation of stiffness matrix of individual strips.
3) Assembling of structure stiffness matrix
4) Forming the load vector
5) Solving the assembled equations for the displacements.
6) Computing the internal forces in the members and reactive forces at the support.
7) Presentation of the results.

Features of the Program

1. The programming language used is FORTRAN77.

2. The program can handle any number of joints and members depending upon memory allocations available with PC.
3. The program can handle yielding of the support in all three directions. Also it can handle symmetric structures in-plane, point load loads etc.

3. VARIATION OF FLEXURAL STRAIN:

The parametric study to know strain distribution in case of deep beam is performed here. It is found that the smaller the span/depth ratio (i.e., less than 2.0), the more pronounced the deviation of the strain pattern from that of Euler Bernoulli theory[12]. Figure 1 & Figure 2 shows that the flexural strain at mid span of simply supported deep beam for two different shear span –to-depth ratios. The beams have disturbed region in flexural strain distribution. Deep beams behave differently from shallow beams. In these members, the distribution of strain across the depth of the cross section is nonlinear and a significant amount of load is carried to the supports by a compression strut joining the load and the reaction. These structural elements belong to D (disturbed) regions. Structural members can be broadly divided into two regions, namely, B (or Bernoulli) regions where the strain distributions are linear, and D (or Disturbed) regions where the strain distributions are non–linear. While well defined theories are available for designing B regions, thumb rule or empirical equations are still being used to design D regions, though B and D regions are equally important. Schlaich et al. (1987) identified deep beams as discontinuity regions where the strain distribution is significantly nonlinear and specific strut-and-tie models need to be developed, whereas shallow beams are characterized by linear strain distribution and most of the applied load is transferred through a fairly uniform diagonal compression field.

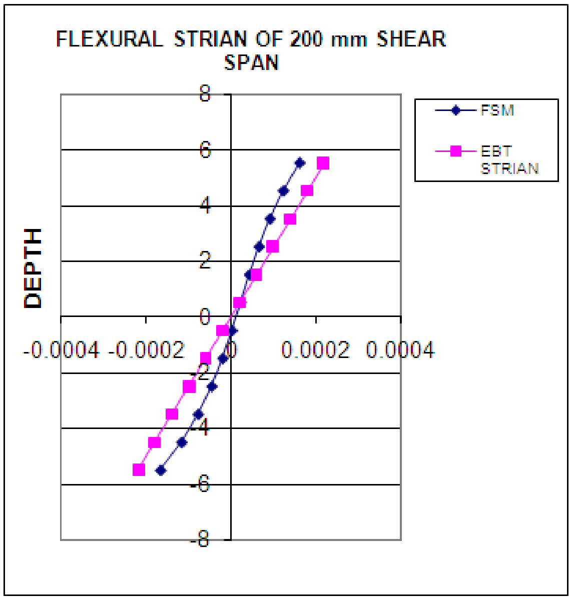

Figure 1 Flexural Strain Distribution shear span-to-depth ratio 0.57

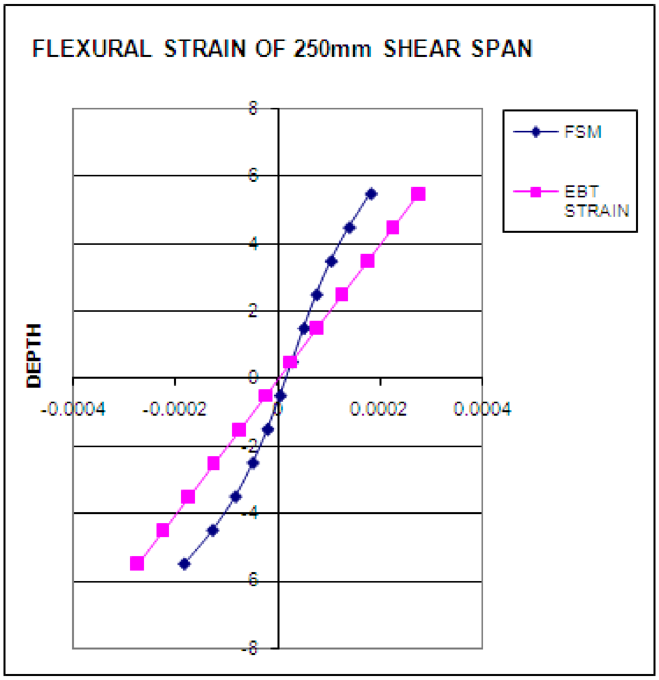

Figure 2 Flexural Strain Distribution shear span-to-depth ratio 0.71

From the variation of flexural strain graphs the definition of simply supported deep beam as per IS 456:2000 i.e. L/D ratio is less than or equal to 2.0 is reasonably accurate.

4 VARIATION OF FLEXURAL STRESS:

The stresses in isotropic homogeneous deep beams can be determined using finite strip analysis. It is found that the smaller the span/depth ratio (i.e., less than 2.0), the more pronounced the deviation of the stress pattern from that of Euler Bernoulli theory. Figure 3 & Figure 4 shows the flexural stress at mid span of simply supported deep beam for two different shear span –to-depth ratios. The tensile stresses increase rapidly at the bottom and neutral axis moves towards soffit of the beam[11].

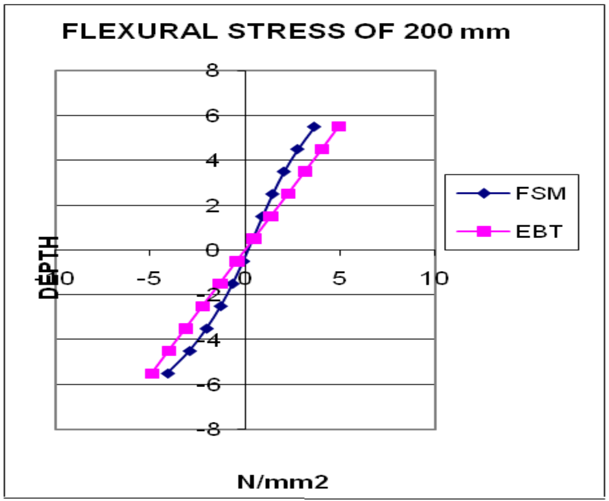

Figure 3 Flexural Stress Distribution shear span-to-depth ratio 0.57

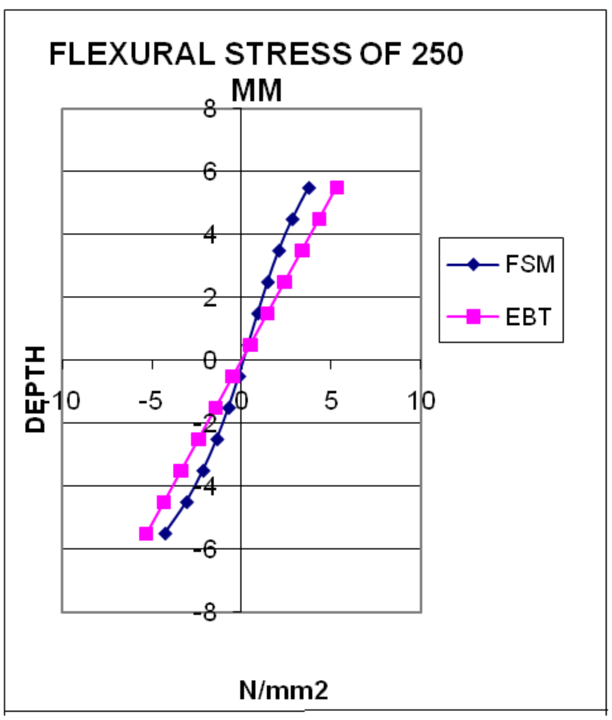

Figure 4 Flexural Stress Distribution shear span-to-depth ratio 0.71

From the variation of flexural stress graphs the definition of simply supported deep beam as per IS 456:2000 i.e. when L/D ratio is less than or equal to 2.0 is reasonably accurate.

5 VARIATION OF SHEAR STRESS:

Figure 5 & Figure 6 shows the shear stress near support of simply supported deep beam for two different shear span –to-depth ratios .The beams have drastic change in shear stress distribution. Deep beams behave differently from shallow beams. The shear stress patterns have also changed in case of deep beam. It is found that the smaller the span/depth ratio (i.e., less than2.0), the more pronounced the deviation of the shear stress distribution from that of Euler Bernoulli theory[12].

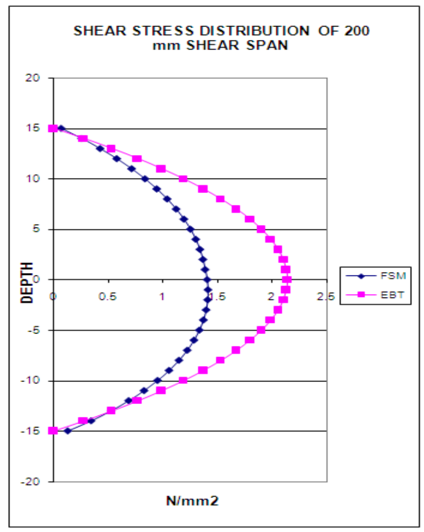

Figure 5 Shear Stress Distribution shear span-to-depth ratio 0.57

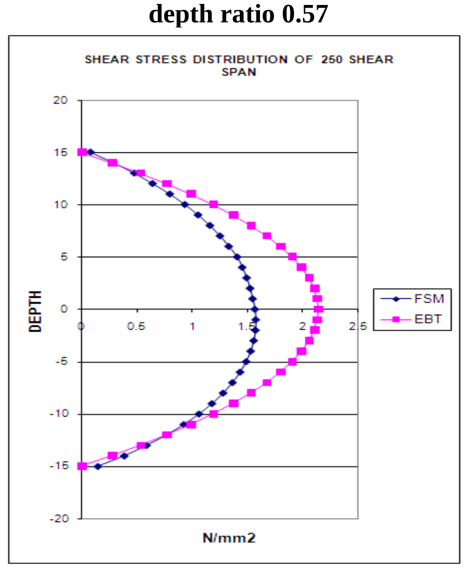

Figure 6 Shear Stress Distribution shear span-to-depth ratio 0.71

From the variation of shear stress graph it is clear that shear effect is predominant in beams having L/D ratio less than or equal to 2.0 which may lead to warping of the section.

6. TENSION REINFORCEMENT CALCULATIONS FROM GRAPH:

Tension Reinforcement required is calculated from the flexural stress graphs which are plotted by using Finite Strip program.

Reinforcement required for Shear span 200 mm:

Sample calculation of reinforcement for bottom most strip

Area of steel required =

$$\frac{\text{Flexural stress in strip x Area of Strip}}{\text{Design stress in steel}}$$

$$= \frac{\sigma_y \times A_{strip}}{0.87\, f_y}$$

$$= \frac{4.10687 \times 31.819 \times 150}{0.87 \times 415}$$

$$= 54.29\ mm^2$$

Similarly calculations for all the strips are done and tabulated.

Table 1: Reinforcement required as per FSM for Shear span 200 mm:

Sr.No.	Strip No.	Reinforcement required mm^2
1.	1	0.86
2.	2	8.99
3.	3	17.3
4.	4	27.183
5.	5	38.973
6.	6	54.293
Total		**147.6 mm^2**

Reinforcement required for Shear span 250 mm:

Sample calculation of reinforcement for bottom most strip

Area of steel required =

$$\frac{\text{Flexural stress in strip x Area of strip}}{\text{Design stress in steel}}$$

$$= \frac{\sigma y \times A_{strip}}{0.87\, f_y}$$

$$= \frac{4.58062 \times 31.819 \times 150}{0.87 \times 415}$$

$$= 60.55\ mm^2$$

Similarly calculations for all the strips are done and tabulated.

Table 2: Reinforcement required as per FSM for Shear span 250 mm:

Sr.No.	Strip No.	Reinforcement required mm^2
1.	1	0.96
2.	2	10.032
3.	3	19.296
4.	4	30.32
5.	5	43.805
6.	6	60.55
	Total	**164.97 mm^2**

7. DESIGN OF DEEP BEAMS[3]

7.1 INTRODUCTION:

Deep beams are designed for two points loading and for two shear spans viz. 200 mm and 250 mm. Point loads of 50 kN are applied on deep beams. Dimensions of deep beams chosen for design purpose are,

Length = 700 mm,
Depth = 350 mm,
Thickness = 150 mm

7.2 DESIGN METHODS

Design of deep beams is done by following methods.

1. Design by using I.S.456-2000 method
2. Design by using B.S.8112 method
3. Design by using ACI-318 method
4. Design by using ACI-Appendix A (Strut & Tie) method

For each method mentioned above, several beams with 200 mm and 250 mm shear spans are designed and cast for experimental study.

8. EXPERIMENTAL WORK

8.1 INTRODUCTION:

Deep beams are designed by using I.S.456-2000, B.S.8112, ACI-318 and ACI-Appendix A (strut & Tie method) for two points loading and for several shear spans. Dimensions of Deep beams chosen for design purpose are,

Length = 700 mm,
Depth = 350 mm,
Thickness = 150 mm

9. Testing in Laboratory and Test Results

Image 7 Deep beam testing

Image 9 Strut formation in deep beams

Mode of failure was found to be shear with diagonal tension & can be categorized as given in table.

Image 8 Diagonal cracking in deep beam

TABLE 3: SAMPLE TEST RESULTS

Loading: Two point loading, each point load of 50 kN (working load)

Beam dimensions: Total Length = 700 mm, Effective Span = 600 mm, Depth = 350 mm, Thickness = 150 mm, Average cube strength = 21 N/mm^2

Beam No.	**B 1/1**	**B 1/2**	**B 2/1**	**B 2/2**	**B 3/1**	**B3/2**	**B 4/1**	**B 4/2**
Design Method	**I.S.456**	**I.S.456**	**B.S.8112**	**B.S.8112**	**ACI 318**	**ACI 318**	**Strut & Tie**	**Strut & Tie**
Shear span (mm)	200	250	200	250	200	250	200	250
Shear span to depth ratio	0.57	0.71	0.57	0.71	0.57	0.71	0.57	0.71

Reinforcement provided (No.of bars)	Flexure steel Required in mm^2	160.74	199.845	160.74	199.85	231.33	231.33	169.52	215.89
	Flexure steel Provided i) 10 mm Φ ii) 08 mm Φ iii) mm^2	2 1 207.24	2 1 207.24	2 1 207.24	2 1 207.24	2 1 235.62	2 1 235.62	- 4 200.96	3 - 235.62
	Shear Required in mm^2 Vertical Horizontal	126 105	126 105	113.04 84.78	113.04 84.78	282.6 113.04	282.6 113.04	262.5 72	262.5 72
	6 mm dia. Vertical Horizontal	6 2	6 2	4 3	4 3	9 4	9 4	5 3	5 3
Load at first crack	Total	200kN	190kN	180kN	170kN	220kN	210kN	210kN	200kN
	Each Point load	100kN	95kN	90kN	85kN	110kN	105kN	105kN	100kN
Failure Load	Total	300kN	280kN	285kN	275kN	340kN	334kN	330kN	310kN
	Each Point load	150kN	140kN	142.5kN	137.5Kn	170kN	167kN	165kN	155kN
Deflection at failure	Total	3.4 mm	3.8 mm	3.5 mm	4 mm	3.6 mm	3.75 mm	3.5 mm	3.7 mm
	Permissible deflection	2.4 mm	2.4 mm	2.4 mm	2.4 mm	2.4 mm	2.4 mm	2.4 mm	2.4 mm
	Deflection at 150 kN load	1.03 mm	1.37 mm	1.24 mm	0.9 mm	1.10 mm	1.26 mm	1.33 mm	1.52 mm
Observed mode of failure		Mode II- 3	Mode II- 3	Mode II- 3	Mode II- 3	Mode II- 3	Mode II- 3	Mode II- 3	Mode II- 3

Referring to table nos. 1, 2 and 3, it is found that flexural steel reinforcement as per FINITE STRIP METHOD is less than that specified by codes.

Description of modes of failure as described by Salamy et al[8]:

Failure modes of deep beam can be divided in following two main categories.

a. Flexural failure mode

b. Shear failure mode

Shear failure mode can be sub divided into following three categories.

Mode II-1: Diagonal tension failure, which in the line of thrust become so eccentric and give rise to flexural failure in compressive zone. It is important however to mention that this kind of failure is a result of tensile crack extension in compressive zone due to flexural load[8].

Mode II-2: Shear compression failure where RC beam fails due to the development of diagonal crack into the compressive zone and reduces the area of resisting region excessively and beam crushes once generated compressive stress exceeds compressive strength of concrete[8].

Mode II-3: Shear proper or compressive failure of struts, which is often observed in beams with very small shear span to depth ratio (a/d < 1.5).In this case due to the small a/d ratio, the line of thrust will be so steep and arch action not

only reserve flexural capacity in most cases but also efficiently sustains required shear force. Arch is clearly observed in those beams and finally beams fail due to either sudden tensile crack formation parallel to the strut axes or compressive crush in normal direction to the strut axes[8].

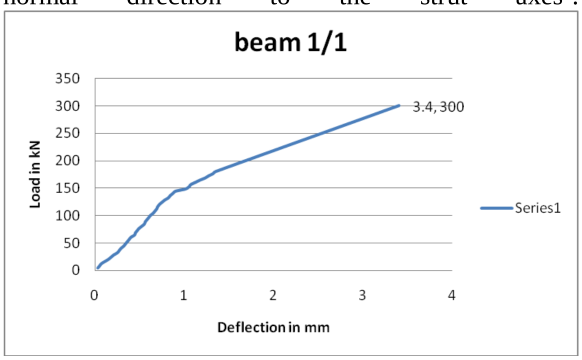

Fig. 10 GRAPHS OF LOAD VS DEFLECTION

10. CONCLUSION

Following conclusions can be drawn from above studies.

1. Failure of deep beams was mainly due to diagonal cracking and it was along the lines joining the loading points and supports.
2. The strength of beams with 250 mm shear span is about 5 % less than that of 200 mm shear span. It is clear from these results that the strength of deep beam is inversely proportional to the shear span for the constant depth of the beam.
3. No separate checking for shear is specified in I.S.456. It is assumed that the arching action of the main tension steel & the web steel together with concrete will carry the shear.
4. All the beams had low deflection at failure as there was no flexural failure.
5. The overall average load at first crack was found approximately half of the ultimate failure load. Therefore in design of deep beams, a load factor of 1.5 seems to be reasonable. As reported by F.K.Kong the shear strength of deep beams is 2 to 3 times greater than that given by usual equations which is true as in our case the strength is about 2 times greater than design loads.
6. Strut-and-tie model is a good approach to design. It is a simple approach but provisions against web cracking are not clearly given in this method. Though it is a conservative method, the area of steel calculated using the STM is nearly equal to that required as per IS456:2000, BS8112-2006 and ACI318-05 codal recommendations.
7. The flexural steel requirement of IS456:2000 & BS 8112-2006 methods are more by a margin of 8.17 % than Finite Strip Method whereas in case of STM method this margin is found to be 12.93 %. But in case of ACI-318 method it is 36.19 % more than FSM method. Therefore it can be concluded that tensile reinforcement requirements of I.S., B.S. & STM methods are near to the FSM whereas the same by ACI -318 methods is more. Therefore the strength of beams designed by ACI -318 method is about 10 % more than other beams.
8. Web steel requirement of ACI-318 method is more than other methods due to specification of minimum spacing of d/5. Due to more web steel, initial cracking load of the beams designed by ACI-318 method is about 7 % more than that of the beams designed by other methods. Even if web reinforcement does not contribute substantially to the strength of deep beams, it prevents initial cracking of loads at low loads.
9. B.S. 8112 takes into account the contribution of horizontal web steel also along with vertical steel. This reduces the vertical steel requirement. The design was found conservative.
10. The flexural tensile force as per the FSM analysis is concentrated in lower 1/3 height for all the beams. Therefore in the deep beams loaded with two point loading, steel for the flexural tensile force may be provided mainly in this height. This is matching with all the codal provisions.

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