Analysis clonal algorithm based window Using Fractional Fourier Transform

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Abstract

An Approximated Exponential Fractional Fourier Transforms(FrFT) Mathematical derivation for clonal algorithm based window is proposed. By control the parameter of FrFT, it is possible to control the Spectral parameters of above windows like Half Bandwidth (**HBW**), Maximum Side Lobe Attenuation(MSLA) and Side Lobe Fall **O**f Ratio(SLFOR). This proposed derivation is also holds good for generalization of FrFT with Fourier Transform(FT).

Index Terms—Fractional Fourier transform, clonal algorithm based window.

NOMENCLATURE

FT: Fourier Transform

FrFT: Fractional Fourier Transform

1.INTRODUCTION

In order to reduce the effects of spectral leakages in Harmonic analysis, windows are used [1]. window functions successfully used in the areas like interpolation factors to design Anti-Imaging filters, speech processing systems, digital filter design and beam forming [2]-[3].windows are also useful to solve reconstructive errors which are objective functions to design the prototype filters [4].windows are essentially Applicable in spectral analysis of signals[5]-[6]. According to [3], as the parameter of FrFTi.e $\alpha = \frac{\pi}{2}$ which could not holds good for generalization of FrFT to FT [7].In this proposed Derivation of FrFT, An attempt is made to study the variations of window parameters like HBW,MSLA and MSLFOR by different values of fluid parameter of FrFT to FT at α = $\frac{\pi}{2}$ This paper is organized as follows. ::section-II gives an overview of FT, and mathematical model of windows by using FT. section-III gives an overview of FrFT, and mathematical model of windows by using FRFT. Later conclusive remarks are discussed in section-IV.

2.Fractional Fourier Transform

Fractional Fourier Transform widely used in quantum mechanics and quantum optics [13].Fractional Fourier Analysis can obtain the mixed time and frequency components of signals[14].it finds various applications like pattern recognisition with some spatial distortion, Image representation , compression and noise removal in signal processing [15]-[17].FrFT used for Interpretation of sinusoidal signals and design of Digital FIR Filters[18]-[19].

The continuous –time Fractional Fourier Transform of a signal

 $\omega(t)$ is defined through an interval [3]

$$\omega_{\alpha}(u) = \int_{-\infty}^{\infty} \omega(t) K_{\alpha}(t, u) dt - - - (1)$$

Where the transform kernel $K_{\alpha}(t, u)$ of the FRFT is Given by

$$K_{\alpha}(t, u) = \sqrt{\frac{1 - jcot(\alpha)}{2\pi}} exp\left[i\left(\frac{t^{2} + u^{2}}{2}\right)cot(\alpha) - iutcosec(\alpha)\right] \text{ if } \alpha \text{ is multiple of } \pi = \partial(t - u) \text{ if } \alpha \text{ is multiple of } 2\pi$$

$$= \partial(t+u)$$
 if $\alpha + \pi$ is a mulpiple of $2\pi - -(2)$

Where α indicates rotation of angle of the Transformed signal for FrFT.

2.1) window function based on Clonal Algorithm

The Expression for clonal Algorithm based window is [16]

$$\int_{t_1}^{t_2} \omega(t) \sqrt{\frac{1-j\cot(\alpha)}{2\pi}} exp\left[i\left(\frac{t^2+u^2}{2}\right)\cot(\alpha) - \frac{1-j\cot(\alpha)}{2\pi}e^{\frac{ju^2\cot(\alpha)}{2}} - ---(2)\right]$$

$$p = \sqrt{\frac{1-j\cot(\alpha)}{2\pi}}e^{\frac{ju^2\cot(\alpha)}{2}} - ---(3)$$

Then equation-(21) becomes

$$\omega_{\alpha}(u) = p \int_{t_1}^{t_2} \omega(t) \exp\left(i\frac{t^2}{2}\cot(\alpha) - iutcosec(\alpha)\right) dt - - - - (4)$$

Substitute equation-(20) in equation-(23) then

$$\omega_{\alpha}(u) = p \int_{t_{1}}^{t_{2}} (0.5154 - 0.4711 \cos(2\pi t)) + 0.0135 \cos(4\pi t)) \exp\left(i\frac{t^{2}}{2}\cot(\alpha) - iutcosec(\alpha)\right) dt - ---(5)$$

Equation-(5) divided into four parts like I_3, I_4, I_5, I_6 where

According to that

$$I_{3}=0.5154\frac{\left(\frac{e^{-iut_{2}cosec(\alpha)}}{-iucosec(\alpha)}-\frac{e^{-iut_{1}cosec(\alpha)}}{-iucosec(\alpha)}\right)}{(t_{2}-t_{1})+\frac{icot(\alpha)}{6}(t_{2}^{3}-t_{1}^{3})} \quad \dots \dots (9)$$

Now multiply both sides with $e^{-i\frac{t^2}{2}\cot(\alpha)}$ which results to

$$e^{-i\frac{t^{2}}{2}\cot(\alpha)}I_{4} = e^{-i\frac{t^{2}}{2}\cot(\alpha)}(0.23555(\left(\int_{t_{1}}^{t_{2}}\exp(i2\pi t)\exp\left(i\frac{t^{2}}{2}\cot(\alpha)-i4t\right)\exp\left(i\frac{t^$$

Integration above equation and applying limits, you will get

$$e^{-i\frac{t^2}{2}\cot(\alpha)}I_4 = 0.23555\left(\frac{\exp(i2\pi t_2 - iut_2cosec(\alpha))}{i2\pi - iucosec(\alpha)} - \frac{\exp(-i2\pi t_1 - iut_1cosec(\alpha))}{-i2\pi - iucosec(\alpha)}\right) - - -(16)$$

Now Integrate both sides with limits t_1 and t_2

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$$\begin{split} &\int_{t_1}^{t_2} \left(e^{-i\frac{t^2}{2}\cot(\alpha)} I_4 \right) dt \\ &= \int_{t_1}^{t_2} \left(0.23555 \left(\frac{\exp(i2\pi t_2 - iut_2 cosec(\alpha))}{i2\pi - iucosec(\alpha)} \right) \\ &- \frac{\exp(-i2\pi t_1 - iut_1 cosec(\alpha))}{-i2\pi - iucosec(\alpha)} \right) \right) dt - - - (17) \end{split}$$

The above equation can be written as

$$\frac{I_{4}}{\int_{t_{1}}^{t_{2}} \left(0.23555\left(\frac{\exp(i2\pi t_{2}-ut_{2}cosec(\alpha))}{i2\pi-iucosec(\alpha)}-\frac{\exp(-i2\pi t_{1}-iut_{1}cosec(\alpha))}{-i2\pi-iucosec(\alpha)}\right)\right)dt}{\int_{t_{1}}^{t_{2}} \left(e^{-i\frac{t^{2}}{2}\cot(\alpha)}\right)dt}$$
(18)

Now equation-(18) can be divided into two parts x_1 and x_2

Where

$$\begin{aligned} x_1 \\ &= \int_{t_1}^{t_2} \left(0.23555 \left(\frac{\exp(i2\pi t_2 - iut_2 cosec(\alpha))}{i2\pi - iucosec(\alpha)} \right) \\ &- \frac{\exp(-i2\pi t_1 - iut_1 cosec(\alpha))}{-i2\pi - iucosec(\alpha)} \right) \right) dt \end{aligned}$$

And

$$x_2 = \int_{t_1}^{t_2} \left(e^{-i\frac{t^2}{2}\cot(\alpha)} \right) dt$$

Now solving for x_1 Integrating x_1 and applying lilits $x_1 = 0.23555 \left(\frac{\exp(i2\pi t_2 - iut_2 cosec(\alpha))}{i2\pi - iucosec(\alpha)} - \frac{\exp(i2\pi t_2 - iut_2 cosec(\alpha))}{i2\pi - iucosec(\alpha)} \right)$ $\frac{\exp(-i2\pi t_1 - iut_1 cosec(\alpha))}{(1 - t_1)} (t_2 - t_1)$ $i2\pi$ - $iucosec(\alpha)$ --- (19) $-i2\pi$ $-iucosec(\alpha)$

Now solving for x_2

$$x_2 = \int_{t_1}^{t_2} \left(e^{-i\frac{t^2}{2}\cot(\alpha)} \right) dt - - - (20)$$

According to [17]

$$e^{-i\frac{t^2}{2}\cot(\alpha)} = 1 - i\frac{t^2}{2}\cot(\alpha) - - - (21)$$

Substitute equation-(21) in equation-(20) to get

$$x_{2} = \int_{t_{1}}^{t_{2}} \left(1 - i \, \frac{t^{2}}{2} \cot(\alpha) \right) dt \qquad --- (22)$$

Integrating and applying limits to get

$$x_2 = \int_{t_1}^{t_2} dt - \frac{icot(\alpha)}{2} \int_{t_1}^{t_2} t^2 dt - - - (23)$$

$$x_2 = (t_2 - t_1) - \frac{i \cot(\alpha)}{6} (t_2^3 - t_1^3) \quad \dots \quad (24)$$

Finally I4

$$\frac{I_{4} = \frac{0.23555\left(\frac{\exp(i2\pi t_{2} - iut_{2}cosec(\alpha))}{i2\pi - iucosec(\alpha)} - \frac{\exp(-i2\pi t_{1} - iut_{1}cosec(\alpha))}{-i2\pi - iucosec(\alpha)}\right)(t_{2} - t_{1})}{(t_{2} - t_{1}) - \frac{icot(\alpha)}{6}(t_{2}^{3} - t_{1}^{3})} - - - (25)$$

Now solving for I5

According to [17]

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$$\cos(4\pi t) = \frac{\exp(i4\pi t) + \exp(-i4\pi t)}{2} - - - (27)$$

Substitute equation-(27) in equation-(26) then

$$I_{5} = \int_{t_{1}}^{t_{2}} ((0.4711)) \left(\frac{\exp(i4\pi t) + \exp(-i4\pi t)}{2}\right) \exp\left(i\frac{t^{2}}{2}\cot(\alpha) - \frac{1}{2}iutcosec(\alpha)\right) dt - \frac{1}{2}iutcosec(\alpha) dt - \frac{1}{2}iutcosec(\alpha) dt + \frac{1}{2}i$$

Now multiply both sides with $e^{-i\frac{t^2}{2}\cot(\alpha)}$ which results to

$$e^{-i\frac{t^2}{2}\cot(\alpha)}I_5 =$$

$$e^{-i\frac{t^2}{2}\cot(\alpha)}(0.23555(\left(\int_{t_1}^{t_2}\exp(i4\pi t)\exp\left(i\frac{t^2}{2}\cot(\alpha)-i4\pi t\right)\exp\left(i\frac{t^2}{2}\cot(\alpha)-i4\pi t\right)\exp\left(i\frac{t^2}{2}\cot(\alpha$$

$$e^{-i\frac{t^2}{2}\cot(\alpha)}I_5 = 0.23555(\left(\int_{t_1}^{t_2}(\exp(i4\pi t - iutcosec(\alpha)))dt) + (\int_{t_1}^{t_2}(\exp(-i4\pi t - iutcosec(\alpha)))dt)\right) - - - (31)$$

Integration above equation and applying limits, you

will get

$$e^{-i\frac{t^2}{2}\cot(\alpha)}I_5$$

= 0.23555 $\left(\frac{\exp(i4\pi t_2 - iut_2cosec(\alpha))}{i4\pi - iucosec(\alpha)} - \frac{\exp(-i4\pi t_1 - iut_1cosec(\alpha))}{-i4\pi - iucosec(\alpha)}\right)$ (32)

Now Integrate both sides with limits t_1 and t_2

$$\begin{split} &\int_{t_1}^{t_2} \left(e^{-i\frac{t^2}{2}\cot(\alpha)} I_5 \right) dt \\ &= \int_{t_1}^{t_2} \left(0.23555 \left(\frac{\exp(i4\pi t_2 - iut_2cosec(\alpha))}{i4\pi - iucosec(\alpha)} \right) \\ &- \frac{\exp(-i4\pi t_1 - iut_1cosec(\alpha))}{-i4\pi - iucosec(\alpha)} \right) \right) dt - - (33) \end{split}$$

The above equation can be written as

$$\frac{I_{5}=}{\int_{t_{1}}^{t_{2}} \left(0.23555\left(\frac{\exp(i4\pi t_{2}-ut_{2}cosec(\alpha))}{i4\pi-iucosec(\alpha)}-\frac{\exp(-i4\pi t_{1}-iut_{1}cosec(\alpha))}{-i4\pi-iucosec(\alpha)}\right)\right)dt}{\int_{t_{1}}^{t_{2}} \left(e^{-i\frac{t^{2}}{2}cot(\alpha)}\right)dt}$$
(34)

Now equation-(34) can be divided into two parts $x_1 \mbox{ and } x_2$

Where

$$x_{1} = \int_{t_{1}}^{t_{2}} \left(0.23555 \left(\frac{\exp(i4\pi t_{2} - iut_{2}cosec(\alpha))}{i4\pi - iucosec(\alpha)} - \frac{\exp(-i4\pi t_{1} - iut_{1}cosec(\alpha))}{-i4\pi - iucosec(\alpha)} \right) \right) dt$$

And

$$x_{2} = \int_{t_{1}}^{t_{2}} \left(e^{-i\frac{t^{2}}{2}\cot(\alpha)} \right) dt$$

Now solving for x_1

Integrating x_1 and applying lilits

$$\begin{aligned} x_1 &= 0.23555 \left(\frac{\exp(i4\pi t_2 - iut_2 cosec(\alpha))}{i4\pi - iucosec(\alpha)} - \frac{\exp(-i4\pi t_1 - iut_1 cosec(\alpha))}{-i4\pi - iucosec(\alpha)} \right) (t_2 - t_1) \quad --- \quad (35) \\ \text{Now solving for } x_2 \end{aligned}$$

$$x_{2} = \int_{t_{1}}^{t_{2}} \left(e^{-i\frac{t^{2}}{2}\cot(\alpha)} \right) dt - - - (36)$$

According to [17]

$$e^{-i\frac{t^2}{2}\cot(\alpha)} = 1 - i\frac{t^2}{2}\cot(\alpha) - - - (37)$$

Substitute equation-(37) in equation-(20) to get

$$x_2 = \int_{t_1}^{t_2} \left(1 - i \frac{t^2}{2} \cot(\alpha) \right) dt - (36)$$

Integrating and applying limits to get

$$x_2 = \int_{t_1}^{t_2} dt - \frac{i \cot(\alpha)}{2} \int_{t_1}^{t_2} t^2 dt - -- (38)$$

$$x_2 = (t_2 - t_1) - \frac{icot(\alpha)}{6}(t_2^3 - t_1^3) - (39)$$

Finally I5

$$\frac{I_{5} = \frac{0.23555\left(\frac{\exp(i4\pi t_{2} - iut_{2}cosec(\alpha))}{i4\pi - iucosec(\alpha)} - \frac{\exp(-i4\pi t_{1} - iut_{1}cosec(\alpha))}{-i4\pi - iucosec(\alpha)}\right)(t_{2} - t_{1})}{(t_{2} - t_{1}) - \frac{icot(\alpha)}{6}(t_{2}^{3} - t_{1}^{3})}$$
(40)

Finally

$$\omega_{\alpha}(u) = I_3 - I_4 + I_5 - - - (41)$$

Thus equation-(41) is the FRFT based clonal algorithm based window.

When substitute $\alpha = a\frac{\pi}{2}$ where a = 1 in equation-(41) results to generalized Fourier Transform based clonal

algorithm based window.

The spectral parameters of above window is shown in Table and it's spectral responses are shown from Fig1 to Fig6.

Table:

Spectral Parameters of FrFT Based clonal algorithm based window for variations in a

А	SLA in dB	HBW in dB	SLFOR in
			dB
0.93	-57	0.0195	-73.94
0.94	-60.8	0.0195	-71.85
0.95	-65.5	0.0185	-71.87
0.96	-63.9	0.0185	-72.55
0.97	-62.2	0.0185	-72.09
0.98	-61.0	0.0185	-70.76
0.99	-60.3	0.0185	-69.49
1	-60	0.0185	-69.1



Fig1:Spectral Response of window for a=0.93



Fig2:Spectral Response of window for a=0.94



Fig3:Spectral Response of window for a=0.95



Fig4:Spectral Response of window for a=0.96



Fig5:Spectral Response of window for a=0.97



Fig6:Spectral Response of window for a=0.98



Fig7:Spectral Response of window for a=0.99



Fig8:Spectral Response of window for a=1

3 Conclusion:

From the study of Exponential derivation of FrFT for clonal algorithm based window, The controllability of window parameters like HBW,MSLA and SLFOR is possible..i.e. The MSLA increases from -60 dB to -65.5 dB for a= 1 to a=0.95 decreases to -57 dB for a=0.93[Table].so that from the table it is observed that the spectral parameters of clonal based window is improved by controlling the 'a' parameter of FrFT. and also one of the property of FrFT is Generalization of FrFTto FT i.e. when $\alpha = \frac{a\pi}{2}$ where a=1;then The FrFT should equals to FT .This proposed Mathematical derivation of FrFT fulfills the Property of FT.

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