

Analysis of MIMO with Zero Forcing Successive Interference Cancellation Equalizer

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Abstract— This paper contains some advanced applications of MIMO systems and some main notes on their implementations, which started with MIMO with ZF technique that investigate the effects of phase noise in centralized and distributed narrowband MIMO systems, and discuss the feasibility of phase and frequency synchronization problem. An equalizer, called successive interference cancellation zero-forcing equalizer (SIC-ZFE) is proposed. The proposed equalizer provides ISI-free communications over the ISI MIMO channels without a long guard period. The simulated results with the 2×2 MIMO system with zero forcing equalizer showed matching results as obtained in for a 1×1 system for BPSK modulation in Rayleigh channel. In this paper, we will try to improve the bit error rate performance by trying out Successive Interference Cancellation (SIC). We will assume that the channel is a flat fading Rayleigh multipath channel and the modulation is BPSK. Simulation results show that even with only one selected antenna at the receiver, performances in terms of BER still satisfactory. Nevertheless, when more antennas are selected, better BER values are achieved thanks to receive diversity.

Key Words ----- MIMO, Successive Interference cancellation, Zero Forcing, Simulation, Algorithm.

I. INTRODUCTION

Multiple antennas communications are a new trend for high speed wireless communications. Algorithms for multiple antennas systems, like V-BLAST [1] and space-time block codes [2,3], were proposed. These algorithms work well with flat fading channels but, with the increasing channel bandwidth, consideration in selective fading becomes viable. Migrating to selective fading channel implies that the space-time receiver will need to eliminate the inter-symbol interference (ISI), and at the

same time resolve the inter-block interference (IBI) problem. IBI can be avoided by inserting a long guard period between each block of transmitted signals [4-6]. However, this guard period is a kind of transmission redundancy, which consumes the system bandwidth. This bandwidth consumption problem will be even more severe for the channel with high dispersion.

Recently, a promising system known as Space-time Modulated Codes (STMC) is proposed by Xia [7]. STMC is a special type of the space time block codes for selective fading MIMO channel. The STMC systems resolve the IBI problem with a relaxed guard period requirement. Hence, it avoids the degradation of system throughput suffered by most of the block based transmission system.

Based on FIR-ZFE, we proposed a new equalizer, called Successive Interference Zero-Forcing Equalizer (SICZFE), which inherited all the advantages of FIR-ZFE. The SIC-ZFE improves the performance of the FIR-ZFE system by introducing the successive interference cancellation technique within each signal vector. As a result, even if there are any detection errors, it will only affect the current signal vector. Therefore, the GSIC-ZFE is free from error propagation problem when compared to the DFE technique considered in [7]. Rather than the traditional element-by-element interference cancellation mechanism, a successive interference cancellation process is proposed in SIC-ZFE for reducing the computational complexity. Consider a STMC system with K sub-channels. If we only detect and cancel one element in each interference cancellation, K iterations will be required for detecting the whole signal vector. Thanks to the nice parallel equalization structure of FIR-ZFE. It is possible to choose any number of elements to be equalized in iteration of interference cancellation. For example, if K elements are detected and canceled in each iteration, the number of iterations will be reduced to $\lceil K/K \rceil$, and hence the computational complexity will be reduced by

approximately K times. Simulation results showed that significant improvement over FIR-ZFE can be obtained, even if only two iterations of interference cancellation are performed for each signal vector.

II. 2x2 MIMO channel

In a 2x2 MIMO channel, probable usage of the available to transmit antennas. Consider that we have a transmission sequence, for example $\{x_1, x_2, x_3, \dots, x_n\}$. In normal transmission, we will be sending x_1 in the first time slot, x_2 in the second time slot, x_3 and so on. However, as we now have 2 transmit antennas, we may group the symbols into groups of two. In the first time slot, send x_1 and x_2 from the first and second antenna. In second time slot, send x_3 and x_4 from the first and second antenna, send x_5 and x_6 in the third time slot and so on. Notice that as we are grouping two symbols and sending them in one time slot, we need only $n/2$ time slots to complete the transmission – data rate is doubled. This forms the simple explanation of a probable MIMO transmission scheme with 2 transmit antennas and 2 receive antennas.

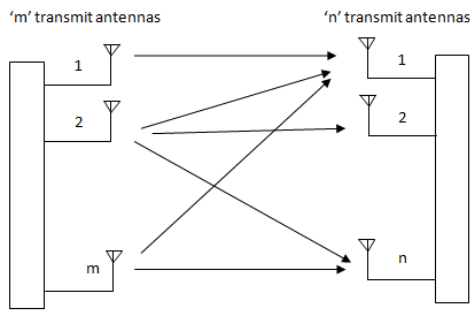


Figure 1: To Transmit to Receive (2x2) MIMO channel

III. Zero forcing equalizer for 2x2 MIMO channel

Let us now try to understand the math for extracting the two symbols which interfered with each other. In the first time slot, the received signal on the first receive antenna is

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \ h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad (1)$$

The received signal on the second receive antenna is

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \ h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad (2)$$

where [8]

y_1, y_2 are the received symbol on the first and second antenna respectively, $h_{1,1}$ is the channel from 1st transmit antenna to 1st receive antenna, $h_{1,2}$ is the channel

from 2nd transmit antenna to 1st receive antenna, $h_{2,1}$ is the channel from 1st transmit antenna to 2nd receive antenna, $h_{2,2}$ is the channel from 2nd transmit antenna to 2nd receive antenna, x_1, x_2 are the transmitted symbol.

The equation can be represented in matrix notation as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (3)$$

Equivalently, $y = Hx + n$ To solve for x , The Zero Forcing (ZF) linear detector for meeting this constraint $WH = I$, is given by,

$$W = (H^H H)^{-1} H^H \quad (4)$$

IV. Zero Forcing with Successive Interference Cancellation (ZF-SIC)

Using [9] the Zero Forcing (ZF) equalization approach described above, the receiver can obtain an estimate of the two transmitted symbols x_1, x_2 i.e.

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (5)$$

Take one of the estimated symbols (for example \hat{x}_2) and subtract its effect from the received vector y_1 and y_2

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,2} \hat{x}_2 \\ y_2 - h_{2,2} \hat{x}_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} x_1 + n_1 \\ h_{2,1} x_1 + n_2 \end{bmatrix} \quad (6)$$

$$r = h x_1 + n \quad (7)$$

The above equation is same as equation obtained for receive diversity case. Optimal way of combining the information from multiple copies of the received symbols in receive diversity case is to apply Maximal Ratio Combining (MRC).

$$\hat{x}_1 = \frac{h^H r}{h^H h} \quad (7)$$

This forms the simple explanation for Zero Forcing Equalizer with Successive Interference Cancellation (ZF-SIC) approach.

Algorithm:

Equalizer Design Procedure

$\tilde{\mathbf{C}}^{(1)} := \tilde{\mathbf{C}}$.
 Set $\Theta^{(0)}$ to be an empty set.
 For $a := 1$ to α ,
 Evaluate $[\tilde{\mathbf{C}}^{(a)}]^\dagger$.
 For $k \in [0, K-1]$ and $k \notin \bigcup_{i=0}^{a-1} \Theta^{(i)}$,

$$\gamma_k := \left\{ \begin{array}{l} \arg \min_{\gamma} \left(\left\| [\tilde{\mathbf{C}}^{(a)}]^\dagger_{\{(\gamma K+k)\text{-th row}\}} \right\| \right) \\ \text{and } [\tilde{\mathbf{C}}^{(a)}]_{\{(\gamma K+k)\text{-th column}\}} \neq \mathbf{0} \end{array} \right\}$$

 $\lambda_k := \left\| [\tilde{\mathbf{C}}^{(a)}]^\dagger_{\{(\gamma_k K+k)\text{-th row}\}} \right\|$.
 end;
 $\Theta^{(a)} := \{k_1^{(a)}, \dots, k_{\beta_a}^{(a)}\}$,
 with $\lambda_{k_1^{(a)}} \leq \lambda_{k_2^{(a)}} \leq \dots \leq \lambda_{k_{\beta_a}^{(a)}} \leq \dots \leq \lambda_{k_{(K-\sum_{i=1}^{a-1} \beta_i)}}$.
 For $b := 1$ to β_a ,
 $\mathbf{f}_{k_b^{(a)}} := [\tilde{\mathbf{C}}^{(a)}]^\dagger_{\{(\gamma_{k_b^{(a)}} K+k_b^{(a)}+1)\text{th row}\}}$.

$$[\tilde{\mathbf{C}}^{(a+1)}]_{\{(k+1)\text{th column}\}} := \begin{cases} \mathbf{0} & , \text{ for } k = \gamma_{k_b^{(a)}} K + k_b^{(a)}, \\ [\tilde{\mathbf{C}}^{(a)}]_{\{(k+1)\text{th column}\}} & , \text{ otherwise.} \end{cases}$$

 end;

Interference Cancellation Procedure

$\mathbf{r}_\ell^{(1)} := \tilde{\mathbf{C}} [\mathbf{x}_{\ell-L_D}^T \dots \mathbf{x}_\ell^T]^T + [\mathbf{n}_{\ell-L_F}^T \dots \mathbf{n}_\ell^T]^T$.
 For $a := 1$ to α ,
 For $b = 1, \dots, \beta_a$,

$$\hat{\mathbf{y}}_{k_b^{(a)}, \ell} := Q \left(\mathbf{f}_{k_b^{(a)}} \mathbf{r}_\ell^{(a)} \right)$$
.
 end;

$$[\mathbf{v}_\ell^{(a)}]_{\{(\gamma_k K+k+1)\text{th row}\}} := \begin{cases} \hat{\mathbf{y}}_{k, \ell} & , \text{ for } k = k_1^{(a)}, \dots, k_{\beta_a}^{(a)}, \\ 0 & , \text{ otherwise.} \end{cases}$$

 $\mathbf{r}_\ell^{(a+1)} := \mathbf{r}_\ell^{(a)} - \tilde{\mathbf{C}}^{(a)} \mathbf{v}_\ell^{(a)}$.
 end;

Figure 2: SIC-ZFE Algorithm

V. SIMULATION

In this section first Generate random binary sequence of +1's and -1's. Group them into pair of two symbols and send two symbols in one time slot then Multiply the symbols with the channel and then add white Gaussian noise. Equalize the received symbols with Zero Forcing criterion and Take the symbol from the second spatial dimension, subtract from the received symbol. Perform Maximal Ratio Combining for equalizing the new received symbol then Perform hard decision decoding and count the bit errors. Repeat for multiple values of $\frac{E_b}{N_0}$ and plot the simulation and theoretical results.

for BPSK modulation with 2x2 MIMO and ZF-SIC equalizer (Rayleigh

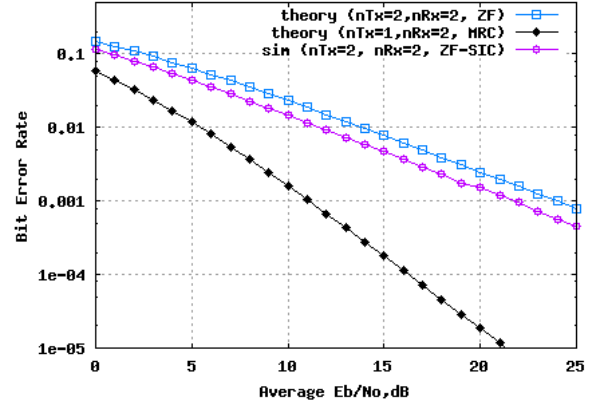


Figure 3: BER plot for BPSK in 2x2 MIMO channel with Zero Forcing Successive Interference Cancellation equalization

Compared to Zero Forcing equalization alone case, addition of successive interference cancellation results in around 2.2dB of improvement for BER of 10^{-3} . he improvement is brought in because decoding of the information from the first spatial dimension (\mathcal{X}_1) has a lower error probability that the symbol transmitted from the second dimension. However, the assumption is that \mathcal{X}_2 is decoded correctly may not be true in general.

VI. CONCLUSIONS

An equalizer for on the SIC and the linear ZFE is proposed. In each iteration, the proposed equalizer detects and cancels a group of elements of the source vector in a receive signal. This SIC process allows the FIR equalizers designed by SIC-ZFE algorithm to be further optimized without error propagation problem. Simulation results showed that significant BER performance gain can be obtained by the proposed SIC-ZFE even if only two interference cancellation iterations are applied to each input signal vector when compared to that of FIR-ZFE.

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