

# Analysis Of Multifractal Behaviour Of Electrocardiograms: DFA Method

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## Abstract

Attempts have been made in this paper to show the multifractal property of the electrocardiograms. With the aid of the detrended fluctuation analysis (DFA), the multifractal nature of the electrocardiograms (ECG) has been studied. Normal electrocardiograms are acquired through the polypara module from two subjects, whereas the diseased electrocardiogram of BIDMC congestive heart failure and atrial fibrillation are obtained from the physionet. Both the ECG signals have been analysed by the Hurst exponent, the partition function and the singular spectrum of multifractal detrended fluctuation analysis (MF-DFA). There is a little diversification between the spectrums of the normal ECGs and the diseased ECGs but the results show a strong degree of multifractality in the time series of the electrocardiograms. The results are all implemented upon the MATLAB platform.

## 1. Introduction

Life is one of the most complex non-linear systems and heart is the core of this life-cycle system. Fractals are fit for signal modelling in the real world, such as electroencephalograms (EEG), electrocardiograms (ECG), as well as turbulent flows, lightning strikes, DNA sequences, and geographical objects which represent some of many natural phenomena and are difficult to be characterized using traditional signal processing theory [1-4]. Electrocardiogram (ECG) is a graphical representation of cardiac activity. In general, ECG signals have unique morphological characteristics (P-QRS-T complex) and it is highly significant than other biological signals. Physiologic signals generate complex fluctuations in their output signals that reflect the underlying dynamics [5-9]. The main features of this physiologic time series such as ECG are non-stationarity, non-linearity and non-equilibrium phenomena. Human cardiac dynamics are driven by complex non-linear interactions of two competing forces: sympathetic stimulation increases the heart rate,

whereas parasympathetic system decreases it [12]. For this type of intrinsically noisy system, the novel technique of detrended fluctuation analysis (DFA) has been developed to study the non-stationary behaviour of ECG signal [15]. Normal ECG signals have been acquired through the polypara module. The data of the diseased patient is collected from the patients suffering from BIDMC congestive heart failure and atrial fibrillation [16-17].

## 2. DFA Algorithm

The DFA algorithm [2] quantifies fractal-like correlation properties by calculating the scaling property of the root-mean-square fluctuation of the integrated and detrended time series data. The steps of the DFA algorithm are as follows:

Step 1: At first the ECG time series is taken which is denoted by  $\{x(i)\}$ . Then the profile  $\{Y(i)\}$  is determined.

$$Y(i) = \sum_{k=1}^i x(k) - \langle x \rangle \quad (1)$$

Where ' $\langle x \rangle$ ' is the mean of the record

Step 2: The profile  $\{Y(i)\}$  is divided into  $N_s \equiv \lfloor \frac{N}{S} \rfloor$  boxes of the same size 'S'.

Step 3: In each box, the integrated time series is fitted by using a polynomial function,  $p_v(i)$ , which is the local trend.

Step 4: The local trend is subtracted and the detrended fluctuation function is given by ,

$$Y_s(i) = Y(i) - P_v(i) \quad (2)$$

Step 5: In each box of size 'S', the variance is determined,

$$F_s^2 = \frac{1}{S} \sum_{i=1}^S \{Y[(v-1)S+i] - P_v(i)\}^2 \quad (3)$$

Step 6: The qth order fluctuation function  $F_q(S)$  is calculated,

$$F_q(S) = \sqrt[q]{\frac{1}{N_S} \sum_{v=1}^{N_S} (F_S^2(v))^q} \quad (4)$$

Step 7: The procedure is repeated for different box sizes (different scales).

### 3. Parameters to study MF-DFA

DFA [3] is a well-established method for determining data scaling behaviour in the presence of possible trends without knowing their origin & shape. Repeating the procedure for several scales, as discussed above,  $F_q(S)$  will increase with increasing 'S'. If the time series is of long range correlation, then

$$F_q(S) \propto S^{h(q)} \quad (5)$$

The 'h(q)' is called the generalized Hurst exponent

#### 3.1. Generalised Hurst Exponent

This parameter determines whether the time series is monofractal or multifractal. For monofractal time series 'h(q)' is constant. On the other hand, for multifractal time series, 'h(q)' depends on the value of 'q', the fluctuation function. Therefore, the exponent 'h(q)' is called the generalized Hurst exponent [5].

#### 3.2. Partition Function

The partition function  $\zeta(q)$  is regarded as a characteristic function of the fractal behaviour. The partition function is given by,

$$\zeta(q) = qh(q) - 1 \quad (6)$$

If  $\zeta(q)$  versus q is linear, the time series is monofractal. If  $\zeta(q)$  versus q is convex, the time series has a multifractal property [5].

#### 3.3. Singular Spectrum

The singular spectrum is also an important tool for fractal investigation in time series. It is denoted by ' $f(\alpha)$ '. In fact, the curve of the spectrum is single-humped for a multifractal, while it reduces to a point for monofractal time series [6]. For a multifractal, the maximum of the spectrum denotes the dominant fractal exponent, and the width of the spectrum provides the range of the fractal exponents. The singular spectrum is calculated by Legendre Transform:

$$\alpha(q) = \frac{d(\zeta(q))}{dq} \quad (7)$$

$$f(\alpha) = q\alpha(q) - \zeta(q) \quad (8)$$

where  $\zeta(q) = qh(q) - 1$  is the partition function

## 4. Results And Discussion

In order to study the multifractal behaviour of the electrocardiograms, the singular spectrum, the Hurst exponent and the partition function of both normal & diseased ECGs are studied. The diseased ECG is collected from the physionet.

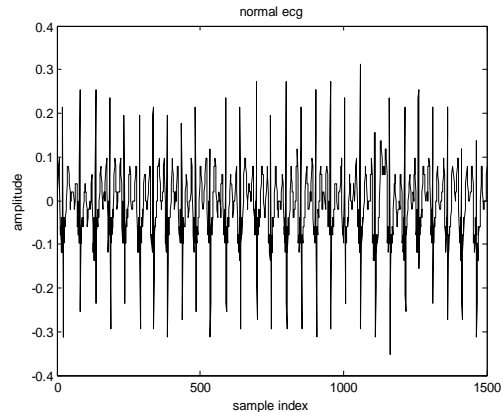


Figure 1: Normal ECG of subject1

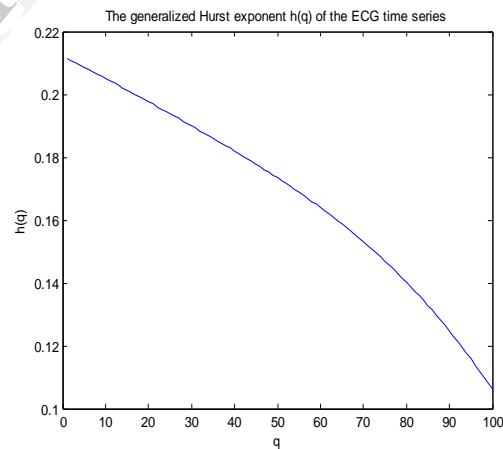
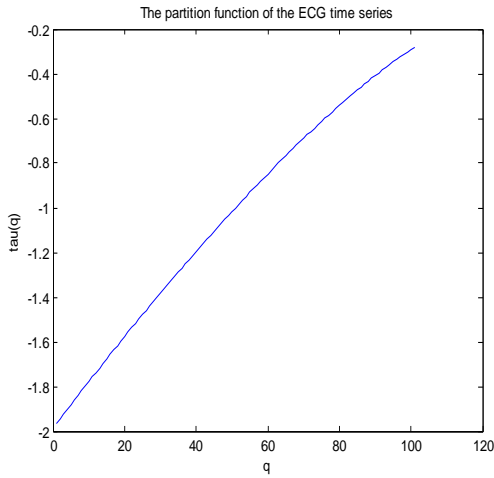
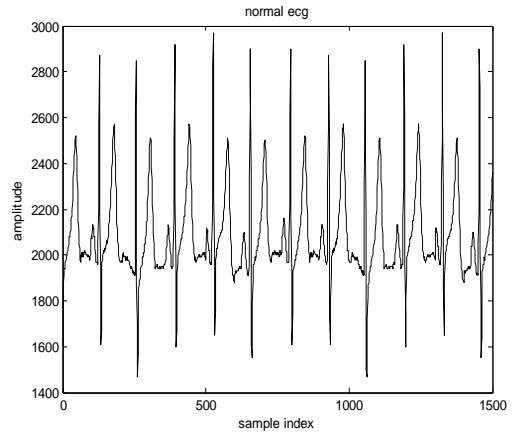


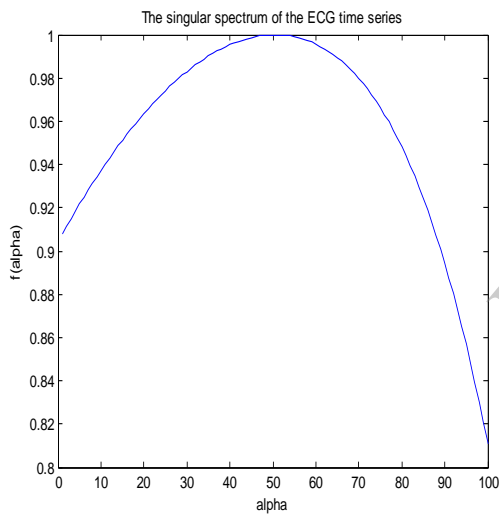
Figure 2: The generalized Hurst exponent of normal electrocardiograms of subject1



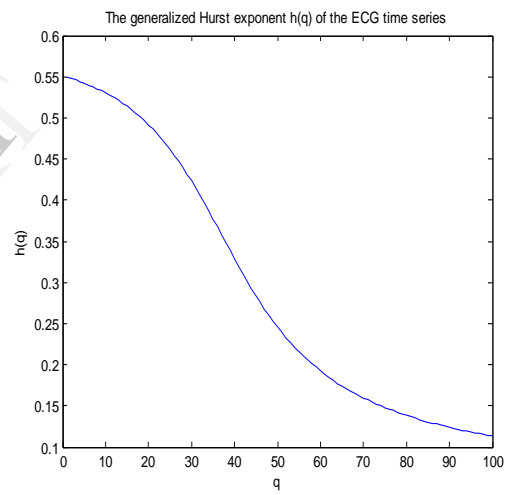
**Figure 3: The partition function of the normal electrocardiograms of subject1**



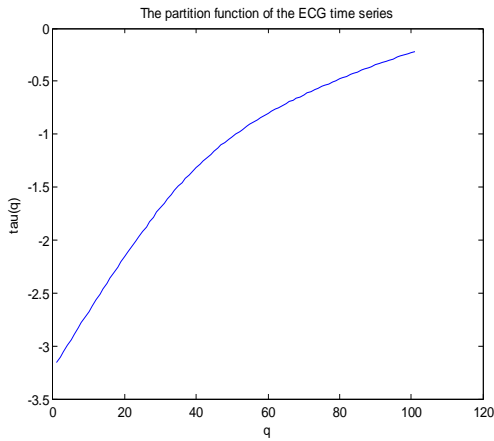
**Figure 5: Normal electrocardiogram of subject 2**



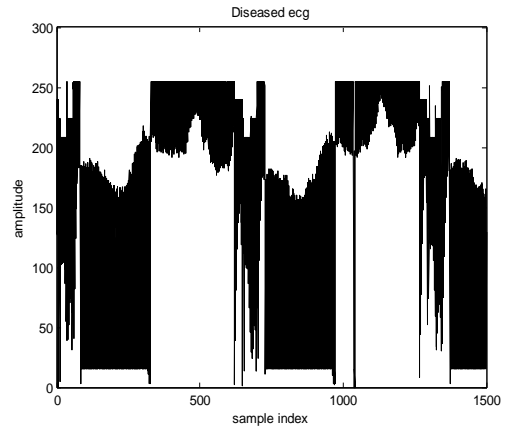
**Figure 4: The singular spectrum of normal electrocardiogram of subject1**



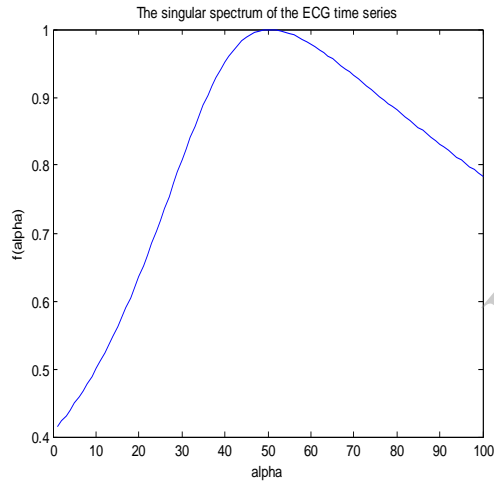
**Figure 6: The generalized Hurst exponent of normal electrocardiograms of subject2**



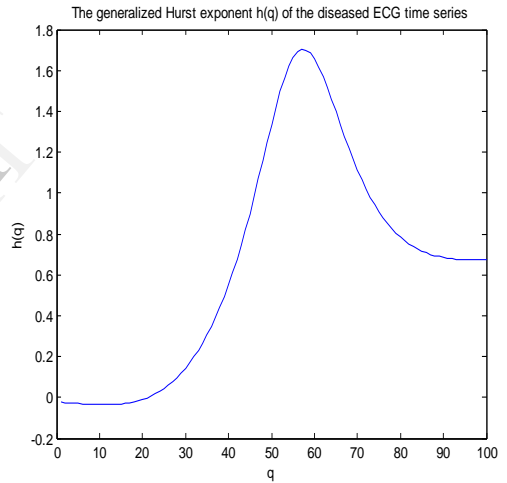
**Figure 7: The partition function of the normal electrocardiograms of subject2**



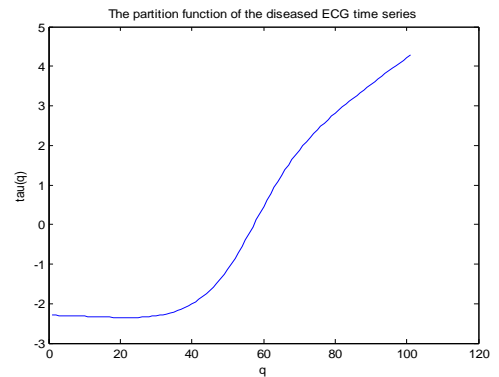
**Figure 9: Diseased electrocardiogram of BIDMC congestive failure (chf01)**



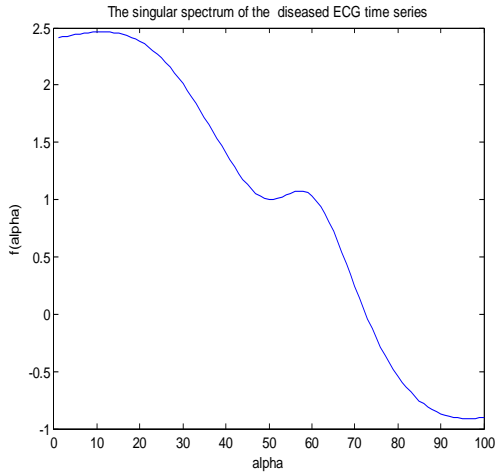
**Figure 8: The singular spectrum of normal electrocardiogram of subject2**



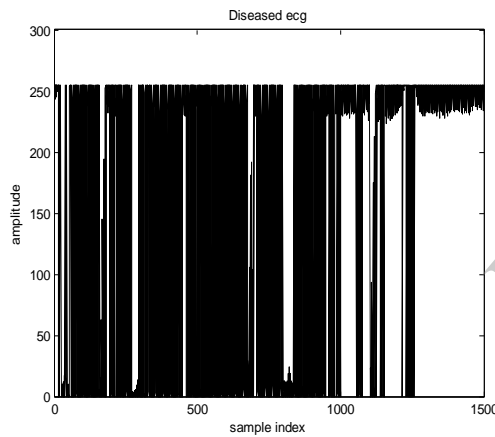
**Figure 10 : The generalized Hurst exponent of diseased electrocardiograms (chf01)**



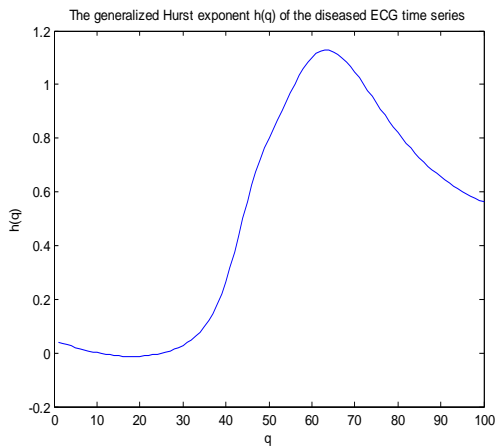
**Figure 11: The partition function of diseased electrocardiograms (chf01)**



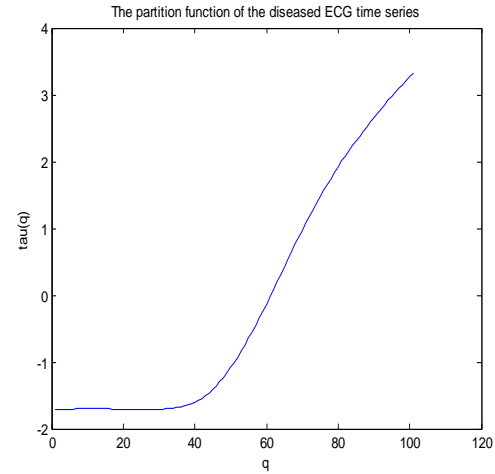
**Figure 12: The singular spectrum of diseased electrocardiograms (chf01)**



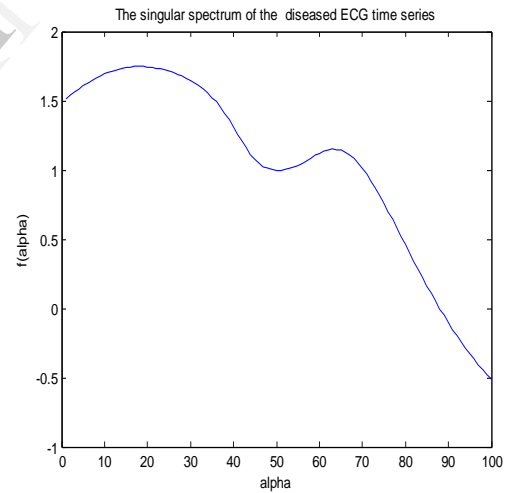
**Figure 13: Diseased electrocardiogram of atrial fibrillation (n01)**



**Figure 14: The generalized Hurst exponent of diseased electrocardiograms (n01)**



**Figure 15: The partition function of diseased electrocardiograms (n01)**



**Figure 16: The singular spectrum of diseased electrocardiograms (n01)**

Figure 1 shows the normal ECG signal of subject1, Figure 2 shows the generalized Hurst exponent of the normal electrocardiograms of subject1, Figure 3 shows the partition function of the normal electrocardiograms of subject1 and Figure 4 shows the singular spectrum of the normal electrocardiograms of subject1. Similarly, Figure 5, Figure 6, Figure 7 and Figure 8 shows the respective results of normal electrocardiograms of subject2. The diseased electrocardiogram is taken from the physionet. It contains the data of BIDMC congestive heart failure

and atrial fibrillation. Figure 9 shows the diseased electrocardiogram of BIDMC congestive heart failure, Figure 10 shows the generalised Hurst exponent of the diseased electrocardiogram of atrial fibrillation, Figure 11 shows the partition function of the diseased electrocardiogram of atrial fibrillation and Figure 12 shows the singular spectrum of the diseased electrocardiogram of atrial fibrillation. Similarly, Figure 13 shows the diseased electrocardiograms of atrial fibrillation, Figure 14, Figure 15 and Figure 16 shows the respective results of the diseased electrocardiograms of atrial fibrillation.

It is obvious from the results that the relation between  $\zeta(q)$  and  $q$  of the normal and diseased both electrocardiograms is non-linear which shows that the ECG time series is multifractal in nature. Also, the singular spectrum of the normal and diseased electrocardiograms are more-or-less single-humped, which shows the multifractality of the ECG time series.

## 5. Conclusion

In this paper, a detailed analysis of the fractal properties of the ECG time series is studied, which is analysed by the Hurst exponent, partition function and the singular spectrum. The results from all these methods show that the ECG time series shows multifractal properties both in the case of normal and diseased electrocardiograms.

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