# Analysis of Proposed Power Delay Profile (PDP) for MIMO-OFDM Systems

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# Abstract

The aim of this paper is to investigate the OFDM scheme, and realize a fully functional system and analyzing how it is reducing the inter-symbol interference caused by the multipath fading channels and different effects and estimating, evaluating the performance of it. The paper proposes a power delay profile (PDP) estimation technique for linear minimum mean square error (LMMSE) channel estimator of multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems. For practical applications, only the pilot symbols of all transmit antenna ports are used in estimating the PDP. The distortions caused by null subcarriers and an insufficient number of samples for PDP estimation is also considered. The proposed technique effectively reduces the distortions for accurate PDP estimation. Simulation results show that the performance of LMMSE channel estimation using the proposed PDP estimate approaches that of Wiener filtering due to the mitigation of distortion effects.

Index Terms: Power delay Profile (PDP), MIMO-OFDM, linear minimum mean square error (LMMSE)

# **1. INTRODUCTION**

OFDM depends on Orthogonality principle. Orthogonality means, it allows the sub carriers, which are orthogonal to each other, meaning that cross talk between co-channels is eliminated and inter-carrier guard bands are not required. This greatly simplifies the design of both the transmitter and receiver, unlike conventional FDM; a separate filter for each sub channel is not required.

Orthogonal Frequency Division Multiplexing (OFDM) is a digital multi carrier modulation scheme, which uses a large number of closely spaced orthogonal sub-carriers.

A single stream of data is split into parallel streams each of which is coded and modulated on to a subcarrier, a term commonly used in OFDM systems. B.Sashi Kanth<sup>2</sup> Asst. Professor Department of ECE, Dadi Institute of Engineering & Technology

Each sub-carrier is modulated with a conventional modulation scheme (such as quadrature amplitude modulation) at a low symbol rate, maintaining data rates similar to conventional single carrier modulation schemes in the same bandwidth [3]. Thus the high bit rates seen before on a single carrier is reduced to lower bit rates on the subcarrier.

In practice, OFDM signals are generated and detected using the Fast Fourier Transform algorithm. OFDM has developed into a popular scheme for wideband digital communication, wireless as well as copper wires.

Since orthogonality is important for OFDM systems, synchronization in frequency and time must be extremely good. Once orthogonality is lost we experience **inter-carrier interference** (ICI). This is the interference from one subcarrier to another. There is another reason for ICI. Adding the guard time with no transmission causes problems for IFFT and FFT, which results in ICI. A delayed version of one subcarrier can interfere with another subcarrier in the next symbol period. This is avoided by extending the symbol into the guard period that precedes it. This is known as a **cyclic prefix [5]**. It ensures that delayed symbols will have integer number of cycles within the FFT integration interval. This removes ICI so long as the delay spread is less than the guard period.

The transmission is generated in such a way that the carriers used are orthogonal to one another, thus allowing them to be packed together much closer than standard frequency division multiplexing (FDM) [2]. This leads to OFDM/COFDM providing a high spectral efficiency. the OFDM uses different multiple access unit for the transmission of data.

Orthogonal Frequency Division Multiplexing is a scheme used in the area of high-data-rate mobile wireless communications such as cellular phones, satellite communications and digital audio broadcasting. This technique is mainly utilized to combat inter-symbol interference [4].

## **1.1 Multiple Techniques**

Multiple access schemes are used to allow many simultaneous users to use the same fixed bandwidth radio spectrum. In any radio system, the bandwidth, which is allocated to it, is always limited. For mobile phone systems the total bandwidth is typically 50 MHz, which is split in half to provide the forward and reverse links of the system.

Sharing of the spectrum is required in order increase the user capacity of any wireless network. FDMA, TDMA and CDMA are the three major methods of sharing the available bandwidth to multiple users in wireless system [2]. There are many extensions, and hybrid techniques for these methods, such as OFDM, and hybrid TDMA and FDMA systems. However, an understanding of the three major methods is required for understanding of any extensions to these methods.

### 1.1.1 Frequency Division Multiple Access (FDMA):

In Frequency Division Multiple Access (FDMA), the available bandwidth is subdivided into a number of narrower band channels. Each user is allocated a unique frequency band in which to transmit and receive on. During a call, no other user can use the same frequency band.



Figure-1 & Figure-2 show the allocation of the available bandwidth into several channels.

### 1.1.2 Time Division Multiple Access:

Time Division Multiple Access (TDMA) divides the available spectrum into multiple time slots, by giving each user a time slot in which they can transmit or receive. **Fig. 1.4** shows how the time slots are provided to users in a round robin fashion, with each user being allotted one time slot per frame. TDMA systems transmit data in a buffer and burst method, thus the transmission of each channel is non-continuous.



Figure-3 TDMA scheme, where each user is allocated a small time slot

#### 1.1.3 Code Division Multiple Access:

Code Division Multiple Access (CDMA) is a spread spectrum technique that uses neither frequency channels nor time slots. In CDMA, the narrow band message (typically digitized voice data) is multiplied by a large bandwidth signal, which is a pseudo random noise code (PN code) [2]. All users in a CDMA system use the same frequency band and transmit simultaneously. The transmitted signal is recovered by correlating the received signal with the PN code used by the transmitter.



Figure- 4 Code Division Multiple Access (CDMA)

#### 2. OFDM GENERATION

To generate OFDM successfully the relationship between all the carriers must be carefully controlled to maintain the orthogonality of the carriers. For this reason, OFDM is generated by firstly choosing the spectrum required, based on the input data, and modulation scheme used. Each carrier to be produced is assigned some data to transmit. The required amplitude and phase of the carrier is then calculated based on the modulation scheme (typically differential BPSK, QPSK, or QAM).

The required spectrum is then converted back to its time domain signal using an Inverse Fourier Transform. In most applications, an Inverse Fast Fourier Transform (IFFT) is used. The IFFT performs the transformation very efficiently, and provides a simple way of ensuring the carrier signals produced are orthogonal.

The Fast Fourier Transform (FFT) transforms a cyclic time domain signal into its equivalent frequency spectrum [8]. This is done by finding the equivalent waveform, generated by a sum of orthogonal sinusoidal components. The amplitude and phase of the sinusoidal components represent the frequency spectrum of the time domain signal.

. The IFFT performs the reverse process, transforming a spectrum (amplitude and phase of each component) into a time domain signal. An IFFT converts a number of complex data points, of length, which is a power of 2, into the time domain signal of the same number of points. Each data point in frequency spectrum used for an FFT or IFFT is called a bin. The orthogonal carriers required for the OFDM signal can be easily generated by

setting the amplitude and phase of each bin, then performing the IFFT. Since each bin of an IFFT corresponds to the amplitude and phase of a set of orthogonal sinusoids, the reverse process guarantees that the carriers generated are orthogonal.



#### Figure- 5 OFDM Block Diagram

The setup for a basic OFDM transmitter and receiver. The signal generated is a base band, thus the signal is filtered, then stepped up in frequency before transmitting the signal. OFDM time domain waveforms are chosen such that mutual orthogonality is ensured even though sub-carrier spectra may overlap [8]. Typically QAM or Differential Quadrature Phase Shift Keying (DQPSK) modulation schemes are applied to the individual sub carriers. To prevent ISI, the individual blocks are separated by guard intervals wherein the blocks are periodically extended.

#### 3. MIMO-OFDM CONCEPT

**MULTIPLE-INPUT** multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) is one of the most promising techniques for wireless communication systems, including the 3rd Generation Partnership Project Long Term Evolution (3GPP LTE), and IEEE 802.16 (WiMAX). MIMO-OFDM provides a considerable performance gain over broadband single-antenna systems by obtaining the spatial diversity or multiplexing gain. Most receiver techniques of MIMO-OFDM systems are designed with the assumption that channel state information (CSI) is available, in order to achieve the maximum diversity or multiplexing gain. The performance gain depends heavily on accurate channel estimation, which is crucial for the MIMO-OFDM systems [8].

The pilot-aided channel estimation, based on the linear minimum mean square error (LMMSE) technique, is optimum in the sense of minimizing mean square error (MSE) when the receiver knows the channel statistics. To obtain the frequency domain channel statistics at the receiver, power delay profile (PDP) estimation schemes have been proposed. These schemes are based on the maximum likelihood (ML) estimation by taking advantage of the cyclic prefix (CP) segment of OFDM symbols. However, the ML PDP estimators require very high computational complexity for obtaining an accurate PDP.

Another approach for improving the performance of LMMSE channel estimation employs an approximated PDP (i.e., uniform or exponential model) with the estimation of second-order channel statistics, which are mean delay and root-mean-square (RMS) delay spread. The channel delay parameters are estimated using pilots with low computational complexity. Therefore, the LMMSE channel estimator with the approximated PDP is appropriate for practical applications such as a WiMAX system [10]. However, the performance degradation is caused by both the correlation mismatch and the estimation error of delay parameters [8]. To reduce the mismatch in the frequency domain, we propose a PDP estimation technique for the LMMSE channel estimator of MIMO-OFDM systems. For practical applications, the proposed technique uses only the pilot symbols of all transmit antenna ports to estimate the PDP with low computational complexity. In addition, the proposed technique effectively mitigates the distortion effects, incurred by null subcarriers and an insufficient number of estimated channel impulse response (CIR) samples [9]. Simulation results show that the performance of LMMSE channel estimation with the proposed PDP estimate approaches that of Wiener filtering.

#### 3.1 SYSTEM MODEL

The system under consideration is a MIMO-OFDM system with P transmit and Q receive antennas, and K total subcarriers. Suppose that the MIMO-OFDM system transmits Kd subcarriers at the central spectrum assigned for data and pilots with K-Kd virtual subcarriers, in order to control interferences with other systems. The CIRs corresponding to different transmit and receive antennas in MIMO systems usually have the same PDP.

Let [kp, np] be the pilot subcarrier for the *p*th transmit antenna at the npth OFDM symbol, which is a QPSK modulated signal from known sequences between the transmitter and receiver. We assume that the pilot subcarriers are distributed over a time and frequency grid as in Fig. 1, to preserve the orthogonality of pilots among different transmit antennas.  $k_p \in \mathcal{F}_p$  and  $n_p \in \mathcal{T}_{p_{\text{represent the index sets for}}$ the pilot subcarriers of the pth antenna port in the frequency and time domains, respectively. At the *npth* OFDM symbol, the number of pilot subcarriers is defined as Kp=Fp The pilot inserted OFDM symbol is transmitted over the wireless channel after performing an inverse fast Fourier transform (IFFT) and adding a CP. It is assumed that the length of CP, Lq, is longer than the channel maximum delay, Lch, making the channel matrix circulant

 $(Lch \le Lg)$ . At the receiver, after perfect synchronization, the removal of CP, and FFT operation, the received pilot symbol for the *q*th receive antenna can be represented as

$$\mathbf{y}_q[n_p] = diag(\mathbf{x}_p)\mathbf{F}_p\mathbf{h}_{p,q} + \mathbf{n}_q$$

Where

 $\mathbf{h}_{p,q} = [h_{p,q}[n_p, 0], h_{p,q}[n_p, 1], \dots, h_{p,q}[n_p, L_{ch}], 0, \dots, 0]^T \dots^2$ is an  $Lg \times 1$  CIR vector at the pth transmit antenna and qth

receive antenna.  $(\cdot)^T$  and  $(\cdot)^H$  represent the transpose operation, and the transpose and conjugate ope ration of a vector or matrix, respectively.

$$\mathbf{x}_{p} = [C_{p}[i_{1}, n_{p}], C_{p}[i_{2}, n_{p}], \dots, C_{p}[i_{K_{p}}, n_{p}]]^{T}$$

denotes a pilot vector at the *np*th OFDM symbol for  $i_k \in \mathcal{F}_n$  and k=1, 2, Kn





### 3.2 PROPOSED METHOD FOR PDP

Derivation of the PDP in MIMO-OFDM systems From (1), the CIR at the (p, q) th antenna port can be estimated approximately using the regularized least squares (RLS) channel estimation with a fixed length of Lg as

$$\hat{\mathbf{h}}_{R,p,q} = \left( \mathbf{F}_p^H \mathbf{F}_p + \epsilon \mathbf{I}_{L_g} \right)^{-1} \mathbf{F}_p^H diag(\mathbf{x}_p)^H \mathbf{y}_q[n_p] \triangleq \mathbf{W}_{RLS,p} \mathbf{y}_q[n_p],$$
 -----3

where  $\epsilon$ =0.001 is a small regularization parameter, and  $\mathbf{I}\mathcal{L}g$  is the  $\mathcal{L}g \times \mathcal{L}g$  identity matrix.  $\mathbf{F}\mathcal{H}p \mathbf{F}p$  in eqn-(1) is illconditioned due to the sparsely of pilot tones in the frequency domain and the presence of virtual subcarriers. To derive the PDP from the estimated CIR in (2), the ensemble average of  $\mathbf{h}R$ ,  $\mathbf{h}HR$ , p,q is given by

$$E\left\{\hat{\mathbf{h}}_{R,p,q}\hat{\mathbf{h}}_{R,p,q}^{H}\right\} = \mathbf{W}\mathbf{R}_{hh}\mathbf{W}^{H} + \sigma_{n}^{2}\mathbf{W}_{RLS,p}\mathbf{W}_{RLS,p}^{H}$$

where

$$\mathbf{R}_{hh} = E\{\mathbf{h}_{p,q}\mathbf{h}_{p,q}^H\} \text{ and } \mathbf{W} = (\mathbf{F}_p^H \mathbf{F}_p + \epsilon \mathbf{I}_{L_g})^{-1} \mathbf{F}_p^H \mathbf{F}_p \dots \mathbf{5}$$

The diagonal elements of the channel covariance matrix, **R***hh*, represent the PDP of multipath channel within the length of *Lg*, and all off-diagonal elements are zeros. Hence, the covariance matrix can be expressed as **R***hh*=*diag* (**p***h*), where **p***h*=[*p*0, *p*1, ..., *pLch*, 0, ..., 0]*T* and

$$p_l = E\{|h_{p,q}[n_p, l]|^2\}.$$

Unfortunately,  $\mathbf{R}\hbar\hbar$  is distorted by  $\mathbf{W}$ , which is an illconditioned matrix due to the presence of  $\mathbf{F}Hp \mathbf{F}p$ . Thus, instead of calculating  $\mathbf{W}$ -1, we investigate the method for eliminating the spectral leakage of  $\mathbf{W}$ . The covariance matrix of the estimated CIR is defined as  $\mathbf{R}_{hh} = \mathbf{W}\mathbf{R}_{hh}\mathbf{W}^{H}$  which can be expressed as

$$\mathbf{R}_{\hat{h}\hat{h}} = \sum_{l=0}^{L_g-1} \mathbf{W} diag(p_l \mathbf{u}_l) \mathbf{W}^H$$
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where  $\mathbf{u}/\mathbf{i}\mathbf{s}$  a unit vector with the *l*th entry being one and otherwise zeros. Let  $\mathbf{p}/l$  and  $\mathbf{t}/\mathbf{b}\mathbf{e}$  the  $Lg \times 1$  vectors defined as

$$p_{\hat{h}} = Dg(\mathbf{R}_{\hat{h}\hat{h}}) \text{ and } \mathbf{t}_{l} = Dg\left(\mathbf{W}diag(\mathbf{u}_{l})\mathbf{W}^{H}\right)_{\dots \mathbf{8}}$$
$$g_{p,q}[n_{p}] = \mathbf{T}^{-1}Dg\left(\hat{\mathbf{h}}_{R,p,q}\hat{\mathbf{h}}_{R,p,q}^{H}\right)_{\dots \mathbf{9}}$$

The received sample vector for estimating PDP at the (p, q)th antenna port on the *np*th OFDM symbol, and

$$\tilde{\mathbf{w}} = \mathbf{T}^{-1} Dg(\mathbf{W}_{RLS,p} \mathbf{W}_{RLS,p}^H).$$

the received samples are known in terms of AWGN, power delay point (PDP) estimation.

$$\langle \mathbf{g}_{p,q}[n_p] \rangle_N \triangleq \frac{1}{N} \sum_{n_p=1}^{|\mathcal{T}_p|} \sum_{p=1}^{P} \sum_{q=1}^{Q} \mathbf{g}_{p,q}[n_p]$$
 ......11

where  $N \triangleq |\mathcal{T}_p| PQ$  represents the total number of samples for PDP estimation.  $|\mathcal{T}_p|$  is the number of pilot symbols at the *kp*th subcarrier in a time slot. When *N* is sufficiently large, the PDP can be perfectly estimated. To improve the accuracy of PDP estimation with insufficient samples, we mitigate the effective noise as follows in below equation.

$$\langle \mathbf{g}_{p,q}[n_p] \rangle_N - \sigma_n^2 \tilde{\mathbf{w}} = \langle Dg \left( \mathbf{h}_{p,q} \mathbf{h}_{p,q}^H \right) \rangle_N + \mathbf{z}_N$$
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The error of PDP estimation with N samples can be calculated as

$$\tilde{\mathbf{e}}_{N} = \left( \left\langle Dg \left( \mathbf{h}_{p,q} \mathbf{h}_{p,q}^{H} \right) \right\rangle_{N} - \mathbf{p}_{h} \right) + \mathbf{z}_{N}.$$
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Since  $[\mathbf{p}_h]_i \geq 0$  for all *i*, the PDP can initially be estimated as

$$\hat{\mathbf{p}}_{init} = \frac{1}{N} \sum_{n_p=1}^{|\mathcal{T}_p|} \sum_{p=1}^{P} \sum_{q=1}^{Q} \mathbf{s}_{p,q}[n_p],$$
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Where sp,q[np] is the sample vector of proposed PDP estimator with the *l*th entry

$$s_{p,q}^{l}[n_{p}] = \begin{cases} g_{p,q}^{l}[n_{p}] - \sigma_{n}^{2}\tilde{w}^{l} & \text{if } g_{p,q}^{l}[n_{p}] > \sigma_{n}^{2}\tilde{w}^{l} \\ 0 & \text{otherwise} \end{cases}$$

To mitigate the detrimental effect of residual noise zN, the proposed scheme estimates the average of residual noise at the zero-taps of ph. At the *l*th entry of  $\hat{p}$ *init*, the zero-tap can be detected as

$$t_z^l = \begin{cases} 1 & \text{if } \hat{p}_{init}^l < \beta_{th} \\ 0 & \text{otherwise} \\ & & \\ \end{bmatrix}$$

Where  $\beta_{th} = \frac{1}{L_a} \sum_{l=0}^{L_g-1} \hat{p}_{init \text{ is defined as a}}^l$ threshold value for the zero-tap detection. Then, the average of residual noise at the zero-taps can be estimated as

$$\hat{n}_{R,avg} = \frac{1}{N_z} \sum_{l=0}^{L_g - 1} \hat{p}_{init}^l t_z^l,$$
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$$\hat{p}_{h}^{l} = \begin{cases} p_{init}^{i} - \hat{n}_{R,avg} & \text{if } p_{init}^{i} > \hat{n}_{R,avg} \\ 0 & \text{otherwise} \end{cases}$$

The estimated PDP in (18) can be used to obtain the frequency-domain channel correlation in the LMMSE channel estimator.

#### 4. SIMULATION RESULTS

The system simulation for the specifications explains the results of the performance in LMMSE using the proposed method of PDP.

The result in Fig-7 explains the performance of the LMMSE technique using PDP in terms of signal to noise ratio (SNR).



Fig-7 Performance of LMMSE Technique using PDP of SNR

The result in Fig-8 explains the performance of the LMMSE technique using PDP with MIMO-OFDM.



Fig-8 Performance of LMMSE Technique using PDP

The result in Fig-9 explains the performance of the MMSE technique using PDP for getting the better MSE value.



**Fig-9** Performance of LMMSE Technique using PDP

The result in Fig-10 explains the performance of the LMMSE technique using PDP in terms of channel estimation.



Fig-10 Simulation of LMMSE channel estimation

### 5. CONCLUSION

We proposed a PDP estimation technique for the LMMSE channel estimator in MIMO-OFDM systems. The CIR estimates at each path of the MIMO channels were used to obtain the PDP. For accurate PDP estimation, we considered the spectral leakage effect from virtual subcarriers, and the residual noise caused by the insufficient number of estimated CIR samples. The proposed technique effectively mitigates both the spectrum leakage and residual noise. Simulation results show that the performance of LMMSE channel estimation using the proposed PDP estimate approaches that of Wiener filtering.

#### REFERENCES

[1] TS 36.211, "Physical channels and modulation," in 3GPP TSG RAN1, Rev. 10.1.0, Jan. 2011.

[2] T.W. Yune, C. H. Choi, G. H. Im, J. B. Lim, E. S. Kim, Y. C. Cheong, and K. H. Kim, "SC-FDMA with iterative multiuser detection: improvements on power/spectral efficiency," *IEEE Commun. Mag.*, vol. 48, no. 3, pp. 164–171, Mar. 2010.

[3] U. K. Kwon, D. Kim, and G. H. Im, "Amplitude clipping and iterative reconstruction of MIMO-OFDM signals with optimum equalization," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 268–277, Jan. 2009.

[4] L. Zheng and D. N. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple antenna channels," *IEEE Trans. Inf. Theory*, vol. 49, pp. 1073–1096, May 2003.

[5] H. C. Won and G. H. Im, "Iterative cyclic prefix reconstruction and channel estimation for a STBC OFDM system," *IEEE Commun. Lett.*, vol. 9, pp. 307–309, Apr. 2005.

[6] T. Yoo and A. Goldsmith, "Capacity and power allocation for fading MIMO channels with channel estimation error," *IEEE Trans. Inf. Theory*, vol. 52, pp. 2203–2214, May 2006.

[7] M. G. Song, D. Kim, and G. H. Im, "Recursive channel estimation method for OFDM-based cooperative systems," *IEEE Commun. Lett.*, vol. 14, no. 11, pp. 1029–1031, Nov. 2010.

[8] S. Omar, A. Ancora, and D. T. M. Slock, "Performance analysis of general pilot-aided linear channel estimation in LTE OFDMA systems with application to simplified MMSE schemes," in *Proc. IEEE PIMRC 2008*.

[9] T. Cui and C. Tellambura, "Power delay profile and noise variance estimation for OFDM," *IEEE Commun. Lett.*, vol. 10, pp. 25–27, Jan. 2006.

[10] X. Gong, C. Zhao, W. Xu, and M. Jiang, "Power delay profile estimation for MIMO-OFDM systems over time-varying multipath channels," in *Proc. IEEE ICCT 2010*.

[11] K. C. Hung and D. W. Lin, "Pilot-based LMMSE channel estimation for OFDM systems with power-delay profile approximation," *IEEE Trans. Veh. Technol.*, vol. 59, pp. 150–159, Jan. 2010.

[12] Y. Li, J. H.Winters, and N. R. Sollenberger, "MIMO-OFDM for wireless communications: signal detection with enhanced channel estimation," *IEEE Trans. Commun.*, vol. 50, pp. 1471–1477, Sep. 2002.

[13] R. Horn and C. Johnson, *Matrix Analysis*. Cambridge University Press, 2009.

[14] TS 36.101, "User equipment (UE) radio transmission and reception," in 3GPP TSG RAN1, Rev. 10.1.0, Jan. 2011.