

Analysis of Steady State Heat Conduction through Tapered Section

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Abstract— It is very difficult to analyze and calculate the thermal behavior of tapered section material. The thermal behavior involves the study of heat transfer rate and heat flux. The tapered section has importance as support or flanges in boilers, chimneys at the lower end. In most of the industries their involvement in the heat loss is considered to be negligible, but it is not so as it appeared to be, so through this paper we analyze the thermal behavior of different material of tapered section according to their thermal conductivity to withstand on the particular heat production without any failure. For finding heat flow rate through the section, we are using Fourier law and comparing the result with the Solid work (simulation) 2011 Software and Finally, the variation of heat flow rate with respect to other factors.

Keywords— Fourier law, Thermal Conductivity, Tapered Section, Heat Flux, Steady State

1. INTRODUCTION

1.1 Heat Transfer

It is a form of energy which occurs due to temperature difference. It is exchange of thermal energy between physical systems. These objects could be two solids, a solid and a liquid or gas, or even within a liquid or gas. The direction of heat transfer is from a region of high temperature to another region of lower temperature, and is governed by the Second Law of Thermodynamics. Heat transfer changes the internal energy of the systems from which and to which the energy is transferred. Heat transfer will occur in a direction that increases the entropy of the collection of systems. The unit of heat transfer rate is J/s or watt. The amount of internal energy changed ΔU during Δt time interval and let Q amount of heat decreases.

$$\Delta U = \int_0^{\Delta t} Q dt \quad \text{----- (1)}$$

Then heat flux is defined as rate of heat transfer per unit area.

$$q = \frac{Q}{A} \quad \text{----- (2)}$$

1.2 Steady and Unsteady Heat Transfer

There are two modes of heat transfer steady and unsteady state. In case of steady state heat transfer, the temperature within the system does not change with time and temperature, it is a function of space coordinates only, but it is independent of time.

Mathematically it can be expressed as,

$$T = f(x, y, z)$$

Under unsteady state, temperature varies with time. Unsteady state conditions are precursor to steady state conditions.

1.3 Conduction Heat Transfer

It is one of the most fundamental modes of heat transfer which occurs due to microscopic collision of particles and movement of electrons with in a body also known to be as internal energy. It is also called as diffusion as shown in Fig1. The rate at which energy is transferred is directly dependent on the temperature difference. Conduction is mediated by the combination of vibrations and collisions of molecules, of propagation and collisions of phonons, and of diffusion and collisions of free electrons.

1.4 Conduction Rate Equation

As we just now see that conduction is the heat transfer from one to other end. For one dimensional, heat conduction through an area A can be expressed as,

$$Q = -KA \frac{dt}{dx} \quad \text{----- (3)}$$

Where,

Q Is the heat transfer rate in J/s

K Is the thermal conductivity unit $\frac{W}{(m.k)}$

A Is the area of cross section in m^2

$\frac{dt}{dx}$ Is the temperature gradient.

List of commonly used materials and their thermal conductivity

Materials	Thermal conductivity
1.Plain carbon steel	43
2.cast carbon steel	37
3.Cast alloy steel	38
4.Grey Cast iron	45

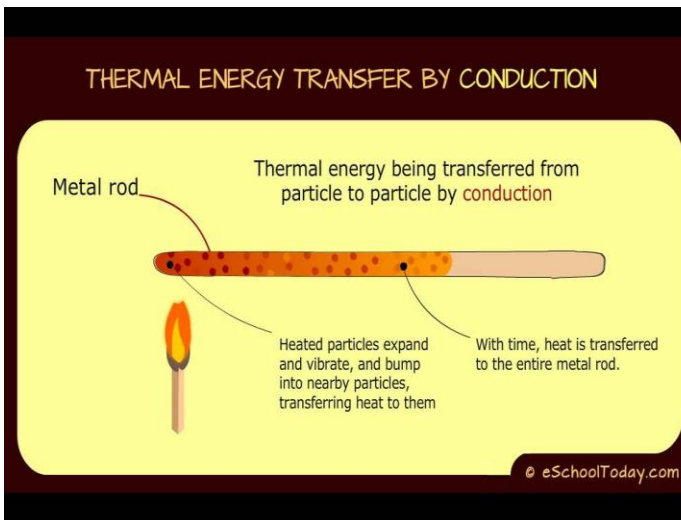


Figure 1

2. RELATED WORK

Variety of research is being conducted on, "To study the heat transfer through materials". The research papers dealing with the thermal analysis of materials in industries have been studied. Some of the research paper reviews are given below H. Baig and M. A. Antar explains the study on Natural convection analysis of heat transfer across multi-layer building blocks. Their study is mostly is used in construction of thermal power plants and in chimneys to prevent from the material failure.

3. MATHEMATICAL ANALYSIS

3.1 Deriving the result.

As we have discussed above the Fourier law of heat conduction in equation (3) now we apply this law in case of tapered section.

Consider a tapered section as shown in the figure 2

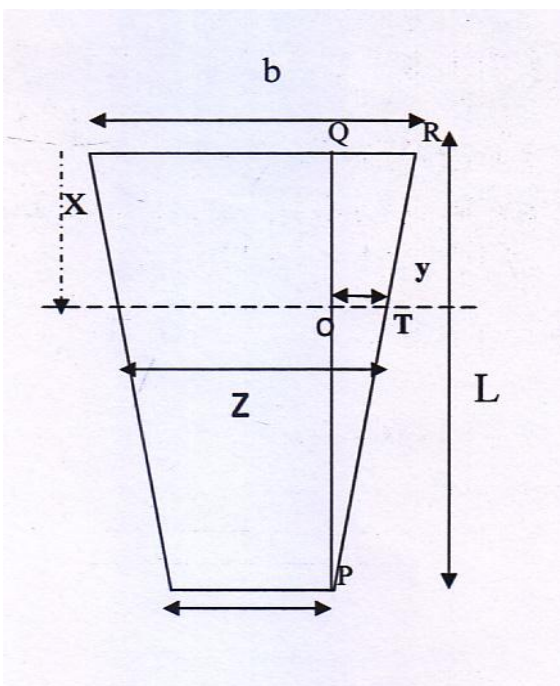


Figure 2

Consider a plane at a distance X from side of width b. Now we are interested to calculate the value of y as shown in the figure. From the similarity of triangles in ΔPQR and ΔPOT , we can say that

$$\frac{PQ}{QR} = \frac{PO}{OT}$$

$$\frac{L}{(b-a)} = \frac{L-X}{y}$$

$$y = \frac{(L-x).(b-a)}{2L} \text{ ----- (4)}$$

So from equation (4) we can find the whole length z as

$$z = a + 2y$$

In order to check this we can use the boundary conditions as

- (i) $X=0$ we get $Z=b$
- (ii) $X=L$ we get $Z=a$

Now, we are using the Fourier law in our case,

$$\frac{Q}{KA} = -\frac{dt}{dx}$$

$$\frac{Q}{K(z.m)} = -\frac{dt}{dx}$$

Where m is the thickness

$$\frac{Q}{K.m[a + \frac{(L-X)(b-a)}{L}]} = -\frac{dt}{dx} \text{ --- (5)}$$

Through the relation (5) we can find heat transfer rate at any point with in the section as shown in the figure 2.

3.2 Assumptions

- The flow of heat to the ground and sided faces due to conduction are considered to be negligible to make the calculation easier.
- The radiation happens is to be negligible as the value of Stefan Boltzmann constant is 5.688×10^{-8} . If we multiply this factor with temperature difference which is in the order of hundreds gives a very less value.
- The convective heat transfer is not considered here. In order to involve it we just need to add the term $hA\Delta T$ in equation (5).
- The trapezium section is considered to be symmetrical about central axis.

4. EXPERIMENTAL AND SOLID WORK SIMULATION RESULT

4.1 Temperature distribution

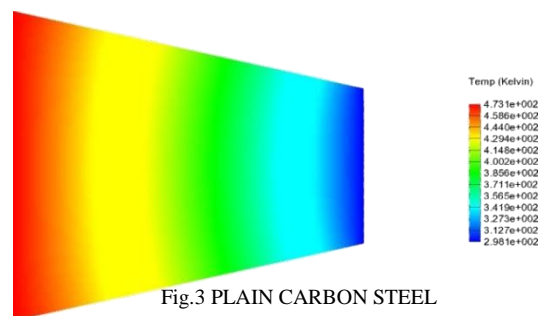


Fig.3 PLAIN CARBON STEEL

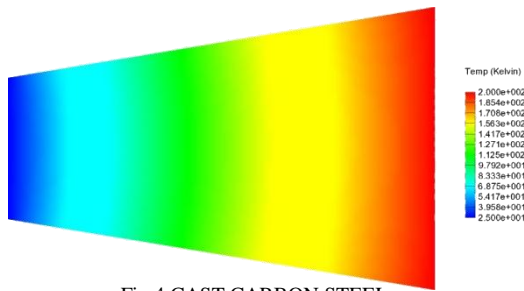


Fig.4 CAST CARBON STEEL

Fig.3 and Fig.4 clearly shows the temperature distribution at 473 K in the furnace and 293K outside that is surrounding temperature. The extreme red part shows the maximum temperature (473K) and blue part shows minimum temperature (293K).

4.2 Graphical Result

The variations of Heat flux at different nodes are shown below and its variation with respect to different axis and thickness can be explained. Here we can easily understand that our assumption the flow of heat through furnace to the ground Y and sides of the support Z is very less in comparison to the direction of X.

Node	X (mm)	Y (mm)	Z (mm)	HEAT FLUX (Cal/(s-cm ²))
2789	67.5555	-20.3729	9.21875	1.14
328	68.663	-20.0688	8.4375	1.10
362	68.663	-20.0688	5.9375	1.10
2927	68.663	-20.0688	4.21875	1.10
2694	69.7704	-19.7648	3.75	1.08
2371	68.663	20.0688	4.21875	-1.10
169	68.663	20.0688	8.4375	-1.12
151	68.663	20.0688	2.5	-1.18
201	68.663	20.0688	5.9375	-

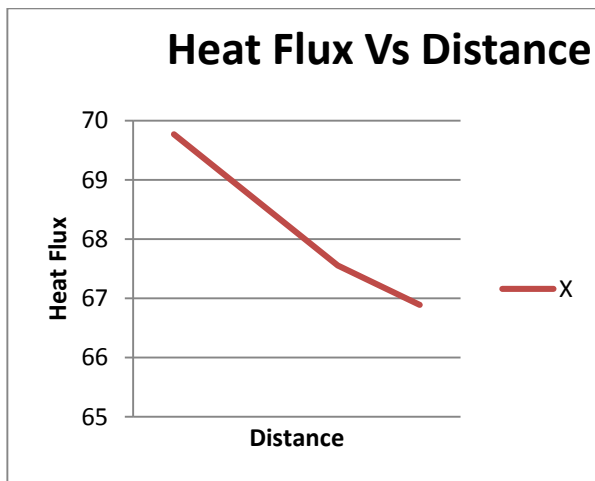


Figure 5

The above graph shows variation of heat flux in x direction as the distance increases Heat flux decreases. This variation is for the material of plain carbon steel.

5. CONCLUSION

Now from the equation derived equation number 5, it can be easily predicted that

$$Q \propto (L - X)$$

That means Q decreases with increase of X which matches with graph shown above. Hence the relation, predicted is correct.

6. FUTURE WORK

To analyze the heat flow through the pipes or boilers of different material in ANSYS and concluding some result from it.

7. REFERENCES

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