Analytical And Finite Element Comparison Of Magnetic Flux Density And Magnetic Field Intensity Of Permanent Magnet

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Abstract

The paper presents calculation the magnetic field and magnetic field intensity of permanent magnet. Consider permanent magnet homogeneously magnetized in Z-axis. In charge method the magnet is reduced to distribution of equivalent charge. Analytical calculation based upon charge method is compared with results obtained in COMSOL MULTYPHYSICS. Magnetic flux density distributions of permanent magnet are also shown in thepaper.

To determine the magnetic field components in vicinity of permanent magnets, itstarts from supposition that magnetization of permanent, M, magnet is known. Thefollowing methods are useful in practical calculation:

- Method based on determining distribution of microscopic Ampere's current or current method;
- Method based on determining magnetic scalar potential or charge method.

Keywords: *Permanent Magnet, Charge Method, Finite Element Method*

1. Analytical Method

Derivation of Magnetic field density from charge method and current method is shown below.

a) CURRENT METHOD:

Now, consider a stationary, homogeneous and isotropic material with a linear constitutive relation $B = \mu_0 (H + M)$. Introduce the vector potential A in equation (1).

$$\nabla . B = 0 \Rightarrow B = \nabla \times A \text{...... (2)}$$

Substitute this into Eq. (2), taking into account the constitutive relation $B = \mu_0 (H + M)$ .These yields

$$\nabla^2 A - \nabla(\nabla . A) = -\mu(J + \nabla \times M) \text{............ (3)}$$

Next, impose the coulomb gauge condition $\nabla . A = 0$ and obtain

$$\nabla^2 A = -\mu_0 (J + \nabla \times M) \text{.................. (4)}$$

If there is no free current (J=0), and if we assume an infinite homogeneous material (no boundaries), then the solution to Eq. (4) can be written in integral form using the free-space Green's function. For operator ∇^2 the green function can be written as,

$$G(x, x') = -\frac{1}{4\pi} \times \frac{1}{|x - x'|}$$

Thus, magnetic vector potential can be written as

$$A(x) = \frac{\mu_0}{4\pi} \int \frac{J_m(x')}{|x - x'|} dv' \text{........ (5)}$$

Now $B = \nabla \times A$ and obtain

$$\nabla \times \frac{J_m(x')}{|x - x'|} = -J_m(x') \times \nabla \frac{1}{|x - x'|}$$

And,

$$\nabla \frac{1}{|x - x'|} = -\frac{(x - x')}{|x - x'|^3}$$

Where, x is the observation point and x' is the source point, ∇ indicates differentiation with respect to the unprimed variables.

So,

$$B(x) = \frac{\mu_0}{4\pi} \int J_m(x') \times \frac{(x - x')}{|x - x'|^3} dv' \text{.............. (6)}$$

If the magnetization **M** is confined to a volume *V* (of permeability μ_0), and falls abruptly to zero outside of *V*, then Eqs. (5) and (6) reduce to

$$A(x)=\frac{\mu_0}{4\pi}\int_v \frac{J_m(x')}{|x-x'|}dv' + \frac{\mu_0}{4\pi}\oint_s \frac{j_m(x')}{|x-x'|}ds' \,..\, (7)$$

And

$$B(x)=\frac{\mu_0}{4\pi}\int_v J_m(x')\times\frac{(x-x')}{|x-x'|^3}dv' + \frac{\mu_0}{4\pi}\oint_s j_m(x')\times\frac{(x-x')}{|x-x'|^3}ds' \,..(8)$$

In these expressions *S* is the surface of the magnet, and $\mathbf{J_m}$ and $\mathbf{j_m}$ are equivalent volume and surface current densities. These are defined in the following:

$$J_m = \nabla \times M \quad \ldots\ldots\ldots\ldots\ldots\ (9)$$
$$j_m = M \times \hat{n}$$

b) CHARGE METHOD:

The derivation of the charge model is as follows: Start with the magnetostatic field equations for current-free regions $\nabla \times H = 0$ and $\nabla . B = 0$. Now, introduce a scalar potential φ_m as,

$$H = -\nabla \varphi_m \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.....\ (10)$$

Finally, substitute Eq. (10) and constitutive relation $B = \mu_0(H + M)$ into $\nabla . B = 0$ and obtain

$$\nabla^2 \varphi_m = \nabla . M \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (11)$$

If there is no free current (J=0), and if we assume an infinite homogeneous material (no boundaries), then the solution to Eq. (11) can be written in integral form using the free-space Green's function.

$$\varphi_m(x) = \int G(x,x')\nabla'.M(x')dv'$$
$$= -\frac{1}{4\pi}\int \frac{\nabla'.M(x')}{|x-x'|}dv' \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ (12)$$

Where, x is the observation point and x' is the source point, ∇' indicates differentiation with respect to the unprimed variables, and the integration is over the volume for which the magnetization exists. If the magnetization **M** is confined to a volume *V* (of permeability μ_0), and falls abruptly to zero outside of *V*, then Eqs. (12) becomes

$$\varphi_m(x) = -\frac{1}{4\pi}\int_v \frac{\nabla' M(x')}{|x-x'|}dv' + \frac{1}{4\pi}\oint_s \frac{M(x').\hat{n}}{|x-x'|}ds' \ \ldots\ (13)$$

Where, S is the surface that bounds V, and $\hat{n}$ is the outward unit normal to S. The form of Eq. (13) suggests the definitions of volume and surface charge densities given in Eq. (14):

$$\rho_m = -\nabla . M \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ldots\ldots\ (14)$$
$$\sigma_m = M . \hat{n}$$

From Eq. (13) and (14) magnetic flux density can be written as

$$B(x) = \frac{\mu_0}{4\pi}\int_v \frac{\rho_m(x')(x-x')}{|x-x'|^3}dv' + \frac{\mu_0}{4\pi}\oint_s \frac{\sigma_m(x')(x-x')}{|x-x'|^3}ds' \ \ldots..\ (15)$$

Now, the magnetic field above a rectangular magnet is find out using charge method is depicted below. The magnet is polarized along its axis with uniform magnetization as shown in figure 1. Assume that magnetization is

$$M = M_s \hat{z} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (16)$$

First determine charge densities. From Eq. (14)

$$\rho_m = -\nabla . M = 0$$

To evaluate σ_m first find surface normals

$n = \hat{z}$	z=0
$= -\hat{z}$	z=-L
$= \pm \hat{x}$	x= $\pm$ b
$= \pm \hat{y}$	y= $\pm$ a

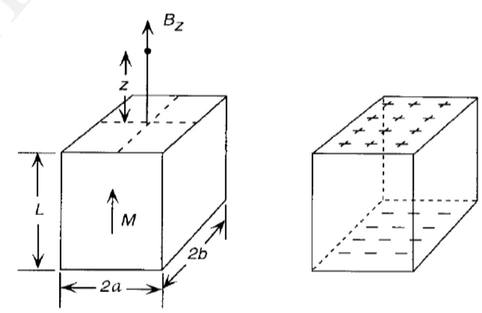


Figure 1: Rectangular bar magnet, (a) physical magnet; and (b) equivalent charge.

From Eq. (16) and the fact that $\sigma_m = M . \hat{n}$. For the top surface $\sigma_m = M$ (z=0), and $\sigma_m = M_s$ for the bottom surface (z=L).

Thus, from Eq. (15)

$$B_z(z) = \frac{\mu_0}{4\pi}\int_{-a}^{a}\int_{-b}^{b} \frac{M_s z}{\left[x'^2 + y'^2 + z'^2\right]^{3/2}}dx'dy' \ \ldots\ (17)$$

Here, x=z $\hat{z}$ and x'=x' $\hat{x}$ +y' $\hat{y}$.

The integrand is even function of x' and y', and therefore

$$B_z(z) = \frac{\mu_0}{4\pi}\int_0^a\int_0^b \frac{M_s z}{\left[x'^2 + y'^2 + z'^2\right]^{3/2}} dx'dy'$$

$$= \frac{\mu_0 M_s z}{\pi}\int_0^a \frac{b}{(y'^2 + z'^2)\sqrt{b^2 + y'^2 + z^2}} dy'$$

$$= \frac{\mu_0 M_s}{\pi}\tan^{-1}\left[\frac{by}{z\sqrt{b^2 + y'^2 + z^2}}\right]_0^a$$

$$= \frac{\mu_0 M_s}{\pi}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{z\sqrt{b^2 + y^2 + z^2}}{ab}\right)\right]$$

In the last step $\tan^{-1}(x) + \tan^{-1}(1/x) = \pi/2$. This is the field due to the top surface. A similar analysis appliesto bottom surface with z replaced by z+L.

The total field is given by

$$B_z(z) = \frac{\mu_0 M_s}{\pi}\left[\tan^{-1}\left(\frac{(z+L)\sqrt{b^2 + y^2 + (z+L)^2}}{ab}\right) - \tan^{-1}\left(\frac{z\sqrt{a^2 + b^2 + c^2}}{ab}\right)\right].$$

(18)

Here, M_s is the saturation magnetization of magnet.

This is required equation to find out magnetic flux density outside the permanent magnet along z-axis.

2. Comparison of results of charge model with COMSOL:

The results of equation (18) are compared with COMSOL. The permanent magnet properties and dimensions are given below.

a) Magnet Properties:

Rare earth permanent magnet is used as source for damping. The damping coefficient is directly square proportion to the magnitude of flux density. The comparison of rare-earth permanent magnet materials are shown in Table-1.

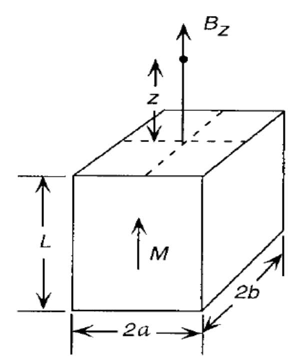
Material	B_r(T)	H_{ci}(kA/m)	BH_{max}(kJ/m^3)	T_c(°C)
Ferrite	0.42	242	33.4	450
$Alnico_9$	1.10	145	75.0	850
$SmCo_5$	1.00	696	196	700
$Nd_2Fe_{14}B$	1.23	947	278	300

Table-1: Permanent magnet materials

MAGNETIC PROPERTY	Sign	Unit	Nom.	Min.
Saturation Magnetization	M_{sat}	A/m	9.70e5	9.70e5
Residual Induction	B_r	kG	12.3	11.9
		mT	1230	1190
Coercive Force	H_{ci}	kOe	11.9	11.3
		kA/m	947	899
intrinsic Coercive Force	H_{ci}	kOe	19	17
		kA/m	1512	1353
Max. Energy Product	$(BH)_{max}$	MGOe	35	33
		kJ/m^3	278	263

Table 2: Properties of NdFeB Magnet

b) Magnet dimensions:



HEIGHT	L	0.012m
LENGTH	b	0.025m
WIDTH	a	0.0125m

Figure 2: Rectangular bar magnet, (a) physical magnet dimensions

3. Results and Conclusion:

Permanent magnet, homogeneously magnetized in known direction is applied to magnet. Method that is used for magnetic field determination is based on superposition of results that are obtained for elementary magnetic dipoles. The tables with magnetic field values, in different points, in vicinity of permanent magnet, are shown. Results obtained by analytical method are satisfactory confirmed using program packet COMSOL MULTIPHYSICS.

DISTA NCE	MAGNET IZATION (M_{sat}) in (A/m)	MAGNETIC FIELD DENSITY(Bz) in TESLA		
Z	Br	COMSOL	CHARGE MODEL	%ER ROR
5	970000	0.24688	0.2461638	0.29
10	970000	0.1676	0.1678099	0.13
15	970000	0.11629	0.1135211	2.38
20	970000	0.07777	0.078147	0.48
25	970000	0.05732	0.0551104	3.85
30	970000	0.03953	0.0398413	0.79
35	970000	0.02958	0.0294894	0.31
40	970000	0.02282	0.0223052	2.26
50	970000	0.01354	0.0135072	0.24
100	970000	0.00228	0.0023047	1.08

Table 3: Analytical and COMSOL result comparison of magnetic flux density

DISTAN CE	MAGNETIC FIELD INTENSITY(H) in A/m		
Z	COMSO L	CHARGE MODEL	%ERRO R
5	196460	195990.3	0.24
10	133371	133606.6	0.18
15	92543.83	90383	2.33
20	61890.9	62218.97	0.53
25	45615.69	43877.7	3.81
30	31454.79	31720.81	0.85
35	23537.94	23478.86	0.25
40	18155.6	17758.9	2.19
50	10775.14	10754.16	0.19

4.References

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