# Analytical Models for the Prediction of Compressive Strength of Lightweight Aggregate Concrete.

## Sule, Samuel

### Faculty of Infrastructural Systems Engineering, University of Port Harcourt, P.M.B. 5323, Rivers State, Nigeria.

#### Abstract

The quality of concrete used in any construction work is a function of its compressive strength. In this paper, analytical models are formulated to predict the compressive strength of lightweight aggregate concrete using the concept of wave propagation in a prismatic rod. The results obtained from the analytical models were compared with those of the measured values obtained from literature which served as the exact values and were found to agree closely. For both the measured and the predicted values, the compressive strength for all the mixes was found to decrease with increase in water-cement ratio and increase with increase in days. The correlation coefficient between the measured and the predicted values was found to be 0.9672 showing the efficiency of the formulated model in the prediction of compressive strength of lightweight aggregate concrete.

Keywords: Analytical models, compressive strength, lightweight aggregate concrete, wave propagation, prismatic rod.

#### 1. Introduction

The use of lightweight aggregate concrete as a structural material is of paramount importance in the construction industry compared to the use of conventional concrete. Lightweight aggregate concrete is a low-density structural material that offers a lot of advantages in the building and construction industries. For instance, it has low thermal conductivity, it is economical both in terms of haulage and handling costs. Lightweight aggregate concrete when used in structural floors reduces the self-weight of reinforced concrete floors.

The evaluation of compressive strength of lightweight aggregate concrete is a task of paramount importance in both the building and construction industries as it is a good indicator of concrete quality [1-6]. Experimental results of the compressive strength of lightweight aggregate concrete are not sufficient to evaluate the quality of concrete as the results are only empirical [7-9].

The production of concrete for building and other civil engineering works is always a difficult task both to civil engineers and project owners due to increasing cost of aggregate as well as the difficulty in locating other materials to serve as substitute materials for civil engineering construction. As a result, the rate of consumption of aggregates in building and other civil engineering works is increasing. According to Kamang and Bingila [10], it is now possible to produce aggregates from almost anything provided it possesses attributes that can be tailored so as to produce the desirable properties in concrete. Lightweight aggregate concrete is indeed a promising structural material. However, the failure which occurs in the concrete made from it may be traceable to the missing knowledge concerning the compressive strength of the concrete made from it. To overcoming this shortcoming, analytical models were formulated to predict the

(4)

compressive strength of lightweight aggregate concrete. It is believed that research into the prediction of compressive strength of lightweight aggregate concrete will help in optimum utilization of lightweight aggregate in concrete production. The formulated models are simple and straightforward and not mathematically cumbersome.

# 2. Development of Analytical Models

Using the concept of wave propagation in an elastic medium, the dynamic modulus of elasticity is given by:

$$E_{d} = V^{2} \frac{(1+\nu)(1-2\nu)}{(1-\nu)} \rho$$
(1)

Where:  $V^2$  = square of velocity of wave in concrete as an elastic medium.

Putting v = poisson's ratio = 0.20 for concrete loaded in compression, equation (1) transform to:

$$E_d = 0.90 \rho V^2 \tag{2}$$

According to Reynolds and Steedman, the volume fraction of cement in the concrete paste using principle of absolute volume is given by:

$$\lambda_{fc} = \left(\frac{c}{c+x+a}\right)^2 \tag{3}$$

Where

 $V_c = c + x + a$ 

c, x and a are cement, water and air content respectively.

= Total volume of concrete.

The concrete compressive strength is a function of cement, water and air content respectively. In compact form, the compressive strength is given as:

$$\sigma_c = f(c, x, a) \tag{5}$$

Dividing both top and bottom of equation (3) by c transforms equation (3) to:

$$\lambda_{fc} = \left(\frac{\frac{c}{c}}{\frac{c}{c} + \frac{x}{c} + \frac{a}{c}}\right)^2 \tag{6}$$

For fully compacted concrete, air content is assumed to be zero.

$$\Rightarrow a = \frac{a}{c} = 0 \quad \text{and} \tag{7}$$

equation (6) transforms to:

$$\lambda_{fc} = \left(\frac{1}{1 + \frac{x}{c}}\right)^2 \tag{8}$$

The higher the modulus of elasticity, the stronger the concrete. Therefore, multiplying equation (8) by the dynamic modulus of elasticity of concrete gives the compressive strength of concrete a particular water-cement ratio as:

$$\sigma_c = E_d \left( \frac{1}{1 + \frac{x}{c}} \right) \tag{9}$$

Substituting for  $E_d$  in equation (9) transforms equation (9) to:

$$\sigma_c = 0.90 \rho V C^2 \left( \frac{1}{1 + \frac{x}{c}} \right)$$
(10)

Concrete develops strength with continued hydration. Therefore, an age factor,  $\alpha$  is incorporated in the compressive strength equation to cater for increase in strength of concrete with age.

$$\alpha = 1$$
 for concrete of age 28 days [3]

and  $\alpha = 1.16$  for 90 days concrete [3]

Incorporating the effect of age on the compressive strength of concrete transforms equation (10) to:

$$\sigma_c = 0.9\alpha\rho V^2 \left(\frac{1}{1+\frac{x}{c}}\right) \tag{11}$$

Equation (11) is used to predict the compressive strength of lightweight aggregate concrete at a particular water cement ratio.

Table 1: Comparison of results of the analytical model with the measured values.

Water cement rat	tio = 0.40			
Compressive Strength	7days	14days	28days	90days
$\sigma_c (N/mm^2)$				
Expt. [ 10]	13.70	14.80	16.61	23.90
Analytical model	13.82	16.32	16.42	22.08

Concrete mix = 1:1.6:2.4

Table 2: Comparison of results of the analytical model with the measured values. Concrete mix = 1:1.6:2.4

Water cement ratio = 0.50

Compressive Strength	7days	14days	28days	90days
$\sigma_{c} (N/mm^{2})$				
Expt. [ 10]	8.30	10.00	10.50	15.0
Analytical model	9.46	10.86	14.65	18.22

Table 3: Comparison of results of the analytical model with the measured values. Concrete mix = 1:1.6:2.4

Water	cement rat	tio = 0.60	
nnragina	Strongth	7 dave	1

Compressive Strength	7days	14days	28days	90days
$\sigma_{c}(N/mm^{2})$			7	
Expt. [10]	8.10	9.20	10.0	15.90
Analytical model	7.56	9.17	11.66	14.65

Table 4: Comparison of results of the analytical model with the measured values. Concrete mix = 1:1.6:2.4

Water cement ratio = 0.70

Compressive Strength	7days	14days	28days	90days
$\sigma_c (N/mm^2)$				
Expt. [ 10]	6.6	6.6	8.3	9.10
Analytical model	5.4	6.71	9.96	12.20

Table 5: Comparison of results of the analytical model with the measured values. Concrete mix = 1:2:2

Water cement ratio = 0.4

Compressive Strength	7days	14days	28days	90days
$\sigma_{c}(N/mm^{2})$				
Expt. [ 10]	10.50	13.1	14.8	21.1
Analytical model	11.67	15.86	18.0	22.4

Table 6: Comparison of results of the analytical model with the measured values. Concrete mix = 1:2:2

water cement ra	10 - 0.30			
Compressive Strength	7days	14days	28days	90days
$\sigma_{c}(N/mm^{2})$				
Expt. [10]	9.7	12.0	13.50	18.50
Analytical model	8.93	12.06	13.24	17.4

Water computing -0.50

Table 7: Comparison of results of the analytical model with the measured values. Concrete mix = 1:2:2

Water cement ratio = 0.60

Compressive Strength	7days	14days	28days	90days
$\sigma_{c} (N/mm^{2})$				
Expt. [ 10]	7.2	8.9	9.80	15.8
Analytical model	7.41	9.17	11.19	14.11

Table 8: Comparison of results of the analytical model with the measured values. Concrete mix = 1:2:2

i atom contone ratio $= 0.7$	Water	cement	ratio	= 0.7
------------------------------	-------	--------	-------	-------

water cement ra	10 = 0.7			
Compressive Strength	7days	14days	28days	90days
$\sigma_{c}(N/mm^{2})$				
Expt. [10]	5.2	6.9	8.30	13.0
Analytical model	5.36	7.92	9.40	11.96

Table 9: Comparison of results of the analytical model with the measured values. Concrete mix = 1:2.4:1.6

Water cement ratio = 0.4

Compressive Strength	7days	14days	28days	90days
$\sigma_c (N/mm^2)$				
Expt. [10]	13.20	13.5	14.01	19.20
Analytical model	11.32	15.2	18.3	23.79

Table 10: Comparison of results of the analytical model with the measured values. Concrete mix = 1:2.4:1.6

Concr	ctc mnx = 1.2.7.1	
Water	cement ratio $= 0$	0.50

Compressive Strength	7days	14days	28days	90days	
$\sigma_{c} (N/mm^{2})$					
Expt. [ 10]	9.6	10.15	11.10	17.60	
Analytical model	9.16	12.10	14.3	18.22	

Water cement ratio = 0.60Compressive Strength 14days 90days 7days 28days  $\sigma_c (N/mm^2)$ Expt. [10] 7.9 9.0 10.90 16.5 Analytical model 7.5 10.25 11.30 14.62

Table 11: Comparison of results of the analytical model with the measured values. Concrete mix = 1:2.4:1.6

Table 12: Comparison of results of the analytical model with the measured values. Concrete mix = 1:2.4:1.6Water cement ratio = 0.70

Compressive Strength	7days	14days	28days	90days
$\sigma_c (N/mm^2)$				
Expt. [ 10]	7.4	8.10	10.2	15.70
Analytical model	5.74	8.80	10.2	12.30

### 3. **Discussion of Result**

Both the experimental and predicted values of compressive strength agreed closely. From Tables 1 -12, it can be seen that both the experimental and predicted values at water-cement ratios of 0.4, 0.5, 0.6 and 0.7 agreed closely.

Results from Table 1 to 12 showed continued strength growth up to 90days for each mix proportion for both experimental and predicted values. In all, the mixes with 0.4 water-cement ratio showed the highest value-for the compressive strength for both the measured and the predicted values. For both the experimental and predicted values, the compressive strength for all concrete mixes decreased with increase in water-cement ratio. This decrease may be attributed to free water content of the fresh concrete which increased the volume of the pores left in the hardened concrete thereby reducing the gel - space ratio which contributes to the strength of hardened concrete.

### 4. **Conclusion**

Based on the comparison between the results of the formulated analytical model and experimental results the following conclusions are drawn.

(i) The proposed analytical model results showed close agreement with the experimental values showing that it can be used as a tool to predict concrete compressive strength from ultrasonic pulse velocity.

- (ii) For both the measured and predicted values, the compressive strength decreased with increase in water-cement ratio and increased with increase in concrete age or maturity.
- (iii) The formulated model can be used to predict the compressive strength of normal concrete.

#### References

- [1] Hong Guang N. and Wang J.Z., Prediction of Compressive Strength of Concrete by Neural Networks. Cem. and Concr. Research, Vol.30 No.8, 2000, pp. 1245 1250.
- [2] Neville, A.M., properties of concrete, second edition, Pitman Publishing Company, London, 1973.
- [3] Neville, A.M. and Brooks, J.J., Concrete Technology, ELBS edition, Longman, London, 1990.
- [4] Okafor, F.O. and Sule, S., Models for Prediction of Structural Properties of Palmnut Fibre - Reinforced Cement Mortar Composite. Nig. Journal of Technology, Vol. 27, No.2, pp. 13 – 20.
- [5] Okafor, F.O. and Sule, S., Improved Analytical Models for the Prediction of Structural Properties of Palmnut Fibre - Reinforced Cement Based Composites, Journal of Engineering Science and Applications (JESA), Volume 7, No.1, June 2010, pp. 1-10.
- [6] Okafor, S.O., Eze- Uzomaka, O.J. and Egbuniwe, N., Structural Properties and Optimum Mix Proportions of Palmnut Fibre Reinforced Mortar Composite. Cem. Concr. Res., Vol. 26, No.7, 1996, pp. 1045 1055.
- [7] Colak, A., A New Model for the Estimation of Compressive Strength of Portland Cement Concrete. Cem. & Concr. Research, Vol. 36, No.7, pp. 1409 -1413, 2006.
- [8] Bhanja and Sengupta B., Investigations on Compressive Strength of Silica Fume Concrete using Statistical Methods. Cem. and Concr. Research, Vol. 32, No.9, pp. 1391 – 1394, 2002.
- [9] Saridemir, M., Prediction of Compressive Strength of Concretes Containing Metakaolin and Silica Fume by neural networks. Advances in Engineering Softwares, Vol.40, No.5, pp. 350 -355, 2009.
- [10] Kamange, E.E.J., Bingila, M.I., Strength and Elastic properties of local plumier (Gaze Bathing) Stone Aggregate Concrete. Nigeria Journal of Construction Technology, vol.3, No.1, Department of Building, University of Jos, 2000, pp. 8-17.