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Application of Kalman Filter for Estimation of Aerodynamic Parameters of an Aircraft using Simulated Flight Data

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***Abstract*— An attempt is made to estimate aerodynamic parameters using the simulated flight data of various flight vehicles using Kalman filter technique. The present paper demonstrates application of Kalman filter for the estimation of ballistic coefficient of a falling body by considering the effects of process noise. As compared to rigid aircraft the mathematical model of a flexible aircraft involves larger number of stability derivatives. Furthermore, for a flexible aircraft, additional derivatives need to be included due to aero elastic effects. Applicability of Kalman filter for parameter estimation is validated on simulated flight data generated for a rigid aircraft as well as for a flexible aircraft. It is concluded that Extended Kalman Filter method can be advantageously applied to estimate aerodynamic parameters from flight data of a flexible aircraft.**

I. INTRODUCTION

All modern aerospace vehicles rely upon an understanding of dynamics and control to improve system performance. An understanding of dynamic elements and the trade-off between vehicle dynamic characteristics require for successful system design, control system properties and system performance. Aircraft parameter estimation is one of the most outstanding and illustrated example of the system identification methodologies. The success of the system identification of the flight vehicle has been possible due to better measurement techniques and data processing capabilities provided by digital computers. Other factors that contribute to system identification are the developments in the fields such as estimation and control theory; the design of appropriate flight test and well understood basic principles of aerodynamic modeling[1,2,3].. Kalman filter is a linear, optimal estimator of state variables of a linear, time-varying system, operating in a Gaussian stochastic environment. Filtering approach is an extension of Kalman filter. Optimal estimator here is referred to a computational algorithm that processes measurements to deduce a minimum error covariance of the state of a system combining all the information available. Generally, Kalman filter[4, 5] is applicable and optimal for linear systems only. When either the system or the measurement equations are non-linear, the same algorithm can still be applied by local linearization of the system about the current state. Such filter applied to nonlinear systems is called Extended Kalman Filter (EKF)[6], and it need not be optimal. EKF produces estimates of the parameters that approximately minimize the mean square error in the parameter estimates themselves as opposed to ML and least squares which minimize a cost function that is based on matching the output variable behavior given a specific input trajectory. Parameter estimation of an aircraft is done with many measurements like, acceleration (both linear and angular), angular orientation, speed, angle of attack, etc. But from cost effectiveness point of view, it may not be feasible to use many sensors for air borne vehicles that go through many development trials.

In the present work, EKF method is used for parameter estimation from flight data of air borne vehicles. The motivation here was to conduct a study on the applicability of the method in extracting aerodynamic parameters by processing flight data obtained for different class of flight vehicle/store. The method has been applied to flight data of a one-dimensional ballistic target, flight data of a rigid aircraft in longitudinal short period mode and also to flight data of a flexible aircraft in longitudinal short period mode. Any parameter estimation method requires adequate information about vehicle dynamics to estimate aerodynamic parameters correctly. Flight data of one-dimensional ballistic target contains very limited information regarding its motion/control variables. The flight data obtained through aircraft maneuvers contains more information regarding its motion and control variables. The applicability of the EKF is tested for these three different classes of flight data. It is observed that EKF can be advantageously applied on flight data of flexible aircraft to estimate few aerodynamic parameters with acceptable level of accuracy.

II. KALMAN FILTER

Kalman filter is an optimal recursive data processing algorithm and it incorporates all information that is available to the filter. It processes all available measurements, regardless of their precision, to estimate the current values of the variables of interest with use of

- knowledge of the system and measurement device dynamics
- the statistical description of the system noises, measurement errors, and uncertainty in the dynamic models

- any available information about initial conditions of the variables of interest

This filter performs the conditional probability density propagation for problems in which the system can be described through a linear model and in which system and measurement noises are white and Gaussian. The three basic assumptions in Kalman filter formulation are

- the model is considered to be linear .
- the measurement noise and system noise are white.
- the measurement noise and system noise are Gaussian.

The physical implications of these assumptions are discussed. A linear model is justifiable because when nonlinearities do exist, the typical engineering approach is to linearize about some nominal point or trajectory, achieving a perturbation model or error model.

"Whiteness" implies that the noise value is not correlated in time. The knowledge of the present noise value is no way helpful to predict the noise value of any other time. Whiteness also implies that the noise has equal power at all frequencies. A system will be driven by a wideband noise, noise having power at all frequencies above the system bandpass and essentially constant power at frequencies within the system bandpass. The assumption of noise as white would extend this constant power level out across all frequencies. Within the bandpass of the system of interest, the fictitious white noise looks identical to the real wideband noise. The mathematics involved in the filter is vastly simplified by replacing the real wideband noise with a white noise which from the system's "point of view" is identical. Gaussianness is related with amplitude whereas whiteness is related with frequency. The probability density of Gaussian noise amplitude takes on the shape of a normal bell -shaped curve. This assumption is justified physically by the fact that a system or measurement noise is typically caused by a number of small sources. Mathematically, when a number of independent random variables are added together, the summed effect can be described very closely by a Gaussian probability density, regardless of the shape of the individual densities. The first and second order statistics (mean and variance or standard deviation) of a noise process can be easily known. In the absence of any higher order statistics, there is no better form to assume than the Gaussian density. The first and second order statistics completely determine a Gaussian density, unlike most densities which require endless number of orders of statistics to specify their shape entirely. The Kalman filter, which propagates the first and second order statistics, includes all information contained in the conditional probability density.

The real world situation is described by a set of nonlinear differential equations to apply extended Kalman filter techniques. These equations are expressed in nonlinear state-space form as a set of first-order non linear differential equations as

$$\dot{x} = f(\vec{x}) + w \tag{1}$$

where '$\dot{x}$' represents system space, '$f(\vec{x})$' is a nonlinear function of those states and 'w' is a random zero-mean process. The continuous process-noise matrix describing the random process *w* for the preceding model is given by

$$[Q] = E\left(ww^T\right) \tag{2}$$

The measurement equation, required for the application of extended Kalman filtering, is considered to be a nonlinear function of the states according to the equation

$$\vec{z} = h(x) + v \tag{3}$$

where '*v*' is a zero-mean random process described by the measurement noise matrix [R], which is defined as

$$[R] = E\left(vv^T\right) \tag{4}$$

For systems in which the measurements are discrete, the nonlinear measurement equation is written as

$$\vec{z}_k = h(\vec{x}_k) + v_k \tag{5}$$

The discrete measurement noise matrix $[R]_k$ consists of a matrix of variances representing each measurement noise source. Since, the system equation, Eq. (1) and measurement equation, Eq (3) are nonlinear, a first order approximation is used in the continuous Riccati equations for the manipulation of systems dynamics matrix $[F]$ and the measurement matrix $[H]$. The matrices are related to the nonlinear system and measurement equations according to the relations

$$[F] = \left.\frac{\partial f(x)}{\partial x}\right|_{x=\hat{x}} \tag{6}$$

$$[H] = \left.\frac{\partial h(x)}{\partial x}\right|_{x=\hat{x}} \tag{7}$$

The fundamental matrix [ϕ_k], required for the discrete Ricatti equations, can be approximated by the Taylor-series expansion for exp($[F]$ T_s) and is given by the equation

$$[\phi]_k = [I] + [F]T_s + \frac{[F]^2 T_s^{\,2}}{2!} + \frac{[F]^3 T_s^{\,3}}{3!} + \ldots \tag{8}$$

where 'T_s' is the sampling time and $[I]$ is the identity matrix. In our applications of extended Kalman filtering, the series is approximated by only the first two terms, because $[\phi]_k$ is only used for the calculation of Kalman gains and the matrix may not necessarily improve the performance of the filter by considering more terms. Therefore $[\phi]_k$ is given by

$$[\phi]_k \approx [I] + [F]T_s \tag{9}$$

The matrix Ricatti equations, required for the computation of the Kalman gains, are given by the equations

$$[M]_k = [\phi]_k [P]_{k-1} [\phi]_k^T + [Q]_k \tag{10}$$

$$[K]_k = [M]_k [H]^T \left([H][M]_k [H]^T + [R]_k\right)^{-1} \tag{11}$$

$$[P]_k = \left([I] - [K]_k [H]\right)[M]_k \tag{12}$$

where $[P]_k$ is the covariance matrix representing errors in the state estimates after an update,$[K_k]$ is the Kalman gain and $[M]_k$ is the covariance matrix representing errors in the state estimates before an update. The discrete process-noise matrix $[Q]_k$ can be found from the continuous process-noise matrix according to equation

$$[Q]_k = \int_0^{T_s} [\phi(\tau)][Q][\phi(\tau)]^T \, d\tau \qquad (13)$$

The preceding approximations for the fundamental and measurement matrices are used in the computation of the Kalman gains. The new state estimate $\hat{x}_k$ is the old state estimate $\bar{x}_{k-1}$ projected forward to the new sampling $\bar{x}_k$ plus a gain times a residual. The residual is the difference between the actual measurement z_k and the nonlinear measurement $h(\bar{x}_k)$.

$$\hat{x}_k = \bar{x}_k + K_k[z_k - h(\bar{x}_k)] \qquad (14)$$

The old estimates that have to be propagated forward do not have to be done with the fundamental matrix but instead can be propagated directly integrating the actual nonlinear differential equations forward at each sampling interval. Euler integration is applied to the nonlinear system of differential equations and is given by the equation

$$\bar{x}_k = \hat{x}_{k-1} + \dot{\hat{x}}_{k-1} T_s \qquad (15)$$

where the derivative is obtained from

$$\dot{\hat{x}}_{k-1} = f(\hat{x}_{k-1}) \qquad (16)$$

In the preceding equation the sampling time Ts is used as an integration interval. In the problems where the sampling time is large, Ts would have to be replaced by a small integration interval, or possibly a more accurate method of integration has to be used.

III. GENERATION OF FLIGHT DATA

Due to the non-availability of real flight data, simulated flight data is generated for different class of flight vehicles for the purpose of parameter estimation. Mathematical models used to generate flight data of ballistic target (FD-BT), longitudinal motion of rigid aircraft (FD-RA/C) and longitudinal motion of flexible aircraft (FD-AE A/C1, FD-AE A/C2) are presented. For the generation of simulated flight data of ballistic target the acceleration equation governing the path of the trajectory was solved using Euler integration. In the case of aircraft, longitudinal equations of motion were solved using fourth-order Runge -Kutta method. Subsequently, the simulated flight-data of the air borne vehicles is used in Kalman filter algorithm and processed for the estimation of aerodynamic parameters. The equations of motion are needed for generating the simulated flight data and also in the Kalman filter algorithm for the propagation of states from one time step to the other.

IV. FLIGHT DATA OF BALLISTIC TARGET (FD-BT)[8]

Knowledge of target ballistic coefficient is used in advance guidance laws such as predictive guidance to relax the interceptor acceleration requirements. In addition, knowledge of the target ballistic coefficient is required for fire control due to the importance of accurate intercept point predictions in launching the interceptor on a collision course. Therefore, accurate estimation of ballistic coefficient of a target re-entering the atmosphere is very important for both guidance and fire control purposes. The flight data for simulating such a vehicle motion is modeled to investigate the applicability of EKF method[8] in extracting parameter (ballistic coefficient) from flight data.

The one-dimensional example of a ballistic target falling on tracking radar is considered. The target was initially at 2, 00,000 ft above the radar and had a velocity of 6000 ft/s towards the radar, which is located in the surface of a flat Earth. The trajectory of the ballistic target is presented in Fig. 18. The radar measures the altitude of the target with 25-ft standard deviation measurement accuracy. The radar picks measurements for every 0.1-sec. The simulation is done for 30 sec. An extended Kalman filter is built to estimate the altitude, velocity and ballistic coefficient.

The two forces acting on the object are the drag force and the gravity force. The equation that governs the motion of the object is given by

$$\ddot{x} = \frac{Q_p g_B}{\beta} - g_B \qquad (17)$$

where '$\ddot{x}$' is the acceleration acting on the object ,'g_B' is the acceleration due to gravity and 'Qp' the dynamic pressure and is given by the equation

$$Q_p = 0.5\hat{\rho}_B \dot{x}^2 \qquad (18)$$

where '$\dot{x}$' is the velocity of the target and the air density 'ρ_B' in Eq. (18) is an exponential function of altitude and is given by the equation

$$\hat{\rho}_B = 0.0034 e^{-x/22000} \qquad (19)$$

The term 'β' in Eq. (17) is the ballistic coefficient and is expressed as

$$\beta = \frac{W_B}{S_{ref} C_{D0}} \qquad (20)$$

where 'W_B' is the weight of the ballistic target, 'S_{ref}' is the reference area of the object and 'C_{D0}' is the coefficient of drag. The acceleration acting on the object is expressed as the nonlinear second order differential equation as

$$\ddot{x} = \frac{0.0034 g_B e^{-x/22000}}{2\beta} - g_B \qquad (21)$$

V. Simulated Flight Data of Rigid Aircraft (FD-RA/C)

In most cases, longitudinal maneuvers predominantly excite the short period mode and not the phugoid mode. In short period mode the flight velocity is essentially constant during the maneuver. 3-2-1-1 control (elevator) input is considered for the excitation of short period mode. The example aircraft is chosen for the present study. Trim flight conditions correspond to straight and level cruise flight at an altitude of 1500m and at a Mach number of 0.6. The rigid body short period longitudinal response to a given elevator input was simulated for 8 seconds. The short period longitudinal response was simulated using the following equations.

$$\dot{\alpha} - q = -\rho u S C_L / 2m \qquad (22)$$

$$\dot{q} = \rho u^2 S c C_m / 2I_Y \qquad (23)$$

The equations that define coefficient of lift (C_L) and pitching moment coefficient (C_m) in Eq. (22) and Eq. (23) to describe the aerodynamic model are presented in Eq. (24) and Eq. (25)

$$C_L = C_{L0} + C_{L\alpha}\alpha + C_{Lq}\frac{q\bar{c}}{2u} + C_{L\delta_e}\delta_e \qquad (24)$$

$$C_m = C_{m0} + C_{m\alpha}\alpha + C_{mq}\frac{q\bar{c}}{2u} + C_{m\delta_e}\delta_e \qquad (25)$$

The pitching moment coefficient (C_m) is with reference to the center of gravity. The Eq. (4) and Eq. (5) were solved using fourth-order Runge-Kutta algorithm with a time step of 0.001 sec to obtain simulated flight data. The contribution due to aeroelastic effect was neglected for rigid aircraft case. The flight data used for parameter estimation is pictorially presented in Fig. 1 and Fig. 2. This flight data will be referred to as FD-RA/C for further purpose. The elevator input (3-2-1-1) used to generate this flight data is shown in Fig. 3.

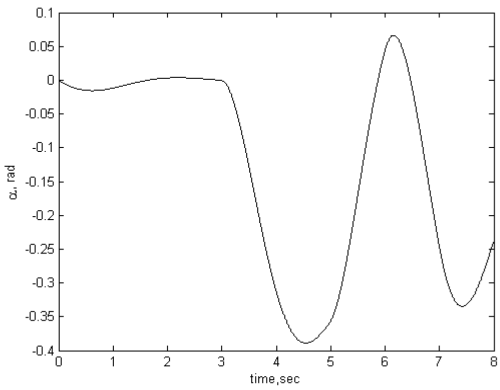

Fig. 1 Simulated α response from the flight data FD-RA/C

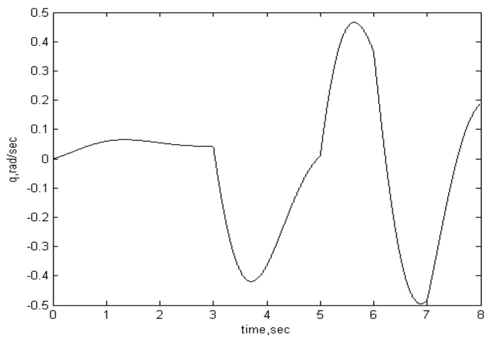

Fig. 2 Simulated q – response from flight data FD-RA/C

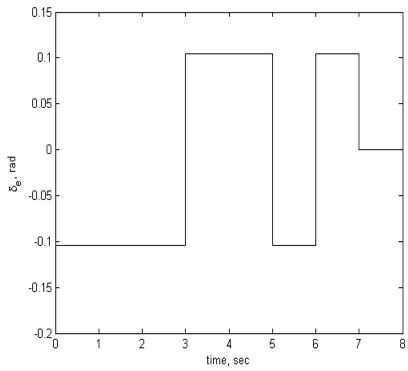

Fig. 3. 3-2-1-1 Elevator Control Input,

$$\delta_{\max} = 0.10471\ rad$$

VI. Simulated Flight Data of Flexible Aircraft FD-AE A/C1 & FD-AE A/C2)

The non linear equations of motion for a flexible aircraft are considered which are obtained from Ref. 7.

$$\dot{\alpha} - q = -\rho u S/2m\left[C_{L0} + C_{L\alpha}\alpha + C_{Lq}\frac{q\bar{c}}{2V} + C_{L\delta_e}\delta_e + \sum_{i=1}^{n}\left(C_{L\eta_i}\eta_i + C_{L\dot{\eta}_i}\dot{\eta}_i c/2u\right)\right] \qquad (26)$$

$$\dot{q} = \rho u^2 S c/2I_Y\left[C_{m\alpha}\alpha + C_{mq}qc/2u + C_{m\delta_e}\delta_e + \sum_{i=1}^{n}\left(C_{m\eta_i}\eta_i + C_{m\dot{\eta}_i}\dot{\eta}_i c/2u\right)\right] \qquad (27)$$

where angle of attack α, pitch rate q and control input δ_e represent the perturbations and η_i and $\dot{\eta}_i$ are the generalized displacement coordinates and their derivatives. $C_{L\alpha}, C_{Lq}$, $C_{L\delta_e}$, $C_{m\alpha}$, C_{mq}, $C_{m\delta e}$, $C_{L\eta_i}$, $C_{L\dot{\eta}_i}$, $C_{m\eta_i}$ and $C_{m\dot{\eta}_i}$ are the aerodynamic coefficients as defined in Ref. 7. The second order differential equation that is satisfied by the generalized displacement coordinates and its derivatives is given in Eq. 28. The additional term $2\xi_i\omega_i\eta_i$ representing the structural damping is also included

$$\ddot{\eta}_i + 2\xi_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = \rho u^2 Sc/2M_i \left[C_\alpha^{\eta_i}\alpha + C_q^{\eta_i}\, qc/2u + C_\delta^{\eta_i}\delta + \sum_{j=1}^{n}\left(C_{\eta_j}^{\eta_i}\eta_j + C_{\dot{\eta}_j}^{\eta_{ii}}\, \eta_j c/2u\right)\right] \quad (28)$$

where ω_i, ξ_i and M_i are the in-vacuo frequency, modal damping, and modal generalized mass respectively and $C_\alpha^{\eta_i}$, $C_q^{\eta_i}$, $C_\delta^{\eta_i}$, $C_{\eta_j}^{\eta_i}$ and $C_{\dot{\eta}_j}^{\eta_{ii}}$ represent the generalized force derivatives due to coupling in elastic and aerodynamic degrees of freedom. Eq. (26), Eq. (27) and Eq. (28) are integrated to generate motion variables α and q. The elevator control input is 3-2-1-1 and is presented in Fig. 3. The aerodynamic model incorporating aero elastic effects is presented in Eq.(29) and Eq.(30)

$$C_L = C_{L0} + C_{L\alpha}\alpha + C_{Lq}\, qc/2u + C_{L\delta_e}\delta_e + \sum_{i=1}^{4}\left[C_{L\eta_i}\eta_i + \left(C_{L\dot{\eta}_i}\dot{\eta}_i\right)c/2u\right] \quad (29)$$

$$C_m = C_{m0} + C_{m\alpha}\alpha + C_{mq}\, qc/2u + C_{m\delta_e}\delta_e + \sum_{i=1}^{4}\left[C_{m\eta_i}\eta_i + \left(C_{m\dot{\eta}_i}\dot{\eta}_i\right)c/2u\right] \quad (30)$$

The flight data obtained for this case is pictorially presented in Fig. 4 and Fig. 5. To study the effect of flexibility of the structure on parameter estimation, two cases are considered. The two set of modal frequencies for the two different configurations. To start with aircraft with moderate flexibility is considered. The flight data obtained with this case will be referred to as FD-AE A/C. In the second case more flexibility is included and the flight data generated for this case is referred to as FD-AE A/C2.

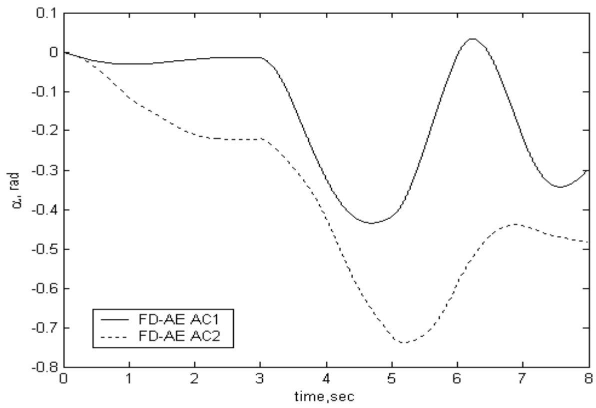

Fig. 4 Simulated α response from the flight data FD-AE AC1 and FD-AE AC2

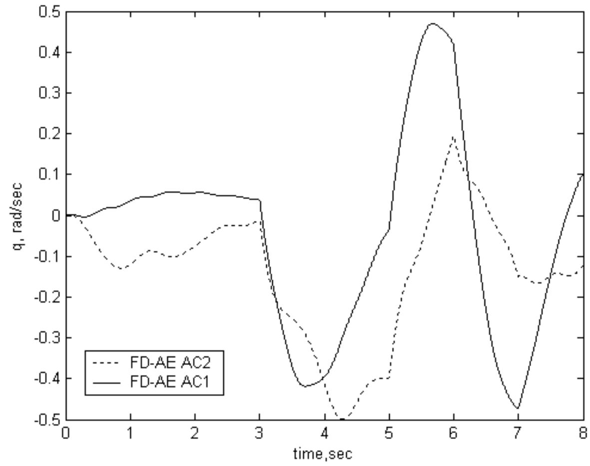

Fig. 5 Simulated q response from the flight data FD-AE AC1 and FD-AE AC2

In this chapter, estimated parameters obtained through EKF method are presented. In the case of ballistic target the factors that affect the behavior of the filter are discussed. The simulated data corresponding to ballistic target (FD-BT), rigid aircraft (FD-RA) and flexible aircraft (FD-AE AC1 & FD-AE AC2) are used as the measured flight data. The estimated parameters are presented along with their standard deviations to assess the accuracy of the estimates of aerodynamic parameters.

VII. Estimation of Ballistic Coefficient from Flight Data of Ballistic Target (FD-BT)

The ballistic target is assumed to fall on a straight line path towards the surface based tracking radar. The ballistic coefficient (β) is estimated through EKF method. It has been observed that there is a negligible change in the estimated value of β by the increase of terms in the Taylor series expansion for the approximation of fundamental matrix $[\phi_k]$. This is valid because the fundamental matrix is actually an infinite Taylor series expansion of the product of sampling time t_s and system dynamics matrix $[F]$.

	True Value	Estimated Value by 2 terms	Estimated Value by 3 terms
Ballistic Coefficient β (Lb/Ft/ sec^2)	500	497.8827 (0.290904)*	499.8196 (0.290046)*

* Standard Deviation

Table 1 Estimated ballistic coefficient without process noise

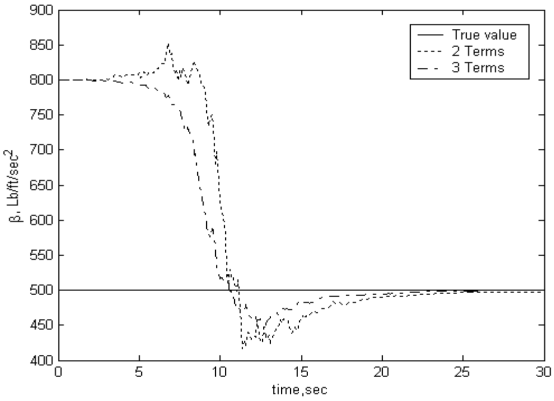

Fig. 6 Estimation of ballistic coefficient β without process noise

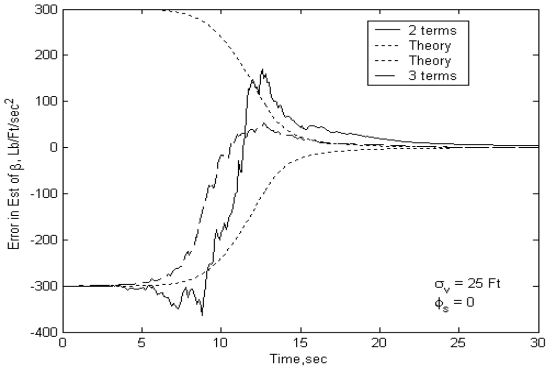

Fig. 7 Error in the estimation of ballistic coefficient β without process noise

For the same case the process noise is included assuming that the filter's knowledge of the real world is in error. It has been observed that addition of process noise to the filter increases the errors in the estimates. Also there is not much difference by increase of number of terms in the Taylor series expansion for the approximation of fundamental matrix $[\phi_k]$. From the Table. 1 and Table. 2 it is evident that by including process noise the accuracy in the estimates deteriorated. Similar observation can be made by comparing Fig. 6 and Fig. 7 with Fig. 8 and Fig. 9.

Table 2 Estimation of ballistic coefficient with process noise

	True Value	Estimated Value by 2 terms	Estimated Value by 3 terms
Ballistic Coefficient β (Lb/Ft/ sec^2)	500	488.9278 (11.07214)	488.7636 (11.23639)

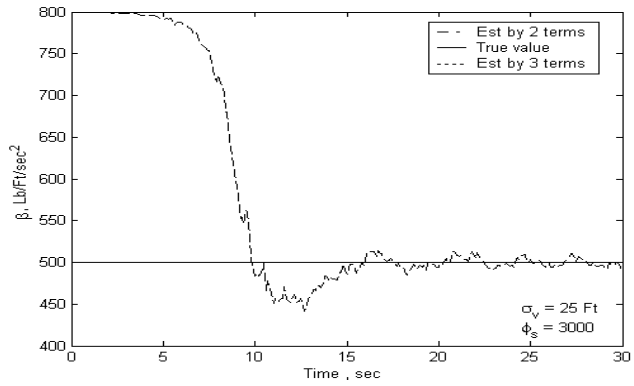

Fig. 8 Estimation of ballistic coefficient β with process noise

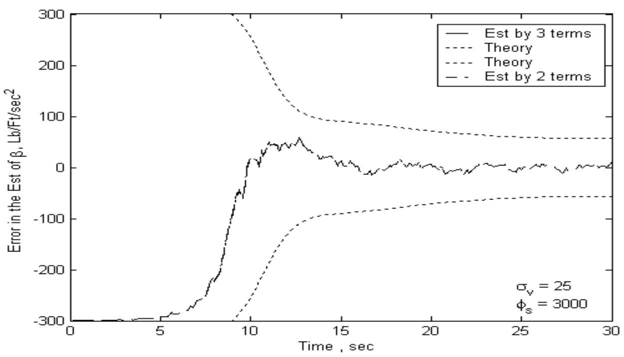

Fig. 9 Error in the estimation of ballistic coefficient β with process noise

VIII. ESTIMATION OF AERODYNAMIC PARAMETERS FROM FLIGHT DATA OF RIGID AIRCRAFT (FD-RA/C)

Knowledge of aerodynamic parameters is of paramount importance, to develop accurate mathematical model which represents the longitudinal dynamics of an aircraft. The aerodynamic parameters needed to develop the aerodynamic model are force derivatives $C_{L\alpha}$, C_{Lq}, $C_{L\delta_e}$ and pitching moment derivatives $C_{m\alpha}$, C_{mq}, $C_{m\delta_e}$. Thus, a study was carried out to explore the possibility of extracting the parameters using EKF method. The simulated data generated, FD-RA/C of the example aircraft is added with Gaussian noise and is used as the measured data in the EKF algorithm. The six aerodynamic parameters $C_{L\alpha}, C_{Lq}, C_{L\delta_e}, C_{m\alpha}, C_{mq}$ and $C_{m\delta_e}$ are considered as state variables along with the flight variables, α and q. The algorithm is run for an elevator input 3-2-1-1 for 8 seconds and the estimated aerodynamic parameters are tabulated along with their standard deviations in Table. 3. The measurement noise variance is 0.016 for both α and q. In Case A the values of diagonal elements of process noise matrix are same and are randomly chosen to be 0.008 and in Case B the values are differently chosen as an attempt to tune process noise.

Table. 3 Estimated Aerodynamic Parameters from FD-RA/C

Parameters	**True Value**	**No Process Value**	**Process Noise Case A**	**Process Noise Case B**
$C_{L\alpha}$	**2.922**	**2.9096 (0.00764)**	**2.8772 (0.06228)**	**2.8887 (0.08767)**
C_{Lq}	**-14.7**	**-22.7217 (0.75708)**	**-16.6971 (1.78776)**	**-16.8602 (1.89439)**
$C_{L\delta_e}$	**0.435**	**0.6756 (0.01976)**	**-0.3212 (0.13688)**	**-0.2328 (0.14417)**
$C_{m\alpha}$	**-1.66**	**-1.6078 (0.00166)**	**-1.8501 (0.03303)**	**-1.6099 (0.0784)**
C_{mq}	**-34.75**	**-38.3583 (6.68809)**	**-39.0636 (6.9463)**	**-41.527 (6.94662)**
$C_{m\delta_e}$	**-2.578**	**-2.25619 (0.00289)**	**-2.7122 (0.10493)**	**-2.8064 (0.0588)**

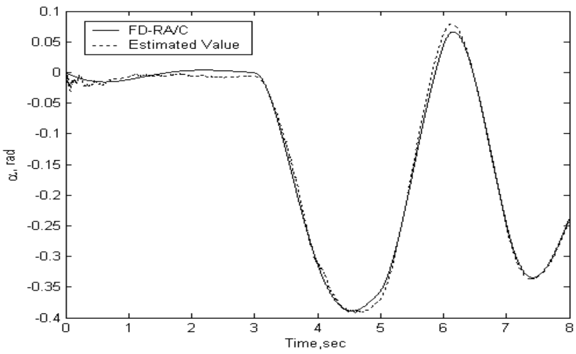

Fig. 10 Comparison of the estimated α value with that of FD-RA/C

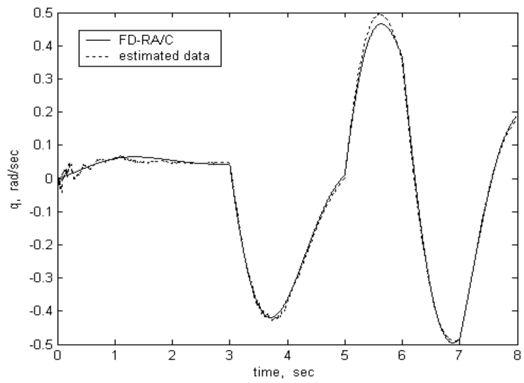

Fig . 11 Comparison of the estimated q value with that of FD-RA/C

Through the values listed in Table 3 it can be observed that by including process noise the estimated parameters show large variance however we can also infer that by proper choice of noise values the estimate of parameters can be improved.

IX. Estimation of Equivalent Aerodynamic Parameters from Flight Data of Flexible Aircraft (FD-AE A/C1 & FD-AE A/C2)

The simulated data from FD-AE A/C1 and FD-AE A/C2 is used in the EKF algorithm by adding Gaussian noise to the data. All the 4 elastic modes of the test aircraft are considered in the generation of the simulated data. A full order model of an aero elastic aircraft has too many parameters to yield satisfactory estimates using any of the conventional parameter estimation methods. In view of this, a study was carried out to identify a simplified model with reduced number of parameters, and to evaluate how the resulting parameters are affected by model simplifications. To start with, a rigid body model was assumed and parameters were estimated from flight data that contain the aero elastic effects. It is expected that the parameter estimates thus obtained would absorb the aero elastic effects. For the convenience of discussion, these parameters are referred by 'equivalent parameters' in Ref. 9. An analytical expression has been proposed to analytically compute the numerical values of the equivalent parameters. It is shown that the numerical value of analytical expression indicates the degree of flexibility of the aircraft, and thereby, a criterion based on it is suggested for deciding adequacy or otherwise of using simpler rigid body models in estimation algorithm. In the EKF algorithm only aerodynamic parameters $C_{L\alpha}, C_{Lq}, C_{L\delta_e}, C_{m\alpha}, C_{mq}$ and $C_{m\delta_e}$ are estimated and presented in Table. 4.

Table 4 Equivalent Aerodynamic Parameters from FD-AE A/C1 and FD-AE A/C2

Parameters	True Value	FD-AE A/C1	FD-AE A/C2
$C_{L\alpha}$	2.922	2.4927 (0.006445)	1.7955 (0.017004)
C_{Lq}	-14.7	-1.5588 (0.6973)	-7.7374 (2.56618)
$C_{L\delta_e}$	0.435	0.9644 (0.0179)	-0.2195 (0.06234)
$C_{m\alpha}$	-1.66	-1.3141 (0.0012)	-0.4582 (0.001378)
C_{mq}	-34.75	-20.6907 (6.6134)	-15.2807 (6.932)
$C_{m\delta_e}$	-2.578	-2.2514 (0.00271)	-1.5197 (0.00752)

For the case of FD-AE A/C 1 the measurement noise for α and q is 0.016 but for the case of FD-AE A/C 2 the measurement noise is 0.09. For the case of FD-AE A/C 2 the aero elastic effects of more flexible aircraft are got included in the algorithm as measurement noise.

XI. Estimation of Aerodynamic Parameters from Flight Data of Flexible Aircraft (FD-AE A/C1 & FD-AE A/C2)

In the final case both the flight data FD-AE A/C1 and FD-AE A/C2 are processed by the EKF algorithm. The algorithm includes only 15 parameters, taking into account only the first elastic mode. Along with the flight variables α and q the generalized displacement coordinate η_1 and its derivative $\dot{\eta}_1$ are also taken as the state variables. The estimated values of aerodynamic parameters are presented in Table. 5. It is seen that the estimation deteriorates as the flexibility increases. Thus for high flexibility aircraft such an approximation may not yield better results Comparison of estimated response of α ,q with that of the flight data FD-AE AC1 and FD-AE AC2 are presented pictorially in Fig.12 to Fig. 15. Based on these Fig. 12 to Fig. 15 it can be easily seen that the estimated response (α, q) closely matches with the simulated response. However, there is a large difference between these parameters; it can be used for simulators and control law specifications to initiate the analysis.

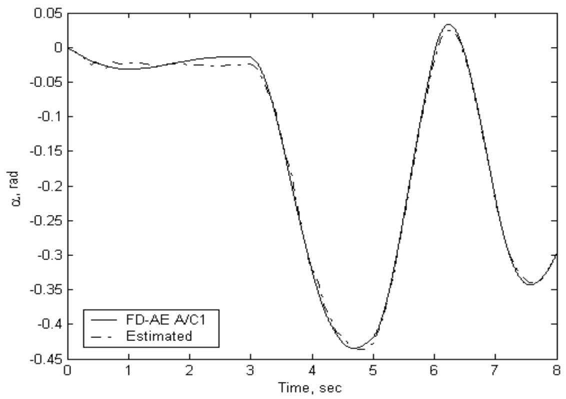

Fig . 12 Comparison of estimated value of α with that of FD-AE A/C1

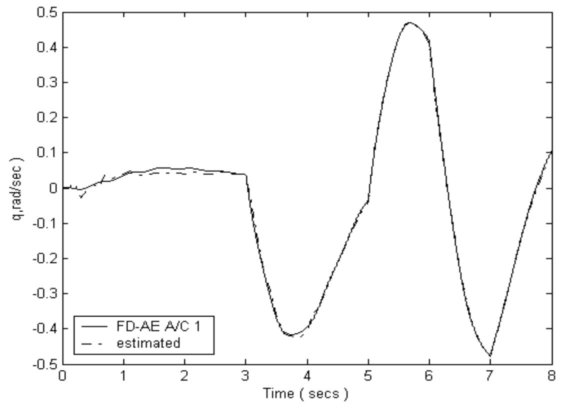

Fig . 13 Comparison of estimated value of q with that of FD-AE A/C1

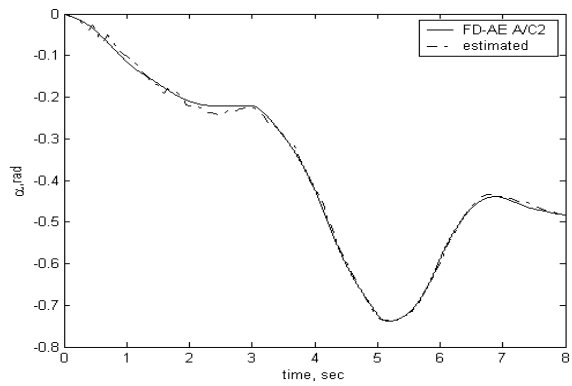

Fig . 14 Comparison of estimated value of α with that of FD-AE A/C2

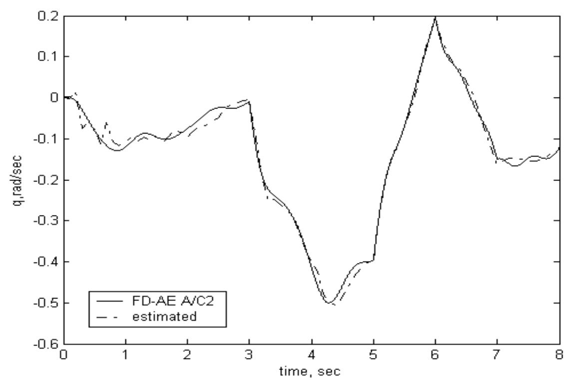

Fig . 15 Comparison of estimated value of q with that of FD-AE A/C2

Table 5 Estimated Aerodynamic Parameters from flight data FD-AE AC1 and FD-AE AC2

	True Values	FD-AE AC1	FD-AE AC2
C_{L_α}	2.922	3.2861 (0.0445)	3.0798 (0.03598)
C_{L_q}	-14.7	-13.2036 (1.02)	-24.6586 (1.7327)
$C_{L_{\delta_e}}$	0.435	0.8184 (0.0312)	0.2619 (0.03821)
C_{m_α}	-1.66	-1.6082 (0.0084)	-1.8725 (0.01857)
C_{m_q}	-34.75	-32.4563 (0.2808)	-41.1266 (0.49461)
$C_{m_{\delta_e}}$	-2.578	-2.565 (0.0145)	-2.1175 (0.01376)
$C_{L_{\eta 1}}$	0.0288	0.0556 (0.0024)	0.012 (0.000479)
$C_{L_{\dot{\eta} 1}}$	0.0848	0.0905 (0.0163)	0.1792 (0.01244)
$C_{m_{\eta 1}}$	0.0025	-0.0411 (0.0025)	-0.0201 (0.000129)
$C_{m_{\dot{\eta} 1}}$	-0.159	-0.0799 (0.022)	-0.1873 (0.00451)
$C_{\eta 1_\alpha}$	-0.014898	-0.017 (0.0018)	-0.0244 (0.000281)
$C_{\eta 1_q}$	-0.0949	-0.1 (0.0186)	-0.0672 (0.009936)
$C_{\eta 1}^{\eta 1}$	0.0000585	0.000062 (0.00001	0.000126 (0.000002)
$C_{\dot{\eta} 1}^{\eta 1}$	-0.00042	-0.000587 (0.00007)	-0.000842 (0.000027)
$C_{\delta_e}^{\eta 1}$	-0.012835	-0.0127 (0.0012)	-0.009864 (0.000217)

CONCLUSION

In the present paper, EKF method has been applied to estimate aerodynamic parameters from simulated flight data. The method has been applied starting from flight data of a one- dimensional ballistic target to flight data of a rigid aircraft and then to the flight data of a flexible aircraft. It is observed that EKF method can be applied successfully to estimate the parameters from the flight data of the three cases. The method also estimates equivalent aerodynamic parameters in which aero elastic effects get absorbed. If appropriate mathematical model of the system is provided then the method can be used advantageously to estimate force and moment derivatives.

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