International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 3 Issue 5, May - 2014

Bending Stress Analysis & Optimization of Spur Gear

Ram Krishna Rathore
Assistant Professor,
Mechanical Engineering Department,
CCET, Bhilai, INDIA

Abhishek Tiwari
Assistant Professor,
Mechanical Engineering Department,
CIT, Rajnandgaon, INDIA

Abstract— **Gear teeth failure due to fatigue is a general incident observed. Yet a minor drop in the root bending stress results in enormous enhancement in the bending fatigue life of a spur gear. If gear fails in tensile fatigue condition the results are cataclysmic and arise with modest or no notification. So in spite of the reasons stated above, this paper is of more practical significance. Up till now the gear design has been enhanced by using better material, surface hardening and carburization, and shot penning for surface finish etc. Some extra efforts have been completed to enhance the durability and strength by changing the various parameters i.e. pressure angle, by asymmetric teeth, by root fillet curve alteration and so on. The majority of these techniques do not provide assurance for the interchangeability of the existing gear design. This work shows the potential by means of the stress concentration methods by inserting the stress relieving features at the most stressed area for the reduction of root fillet or bending stress in spur gear. In this work, circular and elliptical stress relieving holes are employed and better results are obtained than using circular stress relieving holes, which were employed in previous researches. A finite element model of spur gear is considered for analysis and compared with the analytical method and stress relieving features of various sizes are inserted on gear teeth at root area. In this work the optimum size and location of the stress relief features for Spur gear are proposed, which help in reducing the fatigue failure in gears.**

Keywords— Spur Gear, Bending stress, Static analysis, Stress Optimization, FEA

I. INTRODUCTION

Gearing is one of the most essential parts in a mechanical power transmission structure, and in mainly industrial rotating machinery [1]. It is probable that gears will outweigh as the most successful means of transmitting power in upcoming machines due to their high degree of consistency and compactness. Besides, the rapid changes in the business from heavy industries such as ship construction to industries such as automobile developers and office automation tools will impose a distinguished application of gear technology.

The escalating requirement for better power transmission in machines, generators, elevators and vehicles, has formed an increasing demand for an additional accurate examination of the characteristics of gear structures. In the automobile manufacturing, the biggest producer of gears, higher consistency and lighter weight gears are essential as lighter automobiles carry on to be in demand. Additionally, the achievement in engine stress reduction supports the production of better gear pairs for additional stress reduction. However, a lack of these experts exists in the newer, lightweight industries in Japan [14] mainly because smaller amount young people are specializing in gear technology today and conventionally the specialists engaged in heavy industries tend to stay where they are.

A competent system is one in which every elements are consistent. Therefore suitable care must be given to every element in the structure to authenticate the overall reliability. In light of this, a profound examination needs to be performed to recognize the techniques to overcome the failures of the gear and to enhance the reliability of the structure including gear as one of its key element. Designing extremely laden spur gears for power transmission arrangements that are equally strong and silent wants investigation techniques that can simply be employed and moreover give information on bending stresses. The finite element Analysis (FEA) is competent of giving this information, but the time required to make such a model is big [2,3].

Spur gears analyses in the earlier period were performed using analytical techniques [4-7], which requisite several assumptions and generalizations. Normally, gear analyses are multidisciplinary, together with estimations associated to the tooth stress and to tribological failures for example like scoring or wear. In this thesis, static bending stress analyses are carried out, whereas trying to design spur gears to resist bending failure of the teeth. As computers have grown to be more and more potent, people have leaned to use arithmetical approaches to create theoretical models to calculate the effect of anything can be studied [4-9]. This has enhanced gear analyses and computer imitations. Arithmetical techniques can potentially give more precise solutions since they usually need much less uncertain assumptions. The model and the solution techniques, nevertheless, must be selected watchfully to make certain that the outcomes are precise and that the computational time is practical.

There are two principal forms of failure for spur gears in contact with each other: failure by means of bending and failure through contact stress at the gear tooth surface, the contact stress, or pitting stress, between two contacting gears may be calculated by means of the Hertzian contact equation, and is relative to the square root of the applied tooth load (AGMA 2001-D04). The bending stress is estimated by assuming the gear tooth as a cantilevered beam, by a cross section of face width by tooth thickness. The spur gear tooth load is directly proportional to bending stress. Usually, bending failure will happen when the stress on the tooth is superior than or equal to the yield strength of the gear tooth material.

In this work, first, the FE model and solution techniques required for the precise computation of 2D spur gear bending stresses are determined. Afterwards, the bending stresses calculated using FEA was evaluated to the outcomes acquired from existing techniques of AGMA (American Gear Manufacturing Association) [24,25,26].

The objective of this study focuses on the reduction of stresses occurs on Spur gear by means of different stress relief features. The Bending stress has to be analysed by means of analytical and FEA method then further we will try to reduce them by some material removal from the gear. After that some advance optimization techniques are also employed in order to get the refine data's on the basis of previously generated stress relief features

II. ASSUMPTIONS

For developing our gear simulation approach, we make the following assumptions:

- The full load is applied to the tip of a single tooth in static condition.
- The gears are perfectly parallel and transmit equal torque.
- For the static analysis the driven gear is fixed in order to calculate gear strength.
- There is no friction in between the mating gears and forces due to tooth sliding friction and radial component are negligible.
- The load is distributed uniformly across the full face width.
- There is no interference between the tips and root fillets of mating teeth.
- There is nonzero backlash and the root fillets are standard, assumed smooth, and produced by a generating process.

III. PROBLEM FORMULATION

Here the two spur gears are mating; they are required to transmit the power to the parallel shaft. Now the two spur gears are having similar number of teeth and material (steel). The upper gear has subjected to a torque of 15000 lbf-inches at shaft, due to which I am going to analyse the bending stress at lower gear. As per [30] the data's which are given are as follows:

1. pitch radius r_p = 2.5 in
2. pressure angle $\alpha = 20°$
3. number of teeth N = 20
4. radius of addendum r_m = 2.75 in
5. radius of dedendum r_d = 2.2 in
6. shaft radius = 1.25 in
7. root fillet has a radius = 0.1 in
8. Face width = 1.0
9. Torque applied at shaft = 15000 lbf-in
10. Material as Steel
11. Young's Modulus = 200Gpa
12. Poison ratio = 0.3

Now let's take AGMA bending stress modified formula [21],

$$\sigma_b = \frac{F_t}{b \times m \times J} \times K_v \times K_o \times K_m$$

Where,

Face width b =1 in

Pitch diameter P_d = 5 in

Module $m = \frac{Pitch\ dia}{No.of\ teeth} = \frac{5}{20} = 0.25$ in

Tangential force $F_t = \frac{2 \times M_t}{P_d} = \frac{2 \times 15000}{5} = 6000$ lbf

Overload factor K_o = 1.25 *(for uniform light shock)*

Load distribution factor K_m *= 1.3 (face width up to 50mm)*

Dynamic factor K_v = 1.2

Geometry factor *J =0.32 (For* $\phi = 20^o$ *and Z = 20)*

$$\sigma_b = \frac{6000 \times 1.2 \times 1.25 \times 1.3}{1 \times 0.25 \times 0.32} = \frac{11700}{0.08}$$

Bending stress $\boldsymbol{\sigma_b}$ **= 146250 Psi = 1008.358 Mpa**

IV. METHODOLOGY

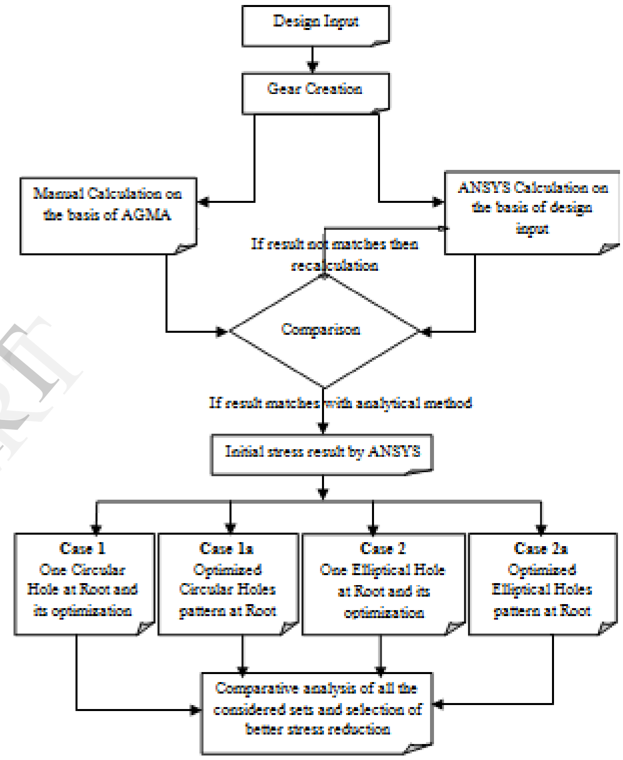

Fig. 1. Flowchart of the methodology

V. ASSESSMENT OF SPUR GEAR

In this section the steel spur gear will be analysed to see the various results from the static analyses. The software used to perform the analysis is ANSYS® 15.

A. *Material properties*

In this paper both the gears are made by steel [30] and there material properties are given in the table 1.

TABLE I. MECHANICAL PROPERTIES OF STEEL SPUR GEAR

PARAMETER	VALUE
Young's Modulus (E)	200GPa
Poisson's Ratio	0.3
Tensile Strength Ultimate	1962 MPa
Tensile Strength Yield	1500 MPa
Density	7850 kg/m3
Thermal Expansion	11x10-6 / oC

B. Boundary and loading Conditions

For checking the stresses in the spur gear we need to consider the assumptions made. For strength checking purpose the boundary condition of the spur gear is fixed at the shaft bore and a tangential force of 6000 lbf is applied on the gear tooth. The boundary and loading condition is shown in figure 2.

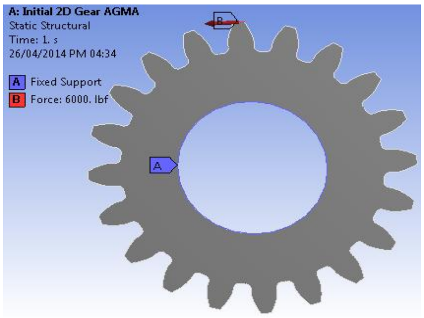

Fig. 2. loading and boundary condition of spur gear

C. FEA analysis result

With all the pre-processing steps the model is now set for the static analysis, where the initial results of the static analysis are shown in figure 3. The results of the static analysis for the spur gear taken are: The Maximum bending stress and minimum fatigue life are 1.4624e5 psi and 232 cycles respectively.

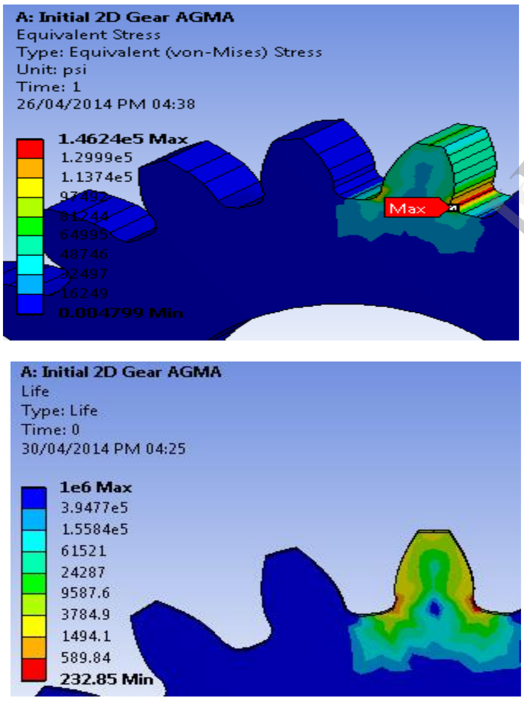

Fig. 3. Maximum Bending stress and fatigue life of spur gear

Now we are going to check the value of stress generated at the root for the bending stress verification. As per the figure 3, it has been clear the results are coming similar as per the AGMA analytical approach, which is shown in table 2.

TABLE II. COMPARISON OF AGMA AND FEA RESULTS

	Analytical method AGMA	**Finite Element Method**	**% error**
Bending Stress	146250 psi Or 1008.358 Mpa	14624 psi Or 1008.3 Mpa	0.01 %

As per the table2, we can say that the analysis which we are doing is considerable for the further analysis and modifications. Now as our prior analysis is correct then we can further move on to achieve the objective as reduction of stress at spur gear tooth. For attaining the goal now we are further taking some cases where we are going to insert some stress relief feature in order to reduce the stresses.

D. CASE 1 –Circular hole at root

The first case we are taking is the, one circular hole at root location. For this case we've chosen one random hole with some specific size and location to observe the result of it; whether it reduces the stress and increase the life or not.

The hole created at the root side is shown in the figure 4. The diameter (*dia*) of the hole is 0.03in and the as per the figure shown below the vertical dimension (*v*) as 2in and horizontal distance (*horz*) from the datum line goes through center and pitch point of the tooth is 0.2in.

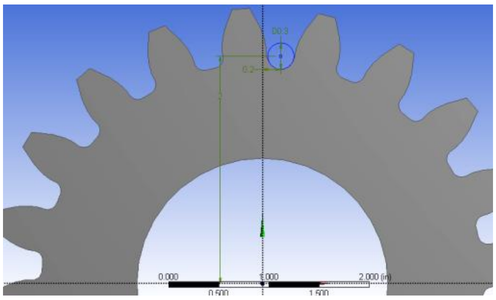

Fig. 4. Geometric location of the hole at the root

The result of the static analysis of the case one is shown in figure 5. The Bending stress generated due to tangential force is 1.4194e5 psi or 978.65 Mpa. Which is less than the initial (without any hole as 1008.3 Mpa) one.

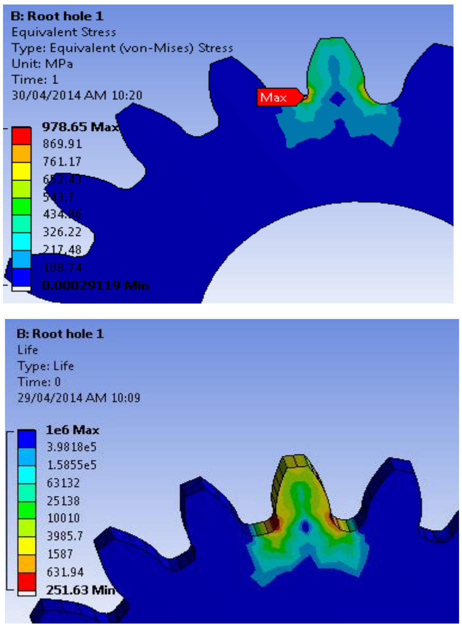

Fig. 5. Bending stress and Fatigue life of case 1 (before optimization)

It has been clearly observed that there is a fair amount of stress reduction with the help of hole created but as we don't know what should be the exact location and the size of the hole created. We can only check it by means of lots of iterations. To reduce the modeling time and enhance the efficiency of the gear, now we are going for the optimization.

For the optimization of the stress and fatigue life we are going to take the size and the location parameters as input and the bending stress and the bending fatigue life are the two response parameter. For the optimization as there are more than one objective, so we've taken the Multi Objective Genetic Algorithm (MOGA) as the optimization method to enhance the performance of the gear.

With the help of MOGA the optimization has been converged after 284 sample point. The candidate points are shown in the Figure 6.

	Candidate Point 1	Candidate Point 2	Candidate Point 3
P1 - dia (in)	0.09367	0.25842	0.089322
P4 - horz (in)	0.16703	0.18403	0.13849
P3 - v (in)	2.0171	2.2667	2.0674
P5 - Equivalent Stress Maximum (MPa)	★★★ 806.04	★ 828.31	– 840.8
P6 - Life Minimum	★★★ 416.78	★ 388.27	– 373.44

Fig. 6. Optimization candidates for case 1

As per the table we can say that the reduction in the stress with respect to the initial (without hole) and with hole of case 1 the stress has been reduced up to great extend.

E. Case 1a -Pattern of Holes

But putting a single hole doesn't make any sense because there are 20 other teeth which are going to be in contact with other gears teeth. So checking a single hole is not a good concept, for this reason we are going to create the pattern of the optimized hole (size and location) for the spur gear. So for this 20 holes are placed in equidistance (as 18º) in 360º at the spur gear. After putting 20 holes in the gear, let us check the variation in the stress and the bending fatigue life, which are shown in the figure 7. The new stress and fatigue life are 813.93 Mpa and 406.36 cycles.

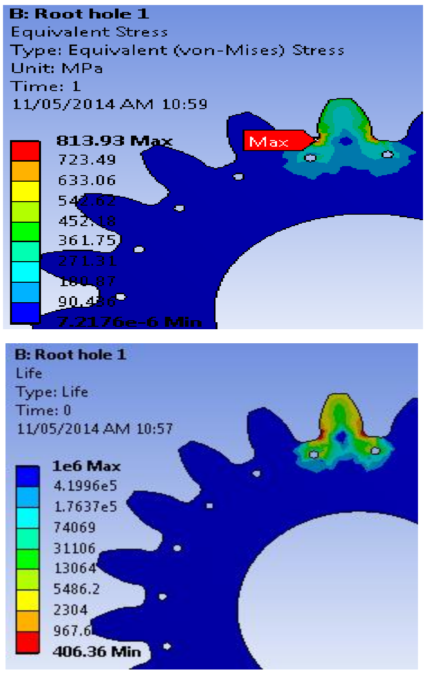

Fig. 7. Optimized stress and life of spur gear in case 1a with holes pattern

F. CASE 2 – Stress Relief Feature – Elliptical hole at root

The second case we are taking is the, one elliptical hole at root location. For this case we've chosen one random elliptical hole with some random size and location to observe the result of it; whether it reduces the stress and increase the life or not.

The hole created at the root side is shown in the figure 8. The input parameters as the maximum diameter (*Dmx*) and the minimum radius (*Rmn*) of the hole is 0.03in and the as per the figure shown below the vertical dimension (*V*) as 2in, horizontal distance (*H*) from the datum line goes through center and pitch point of the tooth is 0.2in and the angle (*Ang*) between the datum line goes through center and pitch point of the tooth and the Major axis is 1º.

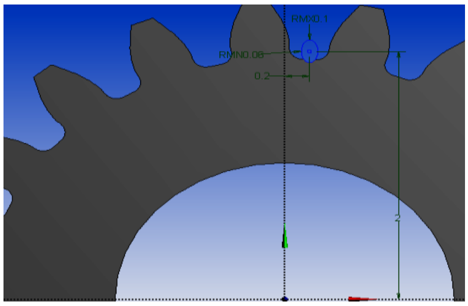

Fig. 8. Geometric location of the elliptical hole at the root

The result of the static analysis of the case one is shown in figure 9. The Bending stress generated due to tangential force is 932.7 Mpa. Which is less than the initial (without any hole as 1008.3 Mpa) one.

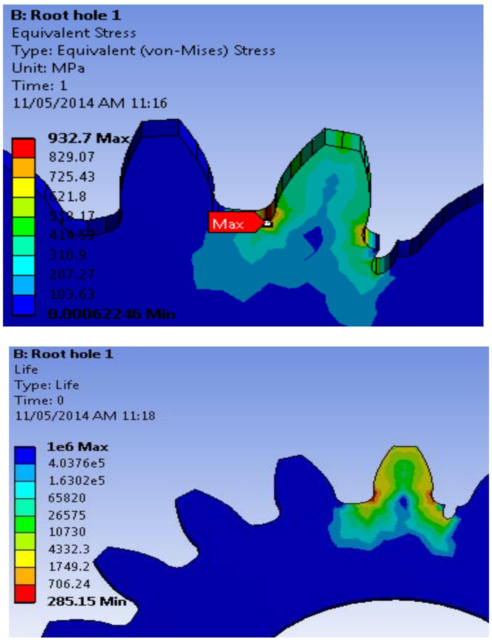

Fig. 9. Bending stress and Fatigue life of case 2 (before optimization)

The minimum fatigue life for one elliptical hole is 285.15 cycles, which is better than the initial spur gear analysis (without holes) as shown in figure 9. From the above shown figures it has been clearly observed that there is a fair amount of stress reduction with the help of elliptical hole created but as we don't know what should be the exact location and the size of the hole created. We can only check it by means of lots of iterations. To reduce the modeling time and enhance the efficiency of the gear, now we are going for the optimization.

With the help of MOGA the optimization has been converged after 240 sample point. The candidate points are shown in the figure 10.

	Candidate Point 1	Candidate Point 2	Candidate Point 3
P7 - Ang (degree)	14.016	33.255	32.798
P8 - H (in)	0.4895	0.33013	0.4841
P9 - V (in)	1.9657	1.8717	1.8717
P10 - Dmx (in)	0.047407	0.09642	0.09642
P11 - Rmn (in)	0.013973	0.08392	0.08392
P5 - Equivalent Stress Maximum (MPa)	846.09	919.96	930.86
P6 - Life Minimum	367.4	295.54	286.62

Fig. 10. Optimization candidates for case 2

As per the table we can say that the reduction in the stress with respect to the initial (without hole) and with hole of *case 1* the stress has been reduced up to great extend.

G. *Case 2a - Pattern of Holes*

But putting a single elliptical hole doesn't make any sense because there are 20 other teeth which are going to be in contact with other gears teeth. So checking a single hole is not a good concept, for this reason we are going to create the pattern of the optimized hole (size and location) for the spur gear. So for this 20 holes are placed in equidistance (as 18º) in 360º at the spur gear.

After putting 20 holes in the gear, let us check the variation in the stress and the bending fatigue life, which are shown in the figure 11. The new maximum stress and minimum fatigue life are 796.16 Mpa and 430.36 cycles.

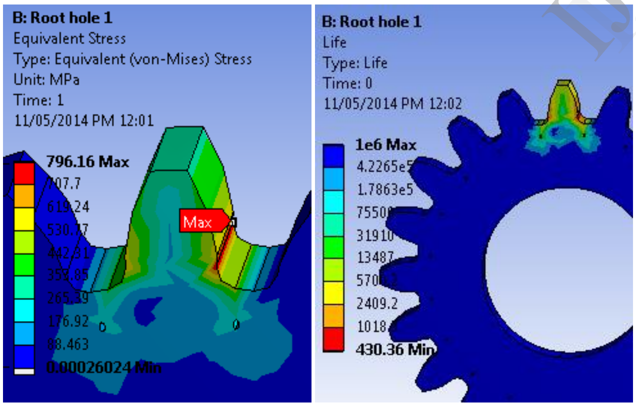

Fig. 11. Optimized stress and fatigue life of spur gear in case 2a with holes pattern

VI. CONCLUSION

We discussed the process detailed in this paper expands upon an important phenomena of Bending Stress and its fatigue life. This paper offers a unique and comprehensive process for the design and optimization of Spur gear bending stress and related fatigue life. By the means of stress relief features we can reduce the bending stress and increase the fatigue life of spur gear, but along with this we discussed about the optimization of the Bending stress and fatigue life with respect to the size and location parameters of the stress relief holes. This will lead us to some unique result where the Bending stress has been reduced by approx. 21% and life has been enhanced approx. just double of initial fatigue life. The summarised results have been shown in the table below. Now we can summaries the paper with the help of the table 3.

TABLE III. TABLE 3 SUMMARY OF SPUR GEAR ANALYSES AND OPTIMIZATION

	Maximum Bending stress (Mpa)	**Minimum Fatigue life (cycles)**
Initial Analyses of spur gear	1008.35	232.85
Optimized Single circular hole of case 1	806.04	416.78
Pattern of optimized circular hole of case 1a	813.93	406.36
%Reduction/enhancement through optimized case 1a pattern w.r.f initial analyses	19.2%	174.5%
Optimized single elliptical hole of case 2	846.84	366.55
Pattern of optimized elliptical hole of case 2a	796.16	430.36
%Reduction/enhancement through optimized case 2a pattern w.r.f initial analyses	21.04%	184.82%

This work presents the results obtained by applying analytical method (AGMA) for spur gear to calculate the bending stress and the same by FEA analyses, which shows a good result in comparison.

The stress analysis of a tooth shows that the trailing fillet which has compressive stresses has higher stress levels than the leading fillet which has tensile stresses. It is concluded from the optimization study of keeping the hole along the profile of the tooth that the effect of any feature like a hole any where above the dedendum and in the tooth leads to an increase in the stresses in the fillets. The choice of the size and location of the elliptical hole is not a simple process, due to the non linear variations in a complex geometry, as the studies have shown. These studies results show a general tendency to have a stress reduction by adding the circular hole but the best results have to be chosen only after an optimization is done as demonstrated.

The introduction of a elliptical hole or circular hole on the dedendum circle reduces the stress levels by a very high percentage with a small loss of rigidity of the tooth. This translates into an exponential increase in the life of the gear due to a better location on the S-N curve for fatigue loading.

REFERENCES

[1] Wilcox, L., and Coleman, W., 1973, "Application of Finite Elements to the analysis of gear tooth stresses", ASME Journal of Engineering for Industry, vol. 95, pp. 1139-1148.

[2] Zeiping Wei', 2004, "Stress and Deformations in involute Spur gears by Finite Element Method", M.Sc. Paper, University of Saskatchewan.

[3] Saxena, Rajul, 2004, "Finite Element Stress Analysis of Spur Gear Teeth", M.Sc. Paper, the University of Texas at Arlington.

[4] Cavdar, K., Karpat, F., & Babalik, F. C. 2005. "Computer Aided Analysis of Bending Strength of Involute Spur Gears with Asymmetric Profile." Journal of Mechanical Design, 127(3), 477.

[5] Kawalec, A., Wiktor, J., & Ceglarek, D. 2006. "Comparative Analysis of Tooth- Root Strength Using ISO and AGMA Standards in Spur and Helical Gears With FEM-based Verification" Journal of Mechanical Design, 128(5), 1141.

[6] Shreyash D Patel, 2010, "Finite Element Analysis of Stresses In Involute Spur & Helical Gear", M.S. paper, The University of Texas At Arlington

[7] Andrew Wright, 2013, "A Comparison of the Tooth-Root Stress and Contact Stress of an Involute Spur Gear Mesh as Calculated by FEM and AGMA Standards", Master of Engineering paper, Rensselaer Polytechnic Institute Hartford, CT

[8] Fredette L. and Brown M., 1997"Gear Stress Reduction Using Internal Stress Relief Features", Journal of Mechanical Design, vol. 119, pp. 518-521.

[9] Spitas, V., Costopoulos, Th. and Spitas, C., 2005, "Increasing the Strength of Standard Involute Gear Teeth with Novel Circular Root Fillet Design", American Journal of Applied Sciences, vol. 2, No. 6, pp. 1058-1064.

[10] Manoj hariharan, 2006, "spur gear tooth stress analysis and stress reduction using stress reducing geometrical features", M.E. paper, Thapar Institute of Engineering And Technology.

[11] M. S. Hebbal, V. B. Math, B. G. Sheeparamatti, 2009, "A Study on Reducing the Root Fillet Stress in Spur Gear Using Internal Stress Relieving Feature of Different Shapes", International Journal of Recent Trends in Engineering, Vol. 1, No. 5, pg 163-165

[12] V.Rajaprabakaran, R.Ashokraj, 2011, "Spur Gear Tooth Stress Analysis And Stress Reduction" IOSR-JMCE, e-ISSN: 2278-1684, p-ISSN: 2320-334X, PP 38-48

[13] Dhavale A.S., Abhay Utpat, 2013, " Study of Stress Relief Features at Root of Teeth of Spur Gear" International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622, Vol. 3, Issue 3, May-Jun 2013, pp.895-899

[14] MackAldener, M., and Olsson, M., 2001, "Interior Fatigue Fracture of Gear Teeth", accepted for publication in Fatigue and Fracture of Engineering Materials and Structures."

[15] I.H.b.W. Nordin, 2012 "Bending Fatigue Strength of Case-Carburized Spur Gears with Boss", International Conference on Applications and Design in Mechanical Engineering 2012 (ICADME 2012) 27-28 Penang, Malaysia.

[16] Guingand, M., de Vaujany, J. P., and Icard, Y., "Analysis and Optimization of the Loaded Meshing of Face Gears", Journal of Mechanical Design, vol. 127, pp. 135-143, 2005.

[17] Shivang S. Jani, "Analysis And Optimisation Of Gear Tooth Profile", International Journal of Mechanical and Production Engineering (IJMPE) ISSN No.: 2315-4489, Vol-2, Iss-1, 2013

[18] K. Deb, A. Pratap, S. Agarwal, T. Meyarivan, 2002 "A fast and elitist multiobjective genetic algorithm: NSGA-II," Evolutionary Computation, IEEE Transactions on, vol. 6, pp. 182-197.

[19] Rathore, Ram Krishna, Amit Sarda, and Rituraj Chandrakar. "An Approach to optimize ANN Meta model with Multi Objective Genetic Algorithm for multi-disciplinary shape optimization.", International Journal of Soft Computing and Engineering (IJSCE), ISSN: 2231-2307, Volume-2, Issue-1, March 2012, Pg. 200-207

[20] R. K. Rathore, Yogesh Chandra Sahu, and Amit Sarda. "Comparative analysis of Direct Adaptive-NSGA-II & NSGA-II Optimization Algorithms", International Journal Of Research In Aeronautical And Mechanical Engineering, ISSN (ONLINE): 2321-3051, Vol.1 Issue.7, November 2013, Pgs: 149-159

[21] http://nptel.iitm.ac.in/courses/IIT-MADRAS/Machine_Design_II/pdf/2_1.pdf

[22] http://www.roymech.co.uk/Useful_Tables/Drive/Gears.html

[23] http://www.scribd.com/doc/9616669/Gear

[24] AGMA standard, 2004, "Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth" ANSI/AGMA 2001--D04 [Revision of ANSI/AGMA 2001--C95], American Gear Manufacturers Association, 500 Montgomery Street, Suite 350, Alexandria, Virginia 22314, ISBN: 1--55589--839--4

[25] AGMA standard Information Sheet "Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth Gear Teeth" AGMA 908---B89, April 1989 (Revision of AGMA 226.01 1984), American Gear Manufacturers Association, 500 Montgomery Street, Suite 350, Alexandria, Virginia 22314, ISBN: 1---55589---525---5

[26] Mitra, G., 2001, "Hand book of gear design", Tata McGraw Hill international edition, New Delhi. ISBN 0-07-460237-3

[27] Norton, R.L., 1996, "Machine Design: An Integrated Approach", New Jersey: prentice- Hall Inc. ISBN-10: 0135554756

[28] Budgell, Peter, 2003, "Finite Element Analysis and Optimization Introduction", ROI Engineering Inc, Ontario, Canada.

[29] Deb, K., 2001, "Multiobjective Optimization Using Evolutionary Algorithms", Wiley, New York. ISBN: 978-0-471-87339-6

[30] Huei-Huang Lee, 2014, "Finite Element simulations with ANSYS Workbench 14", SDC Publications. ISBN 978-1-58503-898-5

[31] Belegundu, Ashok D and Chandrupatla, Tirupathi R. 2005, "Optimization Concepts and Applications in Engineering." Pearson Education, ISBN-13: 9780521878463