BER Performance Comparison of STBC-MIMO with different Equalizers

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Abstract

Multiple-Input Multiple-Output (MIMO) wireless technology has been acknowledged as one of the most potential techniques to sustain high data rate and high performance in distinct channel conditions. Alamouti's space time block coding (STBC) technique for MIMO system has brought tremendous breakthroughs in wireless technologies just because of its simplicity in decoding. Equalization known for mitigating Inter-symbol Interference (ISI) results in interference between successive transmission created by multipath propagation is also been discussed with BPSK modulation over Rayleigh fading channel. In this paper we present performance comparison of space time block coding with different equalizer techniques such as zero forcing (ZF) Equalizer, minimum mean square error (MMSE), maximum likelihood sequence estimation (MLSE). It is observed that the ML equalizer render minimum SNR value for the related BER value. The lower SNR for the same BER implies that it consumes much less power than the other two equalization techniques. This entail that MLSE outperforms ZF and MMSE equalizer.

Keywords- MIMO, MMSE, MLSE, STBC, ZF

1. Introduction

Multiple-Input Multiple-Output (MIMO) systems boost modern communication to a great extent. MIMO systems are simply defined as the system comprising multiple transmitter antennas and multiple receiver antennas. Communication researches show that MIMO system can provide a potentially tremendous capacity, which grows approximately linear with the number of antennas. Implementation of MIMO in wireless communication systems increased instantaneously, especially in wireless LANs (Local Area Networks). The MIMO system expands our focus to turn multi-path propagation, which is an existing obstacle in conventional wireless communication, into an advantage for users [1], [2].

The most important characteristic of MIMO systems is space-time processing. Space-Time Codes (STCs) are the codes intended for the use in MIMO systems. In STCs, signals are coded in both spatial and temporal dimensions which render improved bit error rate (BER) performance. Space time coding also aids in enhancing information rate. Along with dissimilar types of STCs, the one which seize various benefit over other kind of STCs is orthogonal Space-Time Block Codes (OSTBCs). OSTBC is one of the algorithms recognized for MIMO systems, which render high diversity when the channel is unknown at the transmitter. As these codes work independent of receiver antenna thus is form of transmitter diversity. This code comprises the capability of transmitting each symbol per time instant, hence also recognized as full rate code [2]. To combat from the effects of Inter-symbol interference (ISI), various equalization techniques is discussed and simulation study shows BER performance comparison between them.

2. System Model

Considering a MIMO system having two transmit antenna and two receive antenna as shown in Fig.1. At the transmitter end, the data stream are first converted using serial to parallel convertor and then mapped using BPSK modulation technique, after that data stream enters into a Space Time Encoder which is capable of increasing data rate by transmitting various data stream parallel. The encoder outputs are then conveyed to transmit antennas from where the the signal is passed through a wireless propagation channel, which is assumed to be Rayleigh having uniformly distributed phase and Rayleigh distributed magnitude.

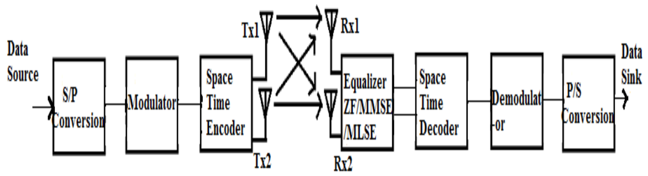

Fig.1. Proposed MIMO Model

The 2 x 2 channel matrix is represented as

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

h_{ij} represents the channel coefficient among i^th receive and j^th transmit antenna.

At the receiver end, receiver performs equalization (this can be ZF/MMSE/MLSE) to reduce or eliminate ISI created by a multipath channel. The data stream can now be decoded and demodulated using space time decoder and then serially converted using parallel to serial convertor.

A. Space Time Block Code

Alamouti has recommended a complex orthogonal space-time block code designed for two transmit antennas. In the Alamouti encoder, two consecutive symbols x_1 and x_2 are encoded with the following space-time coded matrix described by [4]

$$X = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \quad (1)$$

It is obvious that the encoding is done in both the space and time domains. Alamouti encoded signal is transmitted from the two transmit antennas over two symbol instances. At t+T, i.e. during the first symbol instant two symbols x_1 and x_2 are simultaneously transmitted from the two transmit antennas. At t= 2T, i.e. during the second symbol instant these symbols are transmitted again, where $-x_2^*$ is transmitted from the first transmit antenna and x_1^* transmitted from the second transmit antenna. For Maximum Likelihood signal detection of Alamouti's space-time coding scheme, we assume that two channels gains $h_1(t)$ and $h_2(t)$ remain constant over two consecutive symbol periods such that

$$h_1(t)=h_1(t+T)=h_1=|h_1|e^{j\theta 1} \quad (2)$$

$$h_1(t)=h_2(t+T)=h_2=|h_2|e^{j\theta 2} \quad (3)$$

Where $|h_1|$ and $e^{j\theta 1}$ symbolize the amplitude gain and phase rotation over the two symbol periods. At the receiver the received signals y_1 and y_2 at time t and $t+T_s$ can be given as

$$y_1=h_1x_1+h_2x_2+z_1 \quad (4)$$

$$y_2=h_1x_2^* + h_2x_1^* +z_2 \quad (5)$$

where z_1 and z_2 signify the additive noise at time t and $t+T_s$ respectively. In this paper we have proposed Alamouti's space time block code for two transmit antenna and two receive antenna case [4].

3. Equalizer

Equalizer is a digital filter that grants an estimated inverse of channel frequency response. Equalization is used to alleviate the effects of Inter-symbol Interference (ISI) to minimize the likelihood of error that occurs without elimination of ISI, but this dropping of ISI effects has to be balanced with prevention of noise power enhancement [1].

A. ZF Equalizer

Zero Forcing Equalizer (ZF) is a linear equalization algorithm exploit in communication assumption, which reverses the frequency response of the channel. This equalizer was first recommended by Robert Lucky. The ZF Equalizer employs the inverse of the channel to the received signal, to recover the signal before the channel. The name Zero Forcing corresponds to pulling down the ISI to zero in a noise free case. This will be advantageous when ISI is significant compared to noise. Considering a channel containing frequency response F(f) the zero forcing equalizer C(f) is designed such that C(f) = 1 / F(f). Thus the collective effect of channel and equalizer provide a flat frequency response and linear phase F(f)*C(f) = 1. For a certain channel containing frequency response H(s) then the input signal is multiplied by the reciprocal of this [10]. This is expected to remove the effect of channel from the received signal, in precisely the ISI [5]. For minimal complexity let us assume a 2x2 MIMO channel, the channel is designed as,

The signal received on the first receive antenna is,

$$y_1=h_{1,1}x_1+h_{1,2}x_2+n_1=[h_{1,1}\ h_{1,2}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} +n_1 \quad (6)$$

The signal received on the Second receive antenna is,

$$y_2=h_{2,1}x_1+h_{2,2}x_2+n_2=[h_{2,1}\ h_{2,2}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} +n_2 \quad (7)$$

Where y_1, y_2 are the received symbol on the first and second antenna respectively,
$h_{1,1}$ is the channel from 1st transmit antenna to 1st receive antenna,
$h_{1,2}$ is the channel from 2nd transmit antenna to 1st receive antenna,
$h_{2,1}$ is the channel from 1st transmit antenna to 2nd receive antenna,
$h_{2,2}$ is the channel from 2nd transmit antenna to 2nd receive antenna,
x_1, x_2 are the transmitted symbols and

n_1, n_2 are the noise on 1st and 2nd receive antennas.

The equation can be described in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (8)$$

Equivalently,

$$y = H.x + n \quad (9)$$

To solve for x, we need to find a matrix W which satisfies WH = I. The Zero Forcing (ZF) detector for meeting this constraint is given by,

$$W = (H^H H)^{-1} H^H \quad (10)$$

Where W - Equalization Matrix and H - Channel Matrix, This matrix is known as the Pseudo inverse for a general m x n matrix where

$$H^H H = \begin{pmatrix} h_{1,1}^* & h_{2,1}^* \\ h_{1,2}^* & h_{2,2}^* \end{pmatrix} \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} \quad (11)$$

$$= \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \\ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \quad (12)$$

It is seen that the off diagonal elements in the matrix $H^H H$ are not equal to zero, for the reason that the off diagonal elements are non zero in values. Zero forcing equalizer efforts to null out the interfering terms at the moment of acquiring the equalization, i.e. the interference from x_2 is tried to be null out when solving for x_1 and vice versa. But implementation of this causes amplification of noise. Hence Zero forcing equalizer is not the best developed equalizer. However, it is simple and evident to implement [5].

B. MMSE Equalizer

A minimum mean square error (MMSE) estimator illustrates the approach which minimizes the mean square error (MSE), which is a general method of computing the estimator quality [11]. The important feature of MMSE equalizer is that it does not usually eliminate ISI completely but, suppresses the total power of the noise and ISI components in the output [5]. Let x be an unknown random variable, and let y be a known random variable. In [7], an estimator $\hat{X}(y)$ is any function of the measurement y, and its mean square error is given by.

$$MSE = E\{(\hat{X} - X^2)\}, \quad (13)$$

where the expectation is held over both x and y.

The MMSE estimator is then characterized as the estimator attaining least MSE. In a lot of cases, it is impossible to determine a closed form for the MMSE estimator. In these cases, one possibility is to look for the method minimizing the MSE within a specific class, as like as the class of linear estimators. The linear MMSE estimator is a kind of estimator achieving minimum MSE among all estimators of the form AY + b. If the measurement Y is a random vector, A is a matrix and b is a vector [11]. Let us now make an effort to understand the math for extracting the two symbols which interfered with each other. In the first fraction of time, the signal received on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1}\ h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad (14)$$

The signal received on the second receive antenna is

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1}\ h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad (15)$$

Where
y_1, y_2 are the received symbol on the first and second antenna respectively,
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$h_{1,1}$ is the channel from 1st transmit antenna to 1st receive antenna,
$h_{1,2}$ is the channel from 2nd transmit antenna to 1st receive antenna,
$h_{2,1}$ is the channel from 1st transmit antenna to 2nd receive antenna,
$h_{2,2}$ is the channel from 2nd transmit antenna to 2nd receive antenna,
x_1, x_2 are the transmitted symbols and
n_1, n_2 are the noise on 1st and 2nd receive antennas.

Equivalently,

$$y = H.x + n \quad (16)$$

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient W which
Minimizes the

$$E\{[Wy-x][Wy-x]^H\} \quad (17)$$

Criterion,
Where W - Equalization Matrix
H - Channel Matrix and
n - Channel noise
y- Received signal.
To solve for x, we need to find a matrix W which satisfies W*H =I. The Minimum Mean Square Error (MMSE) detector for meeting this constraint is given by, [5]

$$W = [H^H H + N_0 I]^{-1} H^H \quad (18)$$

C. MLSE Equalizer

A maximum likelihood sequence estimator (MLSE) for a single carrier communication systems transmitting N complex symbols $\mathbf{s} = \{s_1, s_2, \ldots, s_N\}^T$, chosen from an alphabet D, having impulse response $\mathbf{h} = \{h_0, h_1, \ldots, h_{L-1}\}$ of length L through a multipath channel, the symbol received on the k^{th} instant is described by [8]

$$r_k = \sum_{j=0}^{L-1} h_j s_{k-j} + n_k, \quad (19)$$

where n_k is the k^{th} zero-mean, σ^2 variance, Gaussian noise sample. To find the most likely transmitted sequence s, the cost function

$$L = \sum_{k=1}^{N} |r_k - \sum_{j=0}^{L-1} h_j s_{k-j}|^2 \quad (20)$$

needs to be minimized. The MLSE equalizer based on the Viterbi Algorithm (VA) minimizes equation (20) optimally by exploiting a trellis, with computational complexity linear in *N* and exponential in *L*. In our future work we will propose that the combination of MIMO with OFDM will make the system more spectrally efficient, here we will also recommend combination of different equalization which will bring more robustness to the system [6].

4. Simulation Model and Result

The simulation test bench has been prepared for the MIMO technique. The parameters used in this are listed below,

Parameter	Values
Modulation technique	BPSK
Channel	Rayleigh
SNR level	0-25 dB
MIMO tech.	STBC
Equalizer	ZF,MMSE,MLSE

The result is given in comparative manner below

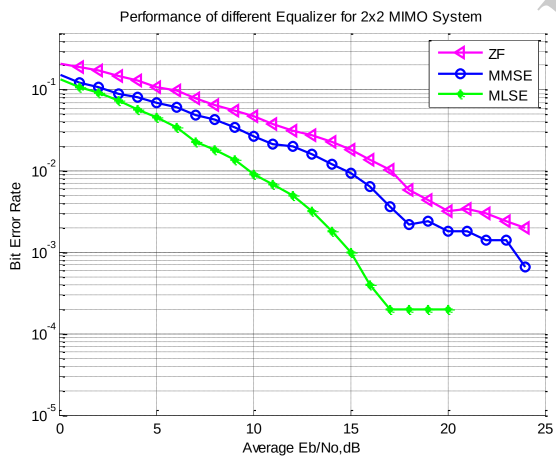

Fig.2. BER performance of MIMO system

5. Conclusion

This paper is a simulation study on the performance comparison of STBC-MIMO with different Equalizer using BPSK modulation technique. The test bench has been developed successfully for simulation and the BER 0.001 has been targeted by different equalizer on 2x2 MIMO System. The SNR levels for BER 0.001 are 10, 14 and 17 dB for MLSE, MMSE and ZF respectively. From the above following observations are made. The ML equalizer is the best of the three equalizers, as it provide minimum SNR value for corresponding BER value. The lower SNR for the same BER implies that it consumes much less power than the other two techniques. Hence we can conclude that BER performance of ML Equalizer is superior to zero forcing Equalizer and Minimum Mean Square Equalizers.

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