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Binary Cyclic Codes in Extending Circular Cliques

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ABSTRACT: **Let G be a coloring graph with circular chromatic number $(G)= \{k/d: G\rightarrow G_{k,d}$, gcd(k,d)=1 and $d\leq|i-j|\leq k-d\}$, $G_{k,d}$ are prime circular cliques. If the two circular cliques $G_{k,d}$ at distance d such that some (k',d')-precolouring of the two cliques is non-extendible. In this section, we examine extending circular colourings of $G_{k,d}\bowtie P_n$, P_n is the path of length $n-1$ with vertex set $\{1,2,\ldots,n\}$. In view of the homomorphism G admits a (k,d)-colouring if and only if, there is a homomorphism f:$G\rightarrow G_{k,d}$. there exist a uniquely extendible homomorphisms between circular cliques.**

KEY WORDS: **Edge coloring, Vertex coloring, Circular chromatic number, Homomorphism, Binary cyclic codes.**

I. INTRODUCTION:

Graph coloring theory has a central position in discrete mathematics — for its own interest as well as for the large variety of applications, dating back to the famous four-color problem stated by Guthrie in 1852 Zhu[9].

Define a(k,d)-colouring of a graph G is an assignment $c:V(G)\rightarrow\{0,1,2,\ldots,k-1\}$ such that for $uv\in E(G)$, $d\leq|c(u)-c(v)|\leq k-d$, d is any positive integer. The *circular complete graph* or *circular clique* $G_{k,d}$ has vertices $\{0,1,\ldots,k-1\}$ and edges $\{ij:d\leq|i-j|\leq k-d\}$. Thus $G_{k,1}$ is simply the (classical) complete graph on k-vertices. Graph coloring is the procedure of assignment of colors to each vertex of a graph G such that no adjacent vertices get same color.

The minimum k for which G admits a k-coloring is called the *chromatic number* of G and denoted by (G).
There are now many papers on colouring extensions. The introduction of [3] provides a nice overview on coloring. We focus on the situation where the precoloured vertices induce a collection of cliques. Let G be a graph with circular chromatic number $(G)=k/d$ [8]is isomorphic to the circular clique $G_{k,d}$. Suppose the vertices of P have been precoloured with a (k',d')-colouring. In [2] Albertson and Moore study the problem of extending a $(k+1)$-colouring of a k-colourable graph where the precoloured components are k-cliques. They also study the problem when the precoloured components are general subgraphs. In the latter case the penalty for having general subgraphs is a larger number of colours may be required for the extension. In this spirit we now turn attention to extending a (k',d')-colouring of a (k,d)-colourable graph where the precoloured components are circular cliques.

We now consider extending (classical) k'-colourings where the precoloured components are $G_{k,d}$. The general problem of extending colourings where the precoloured components are not cliques is considered in [2]. In our work the assumption that the precoloured components are circular cliques

II. PRELIMINARIES:

DEFINITION 2.1: An undirected graph is a type of graph where the edges have no specified direction assigned to the them..

DEFINITION 2.2: A binary code is **cyclic code** if it is a linear [n, k] code and if for every codeword $(c_1, c_2, \ldots, c_n) \in C$ we also have that $(c_n, c_1, \ldots, c_{n-1})$ is again a codeword in C.

- **Vertex coloring** is a concept in **graph theory** that refers to assigning colors to the vertices of a graph in such a way that no two adjacent vertices have the same color..
- In graph theory, **Edge coloring** of a graph is an assignment of "colors" to the edges of the graph so that no two adjacent edges have the same color

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DEFINITION 2.3: Graph coloring is the procedure of assignment of colors to each vertex of a graph such that no adjacent vertices get same color.

DEFINITION 2.4: The **chromatic number** of a graph is the minimal number of colours needed to colour the vertices in such a way that no two adjacent vertices have the same colour.

III. RESULT AND DISCUSSION:

We find Vertex chromatic number, edges chromatic number, Degree and Dimensions of the generator matrix.

A(k,d)-colouring of a graph G is an assignment $c:V(G)\to\{0,1,2,\ldots,k-1\}$ such that for $uv\in E(G)$, $d\le|c(u)-c(v)|\le k-d$, d is any positive integer. The *circular complete graph* or ***circular clique*** $G_{k,d}$ has **vertices** $\{0,1,\ldots,k-1\}$ and **edges** $\{ij: d\le|i-j|\le k-d\}$. Thus $G_{k,1}$ is simply the (classical) complete graph on k-vertices. The circular complete graphs play the role in circular colourings as do the complete graphs in classical colourings. Adopting the homomorphism point of view, see [4], [5], G admits a (k,d)-colouring if and only if, there is a homomorphism $f:G\to G_{k,d}$. Recall, a *homomorphism* $f:G\to H$ is a mapping $f:(G)\to V(H)$ such that $uv\in E(G)$ implies $f(u)f(v)\in E(H)$. We write $G\to H$ to indicate the existence of a homomorphism. It turns out that $G_{k,d}\to G_{k',d'}$ if and only if $k/d\le k'/d'$. Thus, given a graph G, if $G\to G_{k,d}$, then $G\to G_{k',d'}$ for any $k'/d'\ge k/d$ is surjective. Suppose ($k\ge 2d$), d is positive integer and k is prime number with gcd(k,d)=1, the circular chromatic numbers includes all chromatic numbers $\chi(G)=\chi_c(G)$ as well as odd holes see the below figures.

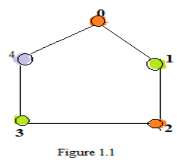
The *circular chromatic number* of a graph G is defined as $\boldsymbol{\chi_c(G)=\mathrm{Inf}\{k/d : G\to G_{k,d}}$ **and gcd(k,d)=1}.**

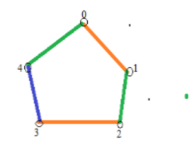
In [4], Bondy and Hell show the infimum may be replaced by a minimum. The proof depends on the fact that optimum colourings must be surjective. The surjective mappings play a key role in our constructions of non-extendible families.

Example 3.1.1: The *circular chromatic number* of a graph G is defined as $\boldsymbol{\chi_c(G)=\mathrm{inf}\{5/1 : G\to G_{5,1}}$ **and gcd(5,1)=1}.**

The adjacency matrix of X is

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$





Vertex coloring graph **Edge coloring graph**

(Fig 1.1)

The polynomial represented by X is $k(x)=1+x^4$

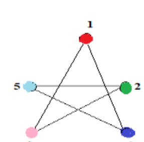
In above Figure 1.1, the vertex chromatic number (G)= 3 and Edge chromatic number is 3.

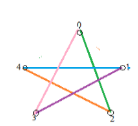
Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is 4, dimension of the code is **5** and has no error correcting codes. $\boldsymbol{(G_{5/1})=3}$

Example 3.1.2: The *circular chromatic number* of a graph G is defined as $\boldsymbol{\chi_c(G)=\mathrm{inf}\{5/2 : G\to G_{5,2}}$ **and gcd(5,2)=1}.**

The adjacency matrix of X is

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$





Vertex coloring graph **Edge coloring graph**

(Fig 1.2)

The polynomial represented by X is $k(x)=x^2+x^3$

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In above Figure 1.2, the vertex chromatic number (G)=5 and Edge chromatic number is 5.

Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is 3, dimension of the code is **5** and has no error correcting codes. **($G_{5/2}$)=5**

Example 3.1.3: The *circular chromatic number* of a graph G is defined as **$\chi_c(G)$=inf{5/3 : $G \rightarrow G_{5,3}$ and gcd(5,3)=1 }.**

The adjacency matrix of X is

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Vertex coloring graph **Edge coloring graph**

(Fig 1.3)

The polynomial represented by X is k(x)= x^2+x^3

In above Figure 1.3, the vertex chromatic number (G)= 5 and Edge chromatic number is 5.

Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is **3**, dimension of the code is **5** and has no error correcting codes. **($G_{5/3}$)=5**

(G)=inf{ 5/1, 5/2 and 5/3} is 5/1=3

Example 3.2.1: The *circular chromatic number* of a graph G is defined as **$\chi_c(G)$=inf{7/1: $G \rightarrow G_{7,1}$ and gcd(7,1)=1 }.**

The adjacency matrix of X is

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Vertex coloring graph **Edge coloring graph**

(Fig 1.4)

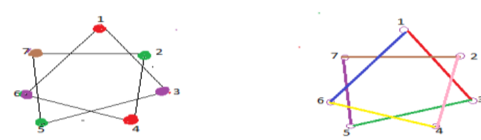
The polynomialrepresented by X is k(x)= $x+x^6$
In above Figure 1.4, the vertex chromatic number (G)=3 and Edge chromatic number is 3.

Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is **6**, dimension of the code is **7** and has no error correcting codes. **($G_{7/1}$)=3**

Example 3.2.2: The *circular chromatic number* of a graph G is defined as **$\chi_c(G)$=inf{7/2: $G \rightarrow G_{7,2}$ and gcd(7,2)=1 }.**

The adjacency matrix of X is

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$



Vertex coloring graph **Edge coloring graph**

(Fig 1.5)

The polynomial represented by X is k(x)= x^2+x^5
In above Figure 1.3, the vertex chromatic number (G)= 4 and Edge chromatic number is 7.

Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is **5**, dimension of the code is **7** and has no error correcting codes. **($G_{7/2}$)=4**

Example 3.2.3: The *circular chromatic number* of a graph G is defined as **$\chi_c(G)$=inf{7/3: $G \rightarrow G_{7,3}$ and gcd(7,3)=1 }.**

The adjacency matrix of X is

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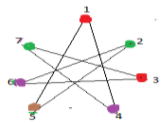
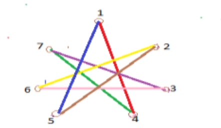
0	0	0	1	1	0	0
0	0	0	0	1	1	0
0	0	0	0	0	1	1
1	0	0	0	0	0	1
1	1	0	0	0	0	0
0	1	1	0	0	0	0
0	0	1	1	0	0	0



Vertex coloring graph



Edge coloring graph

(Fig 1.6)

The polynomial represented by X is k(x)= x^3+x^4

In above Figure 1.3, the vertex chromatic number (G)= 4 and Edge chromatic number is 7.

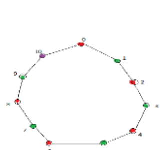
Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is **4**, dimension of the code is **7** and has no error correcting codes. **($G_{7/3}$)=4**

(G)=inf{ 7/1, 7/2 and 7/3} is 7/1=3

Example 3.3.1: The *circular chromatic number* of a graph G is defined as **$\chi_c(G)$=inf{11/1: $G\rightarrow G_{11,1}$ and gcd(11,1)=1 }.**

The adjacency matrix of X is

0	1	0	0	0	0	0	0	0	0	1
1	0	1	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0
0	0	0	0	1	0	1	0	0	0	0
0	0	0	0	0	1	0	1	0	0	0
0	0	0	0	0	0	1	0	1	0	0
0	0	0	0	0	0	0	1	0	1	0
0	0	0	0	0	0	0	0	1	0	1
1	0	0	0	0	0	0	0	0	1	0



Vertex coloring graph

Edge coloring graph

(Fig 1.7)

The polynomial represented by X is k(x)=$x+x^{10}$

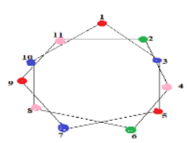
In above Figure 1.7, the vertex chromatic number (G)= 3 and Edge chromatic number is 3.

Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is 10, dimension of the code is 11 and has no error correcting codes. **($G_{11/1}$)=3**

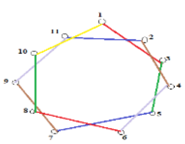
Example 3.3.2: The *circular chromatic number* of a graph G is defined as **$\chi_c(G)$=inf{11/2 : $G\rightarrow G_{11,2}$ and gcd(11,2)=1 }.**

The adjacency matrix of X is

0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	0	1
1	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	0	0	1	0	0	0	1
1	0	0	0	0	0	0	1	0	0	0
0	1	0	0	0	0	0	0	1	0	0



Vertex coloring graph



Edge coloringgraph

(Fig 1.8)

The polynomial represented by X is k(x)=x^2+x^9
In above Figure 1.8, the vertex chromatic number (G)= 4 and Edge chromatic number is 6.

Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is 9, dimension of the code is 11 and has no error correcting codes. **($G_{11/2}$)=4**

Example 3.3.3: The *circular chromatic number* of a graph G is defined as **$\chi_c(G)$=inf{11/3 : $G\rightarrow G_{11,3}$ and gcd(11,3)=1 }.**

The adjacency matrix of X is

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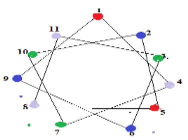
$$\begin{bmatrix} 0&0&0&1&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0&0&1&0\\ 0&0&0&0&0&0&1&0&0&0&1\\ 0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0 \end{bmatrix}$$




Vertex coloring graph **Edge coloring graph**

(Fig 1.9)

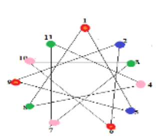
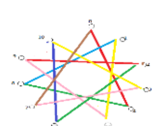
The polynomial represented by X is k(x)= x^3+x^7
In above Figure 1.9, the vertex chromatic number (G)= 4, edge chromatic number is 6.

Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is 7, dimension of the code is 11 and has no error correcting codes. $(G_{11/3})$**=4**

Example3.3.4: The *circular chromatic number* of a graph G is defined as $\chi_c(G)$**=inf{11/4 :** $G \to G_{11,4}$ **and gcd(11,4)=1}**

The adjacency matrix of X is

$$\begin{bmatrix} 0&0&0&0&1&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0&1&0&0\\ 0&0&0&0&0&0&1&0&0&1&0\\ 0&0&0&0&0&0&0&1&0&0&1\\ 1&0&0&0&0&0&0&0&1&0&0\\ 0&1&0&0&0&0&0&0&0&1&0\\ 0&0&1&0&0&0&0&0&0&0&1\\ 1&0&0&1&0&0&0&0&0&0&0\\ 0&1&0&0&1&0&0&0&0&0&0\\ 0&0&1&0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&1&0&0&0&0 \end{bmatrix}$$

Vertex coloring graph **Edge coloring graph**

(Fig 1.10)

The polynomial represented by X is k(x)= x^4+x^7
In above Figure 1.10, the vertex chromatic number (G)= 4 and Edge chromatic number is 6.

Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is 7, dimension of the code is 11 and has no error correcting codes. $(G_{11/4})$**=4**

Example 3.3.5: The *circular chromatic number* of a graph defined as $\chi_c(G)$**=inf{11/5 :** $G \to G_{11,5}$ **and gcd(11,5)=1}**
The adjacency matrix of X is

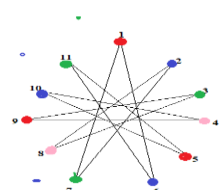
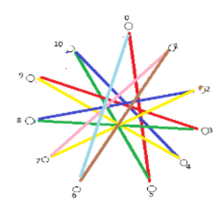
$$\begin{bmatrix} 0&0&0&0&0&1&1&0&0&0&0\\ 0&0&0&0&0&0&1&1&0&0&0\\ 0&0&0&0&0&0&0&1&1&0&0\\ 0&0&0&0&0&0&0&0&1&1&0\\ 0&0&0&0&0&0&0&0&0&1&1\\ 1&0&0&0&0&0&0&0&0&0&1\\ 1&1&0&0&0&0&0&0&0&0&0\\ 0&1&1&0&0&0&0&0&0&0&0\\ 0&0&1&1&0&0&0&0&0&0&0\\ 0&0&0&1&1&0&0&0&0&0&0\\ 0&0&0&0&1&1&0&0&0&0&0 \end{bmatrix}$$

.

Vertex coloring graph **Edge coloring graph**

(Fig 1.11)

The polynomial represented by X is k(x)= x^5+x^6
In above Figure 1.11, the vertex chromatic number (G)= 4 and Edge

chromatic number is 6.

Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is 6, dimension of the code is 11 and has no error correcting codes. $(G_{11/5})$**=4**

$\chi_c(G)$= inf{11/1, 11/2, 11/3, 11/4 and 11/5} is 11/1=3

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We observe that the above graphs , the two circular cliques $G_{k,d}$ at distance d such that some (k',d')-precolouring of the two cliques (Vertex chromatic number and Edge chromatic

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number d ≥ 2) are non-extendible. And the dimensions of generator matrix are same. Also the circular chromatic numbers are

- $\chi_c(G)$= inf{ 5/1, 5/2 and 5/3} is 5/1=3
- $\chi_c(G)$=inf{ 7/1, 7/2 and 7/3} is 7/1=3
- (G)= inf{11/1, 11/2, 11/3, 11/4 and 11/5} is 11/1=3.

HOMOMORPHISM OF A CIRCULAR GRAPHS:

A k-coloring, for some integer k, is an assignment of one of k colors to each vertex of a graph G such that the endpoints of each edge get different colors. The k-colorings of G correspond exactly to homomorphisms from G to the complete graph K_k.[3] Indeed, the vertices of K_k correspond to the k colors, and two colors are adjacent as vertices of K_k if and only if they are different. Hence a function defines a homomorphism to K_k if and only if it maps adjacent vertices of G to different colors (i.e., it is a k-coloring). In particular, G is k-colorable if and only if it is K_k-colorable.[3]

If there are two homomorphisms G → H and H → K_k, then their composition G → K_k is also a homomorphism.[1] In other words, if a graph H can be colored with k colors, and there is a homomorphism from G to H, then G can also be k-colored. Therefore, G → H implies χ(G) ≤ χ(H), where χ denotes the chromatic number of a graph (the least k for which it is k-colorable).[4]

DIRECT PRODUCT GRAPHS: The direct product

G × H of graphs G and H is the graph with the vertex set V (G) × V (H), two vertices (x, y) and (v, w) being adjacent in G × H if and only if xv ∈ E(G) and yw ∈ E(H).

PREPOSITION3.3: Let $\varphi:G\to H$ be a homomorphism and let $v\in V(G)$. The homomorphism φ is *uniquely extendible at* v if whenever $g:G\to H$ is a homomorphism with $g(u)=\varphi(u)$ for all $u\neq v$, then $g(v)=\varphi(v)$. If φ is uniquely extendible at v for all $v\in V(G)$, we simply say φ is ***uniquely extendible****[1]*.

PREPOSITION 3.4: Let G and H be graphs. The *extension product* G X H has as its vertex set $V(G)\times V(H)$ with $(g_1,h_1)(g_2,h_2)$ an edge if $g_1g_2\in E(G)$ and either $h_1h_2\in E(H)$ or $h_1=h_2$. The direct product of G with a reflexive copy (a loop on each vertex) of H.

Proof: The direct product of G and H is the graph, denoted as G×H, whose vertex is V (G)×V (H), and for which vertices (g, h) and(g′, h′) are adjacent precisely if gg′∈E(G) and hh′∈E(H).

Thus, V (G×H) = {(g, h) : g∈V (G) and h∈V (H)},
E(G×H) = {(g, h)(g′, h′) : gg′∈E(G) and hh′∈E(H)}.

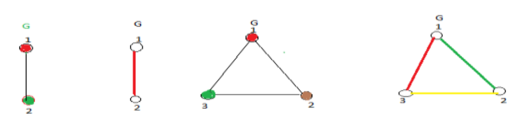
Other names for the direct product that have appeared in the literature are tensor product, Kronecker product, cardinal product, relational product, cross product, conjunction, weak direct product, or categorical product.

A product G×H has a loop at (g, h)if and only if both G and H have loops at g and h, respectively.

Moreover, if G has no loop at g, then the Hlayer H(g,h) is disconnected; whereas if G has a loop at g, then H(g,h) is isomorphic to H.

Suppose (g, h) and (g′, h′) are vertices of a direct product G×H and n is an integer for which G has a g, g′- walk of length n and H has an h, h′- walk of length n. Then G×H has a walk of length n from (g, h) to (g′, h′). The smallest such n (if it exists) equals d((g, h), (g′, h′)). If no such n exists, then d((g, h), (g′, h′)) = ∞.

Example 3.4.1: Let G and H be graphs.The *extension product* $\boldsymbol{\varphi}$**:**$G_{2,2}\to$ **$P_2\times P_2$** has as its vertex set $V(G_2)\times V(H_2)$ with $(g_1,h_1)(g_2,h_2)$ an edge if $g_1g_2\in E(G)$ and either $h_1h_2\in E(H)$ or $h_1=h_2$.



Vertex and Edge coloring graphs of G_2 and G_3

Let $\boldsymbol{\varphi}$**:**$G_{2,2}\to$ **$P_2\times P_2$** the generatoer of a matrix is $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$P_2\times P_2$ – Vertex and Edge graphs (Fig 1.12)

The polynomial represented by X is k(x)=1+x

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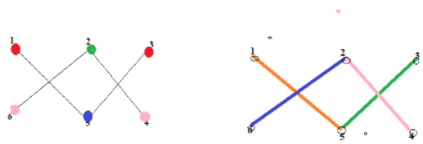
In above Figure 1.12, the vertex chromatic number **(G)= φ:$G_{2,2}$⟶ P_2×P_2** is **4,** edge chromatic number is **2**.

Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is 1, dimension of the code is 2 and has no error correcting codes.

Example 3.4.2: Let G and H be graphs.The *extension product* **P_2×P_3** has as its vertex set $V(G_2)\times V(H_3)$ with $(g_1,h_1)(g_2,h_2)$ an edge if $g_1g_2\in E(G)$ and either $h_1h_2\in E(H)$ or $h_1=h_2$.

Let φ:$G_{2,3}$⟶ P_2×P_3 , the generator of a matrix is $\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$



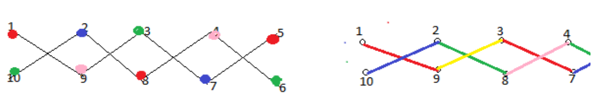
P_2×P_3 – Vertex and Edge graphs
(Fig 1.13)

The polynomial represented by X is k(x)=1 + x+x^2
In above Figure 1.13, the vertex chromatic number **(G)= φ:$G_{2,3}$⟶ P_2×P_3** is **4,** edge chromatic number is **4**.

Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is 2, dimension of the code is 2 and has no error correcting codes.

Example 3.4.3 : Let G and H be graphs.The *extension product* **φ:$G_{2,5}$⟶ P_2×P_5** has as its vertex set $V(G_2)\times V(H_5)$ with $(g_1,h_1)(g_2,h_2)$ an edge if $g_1g_2\in E(G)$ and either $h_1h_2\in E(H)$ or $h_1=h_2$.

Let φ:$G_{2,5}$⟶ P_2×P_5, the generator of a matrix is $\begin{bmatrix}1 & 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1 & 1\end{bmatrix}$



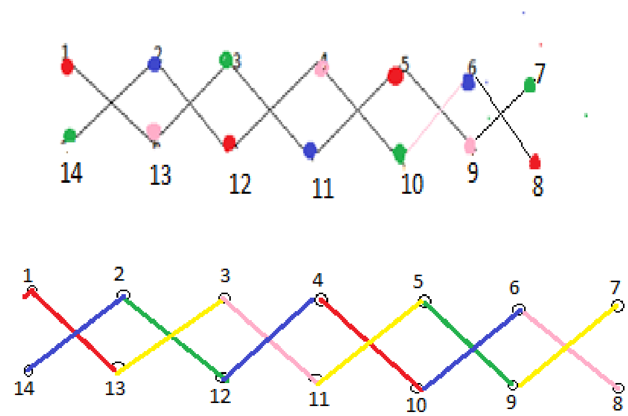
P_2×P_5 – Vertex and Edge graphs
(Fig 1.14)

The polynomial represented by X is k(x)= **1 + x + x^2+x^3+x^4**
In above Figure 1.14, the vertex chromatic number **(G)= φ:$G_{2,5}$⟶ P_2×P_5** is **4,** edge chromatic number is **6**.

Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is 4, dimension of the code is 2 and has no error correcting codes.

Example 3.4.4: Let G and H be graphs.The *extension product* **φ:$G_{2,7}$⟶ P_2×P_7** has as its vertex set $V(G_2)\times V(H_7)$ with $(g_1,h_1)(g_2,h_2)$ an edge if $g_1g_2\in E(G)$ and either $h_1h_2\in E(H)$ or $h_1=h_2$.

Let φ:$G_{2,7}$⟶ P_2×P_7, the generator of a matrix is $\begin{bmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1 & 1 & 1 & 1\end{bmatrix}$



P_2×P_7 – Vertex and Edge graphs
(Fig 1.15)

The polynomial represented by X is k(x)= **1 + x + x^2+x^3+x^4+x^5+x^6**
In above Figure 1.15, the vertex chromatic number **(G)= φ:$G_{2,7}$⟶ P_2×P_7** is **4,** edge chromatic number is **6**.

Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is 6, dimension of the code is 2 and has no error correcting codes.
We observe that from the graphs , the product of two circular cliques $G_{k,d}$ at distance d such that some (k',d ′)-precolouring of the two cliques(Vertex chromatic number and Edge chromatic number d ≥ **2**) are non-extendible. And the dimensions of generator matrix are same.

- $\chi(G)= \varphi:G_{2,2}\rightarrow P_2\times P_2$ is 4
- $\chi(G)= \varphi:G_{2,3}\rightarrow P_2\times P_3$ is 4
- $\chi(G)= \varphi:G_{2,5}\rightarrow P_2\times P_5$ is 4
- $\chi(G)= \varphi:G_{2,7}\rightarrow P_2\times P_7$ is 4

IV.CONCLUSION:

We finish the paper with an extension result for (k,d)-colourings of $G_{k,\mathrm{d}}$ cliques in k-colourable graphs.

1. In above figures, the two circular cliques $G_{k,\mathrm{d}}$ at distance d such that some (k',d')-precolouring of the two cliques(Vertex chromatic number and Edge chromatic number) are non-extendible. And the dimensions of generator matrix are same. The circular chromatic numbers are always same.

- $\chi_c(G)=$ inf{ 5/1, 5/2 and 5/3} is 5/1=3
- $\chi_c(G)=$inf{ 7/1, 7/2 and 7/3} is 7/1=3
- $(G)=$ inf{11/1, 11/2, 11/3, 11/4 and 11/5} is 11/1=3, etc.

2. $\varphi:G_{\mathrm{k,d}}\rightarrow P_n$, fi φ is uniquely extendible at v for all $v\in V(G)$, we simply say φ is *uniquely extendible*. The product of two circular cliques $G_{k,\mathrm{d}}$ at distance d such that some (k',d')-precolouring of the two cliques(Vertex chromatic number and Edge chromatic number d ≥ 2) are non-extendible. And the dimensions of generator matrix are same, but degree of the polynomials is increasing.

- $\chi_c(G)= \varphi:G_{2,2}\rightarrow P_2\times P_2$ is 4
- $\chi_c(G)= \varphi:G_{2,3}\rightarrow P_2\times P_3$ is 4
- $\chi_c(G)= \varphi:G_{2,5}\rightarrow P_2\times P_5$ is 4
- $\chi_c(G)= \varphi:G_{2,7}\rightarrow P_2\times P_7$ is 4

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