

# Blind Image Watermarking Using a Sample Projection Approach

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**Abstract**—this paper presents a robust image watermarking scheme based on a sample projection approach. While we consider the human visual system in our watermarking algorithm, we use the low frequency components of image blocks for data hiding to obtain high robustness against attacks. We use four samples of the approximation coefficients of the image blocks to construct a line segment in the 2-D space. The slope of this line segment, which is invariant to the gain factor, is employed for watermarking purpose. We embed the watermarking code by projecting the line segment on some specific lines according to message bits. To design a maximum likelihood decoder, we compute the distribution of the slope of the embedding line segment for Gaussian samples.

## I. INTRODUCTION

Digital watermarking embeds information within a digital work as a part of the media. Watermarking techniques falls

into three categories of robust, semi fragile and fragile methods according to their specific applications [1]-[3]. Robust watermarking mainly serves for identification purposes while the fragile and semi fragile watermarking are usually employed in authentication applications. Since a good watermarking scheme should always be able to deal with some kinds of attacks, studies in the watermarking research area mostly target robust watermarking problems. Several robust watermarking techniques have been proposed so far. Cox et al. [4] have proposed an additive watermarking approach based on spread spectrum concept which remains highly robust against noise and cropping attacks. Several other studies have improved this approach further [5]-[10]. Based on this observation that boosting the watermarking power increases the barrier against attacks, most of the effective watermarking schemes try to match the characteristics of

the watermark to those of the image asset. Multiplicative watermarking, as an example, has been introduced in [11] and has been widely studied later on using local optimum decoders in multi resolution transform domains such as wavelet and contour let domains [12]-[16]. Besides, a universal optimal detector for scaling based watermarking schemes is presented in [17]. These schemes are highly robust against noise and compression attacks. To satisfy robustness against geometric attacks and reduce the watermark

synchronization problem, Tang and Hang [18] proposed a watermarking scheme which employs a feature extraction and image normalization approach. Based on log polar mapping (LPM) and phase correlation, Zhen et al. [19] proposed an image watermarking technique which embeds the watermark into the LPMs of the image Fourier magnitude spectrum.

## II. SYSTEM MODELING

In this section, we first introduce the model considered for our watermarking algorithm. To this aim, we calculate the distribution of the watermarking variable. We assume to have four samples of an independently and identically distributed (i.i.d) Gaussian random variable as the host signal. We show this signal as  $\mathbf{u} = [u_1, u_2, u_3, u_4]$  with the Gaussian distribution of  $N(0, \sigma^2 u)$ . These four samples form two points  $p$

$= [u_1, u_2]$  and  $q = [u_3, u_4]$  in the 2-D space.

We employ  $c = \frac{u_4 - u_2}{u_3 - u_1}$ , the slope of the line going through these two points as our watermarking variable. We can see that the numerator and the denominator of  $c$  are distributed as  $N(0, 2\sigma^2 u)$ . For independent Gaussian variables  $a \sim N(0, \sigma^2 a)$  and  $b \sim N(0, \sigma^2 b)$ , their ratio  $c = \frac{a}{b}$  which is the ratio of two zero-mean independent Gaussian variables is Cauchy as:

$$f_C(c) = \frac{1}{\pi} \frac{\frac{\sigma_a}{\sigma_b}}{c^2 + \left(\frac{\sigma_a}{\sigma_b}\right)^2} \quad (1)$$

As shown in Appendix A, for the case that the two Gaussian variables  $a$  and  $b$  are correlated with the correlation coefficient

$$r = \frac{E[(a - \mu_a)(b - \mu_b)]}{\sigma_a \sigma_b},$$

the probability density function (PDF) of the variable  $c$  is given

$$f_C(c) = \frac{1}{\pi} \frac{\sigma_a \sigma_b \sqrt{1 - r^2}}{\sigma_b^2 \left(c - \frac{r \sigma_a}{\sigma_b}\right)^2 + \sigma_a^2 (1 - r^2)} \quad (2)$$

And its cumulative distribution function (CDF),  $F_C(c)$  can be computed as

$$F_C(c) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1} \frac{\sigma_b c - r \sigma_a}{\sigma_a \sqrt{1 - r^2}} \quad (3)$$

As we will see in the next sections, we use the probabilistic characteristics of the variable  $c$ , the slope of the mentioned line segment, to design an

optimal decoder and later analyze its performance.

**III. PROPOSED METHOD**

In this section, we introduce our blind watermarking scheme. As discussed in the previous section, we assume the host signal as four-sample i.i.d. Gaussian random signal. In practical applications; these four samples can come from

**A. Watermark embedding**

Let us represent the four samples of the host signal as  $\mathbf{u} = [u_1, u_2, u_3, u_4]$ . We model these four samples as two points  $p = [u_1, u_2]$  and  $q = [u_3, u_4]$  in the 2-D space. Fig.

1(a) illustrates these two points as well as the line segment connecting them. We denote the slope of this line segment as  $\theta$ . The center of this  $(p, q)$  line is located at  $[ \frac{u_1+u_3}{2}, \frac{u_2+u_4}{2} ]$ . By translating the center of this line to the origin, we reach to the points  $p_c$  and  $q_c$ , where

$$p_c = \begin{pmatrix} u_1' \\ u_2' \end{pmatrix} = \begin{pmatrix} \frac{u_1 - u_3}{2} \\ \frac{u_2 - u_4}{2} \end{pmatrix}, \quad q_c = \begin{pmatrix} u_3' \\ u_4' \end{pmatrix} = \begin{pmatrix} \frac{u_3 - u_1}{2} \\ \frac{u_4 - u_2}{2} \end{pmatrix}. \quad (4)$$

Now, to embed the  $M$ -ary watermark code, we project this line segment to one of the  $M$  coding lines ( $L_1$  to  $L_M$  if  $\theta > 0$  or their counterparts  $L'_1$  to  $L'_M$  if  $\theta < 0$ ), shown in Fig. 1(b), depending on the watermark code. We use projection and keep the center of the line segment to impose less distortion and thereby cause more invisibility of the watermarked signal. Fig. 1 (a) illustrates the projection steps in details. We call the resulted points in the mapped line as  $p$  and  $q$ . The slope of the  $i$ th coding

line is  $\alpha_i$  (for  $L_i$ ) if  $\theta > 0$ , the slope of the primary line connecting  $(p, q)$ , is positive and  $\alpha_i < \theta$  (for  $L_i$ ) otherwise, where  $\alpha_i$  is given by

$$\alpha_i = \tan\left(\frac{\pi}{4} - \frac{M - 2i + 1}{2} \beta\right). \quad (5)$$

Here,  $\beta$  is the angle between two consecutive coding lines. The position of  $p$  for the case that the slope of the mapped line is  $k$ , can be computed using the intersection of two following lines:

$$\begin{cases} y = kx \\ y = \frac{1}{k}(x - u_1') \end{cases}$$

With a similar scheme, we can find  $q$ . The obtained points are as follows:

$$p_{\perp} = \begin{pmatrix} \frac{u_1' + ku_2'}{k^2 + 1} \\ \frac{ku_1' + k^2u_2'}{k^2 + 1} \end{pmatrix}, \quad q_{\perp} = \begin{pmatrix} \frac{u_3' + ku_4'}{k^2 + 1} \\ \frac{ku_3' + k^2u_4'}{k^2 + 1} \end{pmatrix}. \quad (7)$$

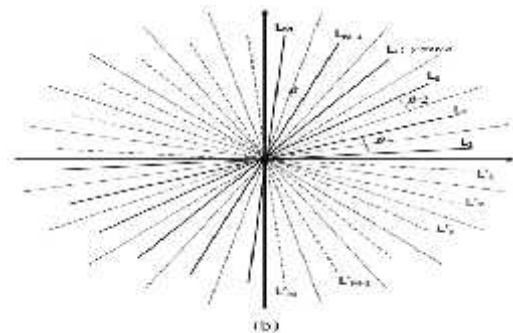
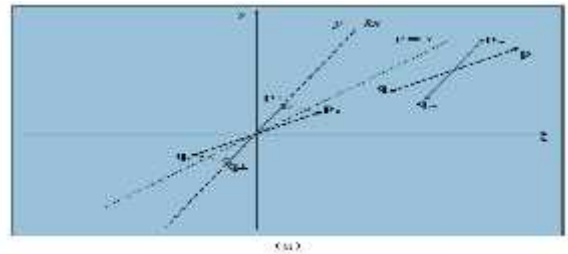


Fig. 1. (a) Steps of the proposed watermarking embedding scheme: translation to the origin, projection, and translation back. (b) Coding space.

Solid lines: coding lines with positive slopes ( $L1 \dots LM$ ); dashed lines: the counterpart coding lines with negative slopes ( $L1 \dots LM$ ); dotted lines:

Decision boundaries. As the final step, we just need to translate back the mapped

line to its original center to obtain points  $pw = [u_1, u_2]$  and  $qw = [u_3, u_4]$  as:

$$\begin{aligned}
 p_w &= \begin{pmatrix} u_1'' \\ u_2'' \end{pmatrix} = \begin{pmatrix} \frac{u_1' + kv_1}{k^2 + 1} & \frac{v_1 + u_2}{2} \\ \frac{kv_1 + u_2}{k^2 + 1} & \frac{u_2 + kv_2}{2} \end{pmatrix} \\
 q_w &= \begin{pmatrix} u_3'' \\ u_4'' \end{pmatrix} = \begin{pmatrix} \frac{u_3' + kv_3}{k^2 + 1} & \frac{v_3 + u_4}{2} \\ \frac{kv_3 + u_4}{k^2 + 1} & \frac{v_4 + u_1}{2} \end{pmatrix} \quad (8)
 \end{aligned}$$

By inserting (4) in (8) we can summarize the whole procedure as

$$\begin{pmatrix} u_1'' \\ u_2'' \\ u_3'' \\ u_4'' \end{pmatrix} = T(k) \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} \quad (9)$$

Using the matrix transfer matrix  $T(k)$  as, given as

$$T(k) = \frac{1}{2k^2 + 2} \begin{pmatrix} k^2 + 1 & k & k^2 & k \\ k & k^2 + 1 & k & 1 \\ k^2 & k & k^2 + 1 & k \\ -k & 1 & k & 2k^2 - 1 \end{pmatrix} \quad (10)$$

Therefore, the watermarking embedding process can be figured as implementation of (9) where  $k$  is defined based on the watermark bit and  $\theta$ , the slope of the primary  $(p, q)$  line, as:

$$k = \begin{cases} \theta & \text{when } \theta \geq 0 \\ -\theta & \text{when } \theta < 0 \end{cases}$$

In Fig. 1(b), we can see that for each coding word we have two lines, a solid one ( $L_i$ ) for positive  $\theta$  and a dashed one ( $L_i$ ) for negative  $\theta$ . In this way, we obtain the watermarked signal  $\mathbf{u} = [u_1, u_2, u_3, u_4]$ .

**B. Watermark decoding**

To extract the hidden bits, an optimum decoder is implemented using M-Hypothesis test as follows. We denote the received signal as  $\mathbf{y} = \mathbf{u} + \mathbf{n}$ , where  $\mathbf{y} = [y_1, y_2, y_3, y_4]$  represents the watermarked signal contaminated with zero

mean AWGN  $n \sim N(0, 2n)$ . For the i.i.d. Gaussian host signal  $\mathbf{u}$ , the watermarked signal  $\mathbf{u}$  is also Gaussian as it is obtained by a linear

transformation through the matrix  $T(k)$ . Thus, the received samples  $\mathbf{y}$  are Gaussian with the variance of  $\sigma_y^2 = \sigma_u'^2 + \sigma_n^2$ . Now, we can use the model given in Section II. We depict the four samples in  $y$  as two points  $pr = [y_1, y_2]$  and  $qr = [y_3, y_4]$  in the 2-D space and calculate the slope of the line segment connecting them as:

$$c = \frac{y_4 - y_2}{y_3 - y_1} = \frac{u_4'' - u_2'' + n_4 - n_2}{u_3'' - u_1'' + n_3 - n_1} \quad (12)$$

As we fixed the slope of the line connecting  $pw = [u_1, u_2]$  and  $qw = [u_3, u_4]$  to  $k$ , defined in (11), we have:

$$k = \frac{u_4'' - u_2''}{u_3'' - u_1''} \Rightarrow u_4'' - u_2'' = k(u_3'' - u_1'') \quad (13)$$

Therefore, by defining  $v = u_3, u_1$ , we can rewrite (12) as:

$$(11) \quad c = \frac{kv + n_4 - n_2}{v + n_3 - n_1} \quad (14)$$

Using (9) and (10), We Have

$$v = u_3'' - u_1'' = \frac{1}{k^2 + 1} (-u_1 - kv_2 + u_3 + kv_4)$$

The Correlation coefficient between  $a$  and  $b$  can be written as

$$r = \frac{kv}{\sqrt{(k^2\sigma_u^2 + 2\sigma_n^2)(\sigma_u^2 + 2\sigma_n^2)}} \quad (15)$$

Now we have  $c = a/b$ , where  $a$  and  $b$  are two correlated zero mean Gaussian random variables. Thus, the distribution function of  $c$  can be given as in (2). Having the distribution of the slope function, we can simply use the Maximum Likelihood detector as:

$$i^* = \arg \max_{i \in \{1, \dots, M\}} f_C(c|i),$$

where  $f_C(c|i)$  represents the distribution function for the  $i^{\text{th}}$  coding line. Here, without loss of generality, we suppose that the received  $c$  is positive. The same argument also holds for the negative case. Thus, the conditional distribution function is

$$f_C(c|i) = \frac{1}{\sigma_{a|i} \sigma_{b|i} \sqrt{1-r^2}} \exp\left\{-\frac{1}{2\sigma_{a|i}^2 \sigma_{b|i}^2 (1-r^2)} \left[ \frac{c^2}{\sigma_{a|i}^2} + \frac{1}{\sigma_{b|i}^2} - \frac{2rc}{\sigma_{a|i} \sigma_{b|i}} \right] \right\} \quad (17)$$

Where

$$\sigma_{a|i}^2 = \frac{2\alpha_i^2}{1-\alpha_i^2} \sigma_a^2 + 2\sigma_n^2, \quad \sigma_{b|i}^2 = \frac{2}{1-\alpha_i^2} \sigma_b^2 + 2\sigma_n^2, \quad (18)$$

$$r = \frac{\frac{2\alpha_i \sigma_a \sigma_b}{1+\alpha_i^2}}{\sigma_{a|i} \sigma_{b|i}} = \frac{\alpha_i \sigma_a \sigma_b}{\sqrt{(\alpha_i^2 \sigma_a^2 + (1-\alpha_i^2) \sigma_n^2)(\sigma_b^2 + (1-\alpha_i^2) \sigma_n^2)}} \quad (19)$$

These parameters are computed by substituting  $k$  as defined in (11) in definitions of variances of  $a, b$  and their correlation coefficient  $r$ .

### IV. PERFORMANCE EVALUATION

Here, we analytically study the error probability of the proposed watermarking scheme in the presence of AWGN. First, we compute  $P^+(e|i)$ , the error probability when the slope of embedding line  $(p, q)$  is positive, given as:

$$P_e^+ = \frac{1}{M} \sum_{i=1}^M P^+(e|i), \quad (20)$$

where  $P^+(e|i)$  represents the error probability when the embedding line is  $L_i$ , the  $i^{\text{th}}$  positive slope coding line. According to (16), for  $i = 2 \dots M-1$ , the error probability can be written as

$$P^+(e|i) = P\left(\tan\left(\phi_i + \frac{\beta}{2}\right) < c < \tan\left(\phi_i - \frac{\beta}{2}\right)\right) + P\left(\tan\left(\phi_i + \frac{\beta}{2}\right) < c | i\right) + P\left(c < \tan\left(\phi_i - \frac{\beta}{2}\right) | i\right) - F_{L_{i+1}}\left(\tan\left(\phi_i + \frac{\beta}{2}\right)\right) - F_{L_{i-1}}\left(\tan\left(\phi_i - \frac{\beta}{2}\right)\right) - 1 + F_{L_{i+1}}\left(\tan\left(\phi_i + \frac{\beta}{2}\right)\right) + F_{L_{i-1}}\left(\tan\left(-\phi_i - \frac{\beta}{2}\right)\right). \quad (21)$$

where,  $c$  is the slope of the detected line, and  $F_C/i(\cdot)$  is its CDF if the embedded line is  $L_i$ . Substituting  $F_C(c)$  from (3) and defining

$$\phi_i' = \phi_i + \frac{\beta}{2} = \frac{\pi}{4} + \frac{M-2i}{2} \beta,$$

$$\phi_{i-1}' = \phi_i - \frac{\beta}{2} = \frac{\pi}{4} - \frac{M-2(i-1)}{2} \beta, \quad (22)$$

and  $d_i = \frac{\sigma_{b|i}}{\sigma_{a|i}}$ , we have:

$$P^+(e|i) = \frac{1}{\pi} \tan^{-1} \left( \frac{d_i \tan(\phi_{i-1}')}{\sqrt{1-r^2}} \right) + \frac{1}{\pi} \tan^{-1} \left( \frac{d_i \tan(\phi_i')}{\sqrt{1-r^2}} \right) + 1 - \frac{1}{\pi} \tan^{-1} \left( \frac{d_i \tan(\phi_i')}{\sqrt{1-r^2}} \right) + \frac{1}{\pi} \tan^{-1} \left( \frac{d_i \tan(\phi_i')}{\sqrt{1-r^2}} \right) \quad (23)$$

we can simplify (23) as:

$$P^+(e|i) = 1 - H_1(\phi_{i-1}') - H_1(\phi_i'), \quad (24)$$

For  $i = 1$  and  $i = M$ , we can see that:

$$P^+(e|1) = 1 - P\left(\tan\left(-\phi_1 - \frac{\beta}{2}\right) < c < \tan\left(\phi_1 + \frac{\beta}{2}\right)\right) = 1 - H_1(\phi_1'), \quad (25)$$

$$P^+(e|M) = 1 - P\left(c > \tan\left(\phi_M + \frac{\beta}{2}\right)\right) - P\left(c < \tan\left(\phi_M - \frac{\beta}{2}\right)\right) = H_M(\phi_{M-1}'). \quad (26)$$

Substituting (24), (25), and (26) in (20), we have:

$$P_e^+ = \frac{M-1}{M} + \frac{1}{M} \sum_{i=2}^M [H_1(\phi_{i-1}')] - \frac{1}{M} \sum_{i=1}^{M-1} [H_1(\phi_i')] = \frac{M-1}{M} + \frac{1}{M} \sum_{i=1}^{M-1} [H_{i+1}(\phi_i') - H_i(\phi_i')]. \quad (27)$$

## IMAGE WATER MARKETING:

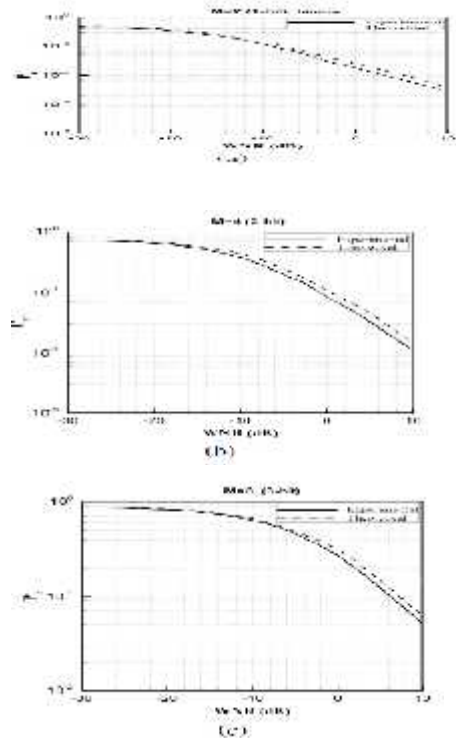


Fig. 2. Comparison between the theoretical error probability and experimental one for a Gaussian random variable with  $u = 40$ . (a) one bit embedding ( $M = 2$ ), (b) two bits embedding ( $M = 4$ ), and (c) three bits embedding ( $M = 8$ ).

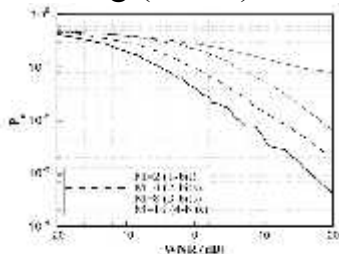


Fig. 3. Generating the probability of error with DVB or 19 dB for different values of M.

With a similar argument we can see that the error probability for the negative slope embedding line is the same as  $P_{e^-}$ ; thesis  $P_{-e} = P_{e^-}$ . Besides, it is easy to show that the

positive and the negative embedding are equally probable in general; Therefore, we can write the error probability of the decoder as

$$P_e = \frac{1}{2} P_{e^+} + \frac{1}{2} P_{e^-} = P_{e^+} \quad (28)$$

Here, we obtained a closed form solution for the error probability of the decoder. In Fig. 2 we compared this theoretical error probability with the experimental case of Gaussian random variable with  $u = 40$  for different capacities of one bit ( $M = 2$ ), two bits ( $M = 4$ ), and three bits ( $M = 8$ ). In this figure, the probability of error is shown versus various noise strengths, measured by the watermark Noise ratio (WNR)

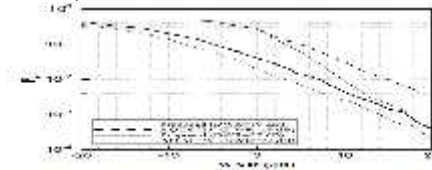


Fig. 4. Comparison between the theoretical and experimental error probabilities for the Gaussian random variable with  $u = 40$ .

## V. EXPERIMENTAL RESULTS

**1) AWGN attack:** As the first attack, we investigate the effect of AWGN to the proposed watermarking scheme. shows the bit error rate (BER) of the proposed method for various images versus different noise powers.

**2) JPEG attack:** Secondly, the proposed technique is tested against JPEG compression with different quality factors. As demonstrated in Fig 8, since low frequency components of the image blocks are used for watermarking, the proposed method is highly robust against JPEG with different quality factor up to 10%.

**3) Filtering attack:** Table I shows the BER results for median filtering, and Gaussian low-pass filtering attacks for different test images. It can be seen that

## VI. CONCLUSION

In this article, we presented a novel blind watermarking approach with the optimal decoder. Watermark embedding is performed by multiplication of  $M$  specific matrices to the vector of samples of size four. These matrices translate and project the vector of samples on the predefined coding lines depending on the message symbol. Assuming the host samples to be i.i.d Gaussian, which is often valid for approximation coefficients of image blocks [15], we obtained a closed form PDF of noisy watermarked samples. Having this distribution function, we designed an optimum ML decoder. We analytically studied and verified the error probability of the proposed decoder in a noisy environment

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