Bond Graph Modeling of Single Stage Reciprocating Air Compressor

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***Abstract*—The Bond Graphs (BG) are used in power simulation since 1983. The power of BG modeling escorts to model physical system systematically. The present work explains modelingof single stage air compressor. The input power is metered at electrical motor and is also sketched at mechanical output and thermodynamic work done. This is a classical paradigm of diverse domains working together. The motor represents electrical domain; the slider crank arrangement represents mechanical;the compression of air represents thermal, and pressure and mass flow rate of air from the inlet to outlet represents pseudopneumatic domain. The modeling software used is 20Sim. The results of simulation validate the power of bond graph modeling in simulation.**

Keywords— Bond Graph Modeling, Compressor, Multi-domain Analysis

I. INTRODUCTION

The modeling and simulation have an increasing importance in the development of complex and large mechanical system, in order to design for optimum performance. There are many ways to simulate the system right from finite element method to MATLAB. All the simulation software solve the governing differential equations, majority of times, are nonlinear, higher order and partial differential equations. An adaptive way of solving these differential equations is reducing them to sequential first order ordinary differential equations and applying a numerical method to solve. This approach is called as State Space Methods. In Bond Graph simulation, the solution variables are power variables as effort and flow such that product of these two variables result in power. While representing or modeling in BG, the dynamics of the system is obtained by basic laws of the physics of respective system or domain. The BG modeling is unified approach especially useful when the system becomes more complicated or multidisciplinary.

The reciprocating air compressor is the one such example of multidisciplinary i.e. combination of the electrical system or domain, mechanical domain, Thermal domain, pneumatic domain and pseudopneumatic domain. Thus it is opted for BG simulation. The three main objectives of the work are (i) to identify fundamental phenomenon that contribute to air compressor e.g. slider crank, etc., (ii) represent each component based on constitutive equations in BG i.e. each subcomponent is represented as BG sub-model (iii) and connect all such sub-models with respect to input, output, casualty, etc., to obtain the BG representation of air compressor system as a whole.

In this work it is attempted to represent the air compressor as a system of different subsystems represented in the form of constitutive elements involved in BG representation like R, C, etc., which will constitute a library so that the next versions in the same domain could be simulated with use of such elements as basic block building elements.

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II. The Bond Graph Review

A BG isa graphical representation ofa physical dynamic system. It is similar to the better known block diagram and signal-flow graph. The major is that the arcs in bond graphs represent bi-directional exchange of physical energy. While those in block diagrams and signal-flow graphs represent uni-directional flow of information [1].

BG were devised by Professor H. Paynter as early as 1959. Professor Karnopp, Professor Margolis and Professor Rosenberg elaborated this graphical model representation into a methodology that has experienced a considerable progress over the decades due to the steady work of many researchers all over the world [1].

In the BG modelling technique, a physical system can be represented by symbols and lines, identifying the power flow paths. Various symbols used in the bond graph are categorized into four groups. First group constitutes of the one port passive elements viz. Inertances (I), compliances (C), and dissipaters (R). The external source inputs to the system are expressed as source of effort (Se) or source of flow (Sf) elements. Two multi-port elements transformer (TF) to perform flow to flow or effort to effort conversion and gyrator (GY) to convert flow to effort or effort to flow. System constraints are represented using '1' junction (representing constant flow) and '0' junction (representing constant effort) elements. [9]

The line segments connecting the elements are called bonds. Each bond carries the associated effort and flow information. The half arrow on the bond represents the

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assignment of power direction to the bond. The power direction assignment in a BG is arbitrary in nature and may be compared with the fixing of coordinate systems in the corresponding physical domain. The notion of causality provides a tool for the formulation of system equations. The notion of causality also enables qualitative analysis of system behaviour.

W. Borutzky [1] developed the bond graph model of the electric motor. J.Granda [2] derived the Bond graph model for the transformation of the piston force into rotary torque of crank. Single TF element shows the transformation of effort and flow from the piston, connecting rod to crank. This single TF element could not tell exact energy transformation between piston, connecting rod and crank. The bond graph of this slider crank mechanism could be improved.

T.Bera et al. [3] developed bond graph model of a single cylinder engine and demonstrated that it is possible to create an integrated model of thermal, pneumatic and mechanical domains. The three port C field element has been used to model combustion process, mechanical process and pneumatic process. The same concept is inherited in the compressor simulation.

P.Feenstra et al. [4] have demonstrated complete Bond graph of engine cooling system, which is taken further for developing intercooler of the air compressor

H. Engja [5] has developed the bond graph model of single stage reciprocating air compressor however in this study the main aim is to author the standard library for different domains. That is after single stage representation, using the same elements next stage is simply added to simulation with little changes in the parameters like bore diameter, etc.

III SINGLE STAGE AIR COMPRESSOR

The process of modeling initiates with identifying the processes involved, their constitutive equations, modeling these components individually in BG. The individual systems with respective inputs are validated for relevant outputs. For example, the transformation of rotational domain to the linear domain is demonstrated independently for simulation in BG. Finally, these BG models for individual components are integrated together to form the overall representation.

The prime mover for reciprocating air compressor is electric motor. The two pulley arrangement converts the electrical energy into mechanical rotary motion. The rotary motion is transmitted into linear motion of the piston by using slider crank mechanism. This linear motion of the piston causes change of volume into the cylinder. At the time of suction air flows inside the cylinder through inlet valves; piston compresses the air and when pressure of air is greater than the critical pressure then pressurized air flows outside the cylinder through outlet valve. For the modeling purpose, the compressor is divided into following subsystem

A. The Input angular velocity
B. The Slider-Crank mechanism
C. The cylinder
D. The Suction and Discharge valve

Each subsystem is modeled independently, and the submodels are then combined into a complete model of the system.

A. *The Input angular velocity*

The rpm developed by electrical motor is taken as input angular velocity. The motor is coupled to crank of the compressor with V-belt and pulleys. The diameter of the motor pulley and flywheel pulley constitute the reduction ratio. The angular velocity at compressor pulley is scaled down while torque scaled up.

This is modeled as SF and TF elements in BG as shown in Fig. 1. The simulation model begins with electrical motor output i.e. angular velocity (rpm) through motor pulley. This is applied to compressor pulley through definite reduction ratio. This ratio is defined in TF. The TF has two ports i.e. input and output. Both these ports are in mechanical rotational domain; i.e. power variables are angular velocity and torque.

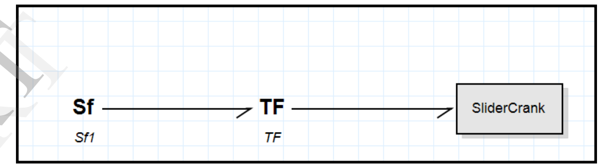

Fig. 1 Bond Graph Model of the Motor

The TF input port is connected to SF. The SF element has preferred flow out casualty. This flow is connected to TF input port. The reduction ratio is defined in TF. The output port of TF has preferred flow out casualty as shown in Fig. 1. The model outputs scaled down angular velocity. Here p1.e & p1.f torque & rpm at input and p2.e & p2.f torque & rpm at output.

```
parameters
      real r = 0.5;
equations
      p1.e = r * p2.e;
p2.f = r * p1.f;
```

B. *Slider-Crank Mechanism*

The crank is coupled to the flywheel pulley as discussed earlier. The angular velocity of the crank is converted into linear velocity of the piston through connecting rod. The pressure on the other side of the piston varies continuously as a result of compression and expansion because of piston motion. Thus, this pressure times area is the force acting on crank through connecting rod through piston. This force times crank length is the torque at pivot.

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BG Model of Crank

Fig. 2 shows schematic of slider crank mechanism. The piston motion is assumed to be y direction and x direction is the horizontal component. The motion of the piston in x direction is zero and motion of the piston in y direction is addition of velocity of the crank end in y direction and velocity of connecting rod end at piston in y direction.

The angular velocity ω of the crank gives the vertical component of the crank end to be $r \sin\theta$, and horizontal component is $r \cos\theta$. The angle θ is the angle subtended by crank it is calculated by integrating angular velocity ω. Similarly the perpendicular force at the crank end gives torque at the compressor crank.

$$\dot{Y}_{CR} = L_{CR}\sin(\theta)\omega \qquad (1)$$

$$\tau = F_{CR} \times L_{CR} \qquad (2)$$

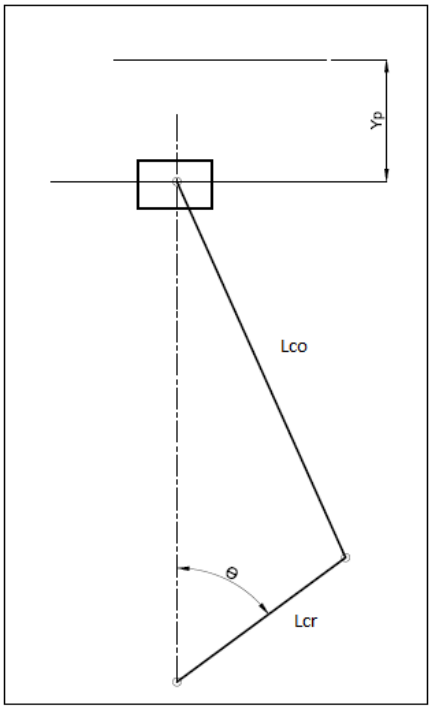

Fig 2 Schematic of Slider Crank Mechanism

These constitutive relations are employed in the BG by element *Crank*. This *Crank* element is similar to the *TF* function. It has two ports i.e. input and output port. The input port is in mechanical rotational domain and output port is in mechanical linear domain. The input port is set to have preferred flow out causality so as the output port. The *Crank* transfers angular motion into linear motion. The reciprocated power variable i.e. effort is torque. The torque is estimated by the force exerted by the compressor air on the piston top. The component of this force perpendicular to crank direction times crank length is nothing but the torque acting on a pivot. This is shown in Fig. 3. The program used in BG of *Crank* are as under.

```
parameters
      real global CrL;
variables
      real global theta;
equations
      theta = int(p1.f);
      if theta > 2 * pi then
            theta = theta - 2*pi;
      else
            theta = theta;
      end;
      p1.e = CrL * p2.e;
      p2.f = CrL*sin(theta)*p1.f;
```

TABLE I Parameters considered for Slider Crank Mechanism

Parameters	Symbol	Value	Units
Length of Crank	L_{cr}	0.0275	M
Length of Connecting Rod	L_{CO}	0.073	M

Where CrL is crank length, p1.f is the angular velocity at the compressor p2.f is a linear velocity of the crank in y direction theta is the angle subtended by crank it is computed by integrating angular velocity. p2.e is the perpendicular force at the crank end, and p1.e is the torque at the crank end.

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BG Model of Connecting Rod

The connecting rod subtends an angle φ at the piston pin; the φ is worked out from the geometrical relationship as

$$\varphi = sin^{-1}\{L_{cr}\sin(\theta)/L_{co}\} \qquad (3)$$

The linear velocity of the connecting rod at the crank end is

$$\dot{Y}_{CO} = -L_{CO}\sin(\Phi)\frac{d\varphi}{dt} \qquad (4)$$

The perpendicular force at the crank end is transferred through connecting rod as it is from the piston. Therefore the force of connecting rod and that of the piston is same. The connecting rod is modeled as single port element. This is because it gives out linear velocity and force across the connecting rod is same. The interface for the port of connecting rod is mechanical linear motion. It works on preferred flow out casualty. The angle *phi* is input for connecting rod which is, in fact, a public variable shared with the slider crank mechanism.

```
parameters
      real global CrL,CoL;
variables
      real global phi;
      real global theta;
equations

phi=arcsin(CrL*sin(theta)/CoL);
p.f = CoL*sin(phi)*ddt(phi);
```


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Where phi is angle φ, CoL is connecting rod length p.f is vertical component of velocity of connecting rod end at the piston side.

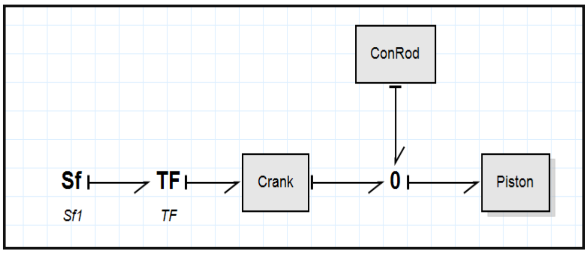

Fig. 3 Bond Graph Model of Slider Crank Mechanism

The linear velocity addition to linear velocity of the crank end which is applied to the piston is calculated based on the value of *phi* using the equation (3) and (4).Since the effort (force) between the piston, connecting rod, and crank end are same, the zero junction is used for model of slider crank mechanism. This means the *Crank* element and the *ConRod*element are connected by '0' junction as shown in Fig. (3) This perpendicular force at the crank produce torque at the center of the crank and this torque is transferred to the motor through *TF* element

Simulation of Slider Crank Mechanism

The parameters of Slider Crank mechanism are mentioned in Table 1. The simulation results of crank end velocity in y-direction, velocity of connecting rod at the piston end and linear velocity of piston is shown in Fig. 4. Simulation is done for the 2 revolution of the Crank i.e. for 0.165 seconds.

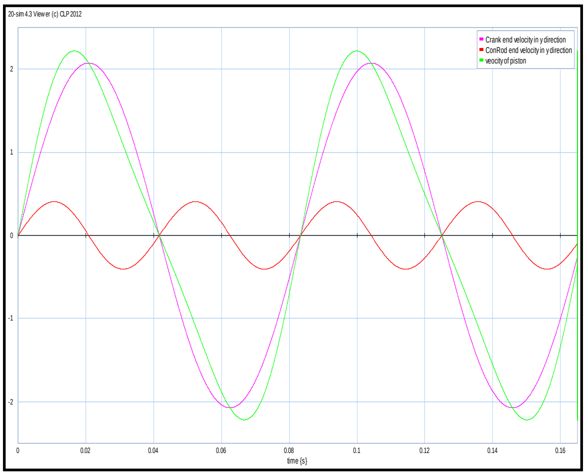

Fig. 4 Response of Slider Crank Mechanism

It is seen that velocity of crank in y direction is sinusoidal with amplitude of 2.07 m/s and velocity of connecting rod at the piston end is also sinusoidal with amplitude 0.40 m/s and linear velocity at the piston is deviated from sinusoidal nature because of effect of both the crank and connecting rod.

BG Model of Piston

The piston is the last element in slider crank mechanism. The piston is the manifestation of the power variables namely force and linear velocity on one side and pressure and volume rate of flow on the other side. The linear velocity of the piston when multiplied by the area of the piston determines the rate of change of volume. This transformation is based on the area of the piston. The linear velocity of piston times area of the piston is rate of change of volume. The constitutive equations for the Piston element is hereunder.

$$Q = A_p \dot{X}_p \qquad (5)$$

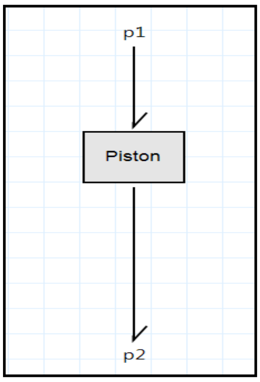

Fig5 BG model of the Piston

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The *Piston*shown in Fig. 5 is two ports element. The input and output port are mechanical linear domain and pneumatic domain respectively. The casualty for the input port is set to preferred effort (force) out, and output port is set to have preferred flow (volume rate of flow) out causality. The program used in the BG of the *Piston* are

```
variables
       real global area;
       real Force;
       real global phi;
equations
       Force = area * p2.e;
       p1.e = Force*cos(phi);
       p2.f = area * p1.f;
```

where p1.f is the velocity of piston p2.f is the volume flow in the cylinder obtained by the multiplying area of the piston. The p2.e is the pressure created in the cylinder and Force is the force generated by the piston, the component of this force along the connecting rod is represented by p1.e.

C Cylinder

The pressure inside the cylinder could be inferred from the polytrophic process $p\ v^n = C$. The mathematical implementation is manifested by

$$v_2 = v_1 - \int Q dt \qquad (6)$$

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$$p_2 = \frac{C}{{v_2}^n} \quad (7)$$

This high pressure of the air produces the force on the piston. There are two components of the force, one along the direction of the piston and other along the direction perpendicular to piston motion. The component of these forces along direction perpendicular to crank produces torque at the crank pivot. The flow is integrated, and the effort is worked out the output of the process it can be represented by C element. The C element is set to have preferred effort out causality as shown in Fig. 6.

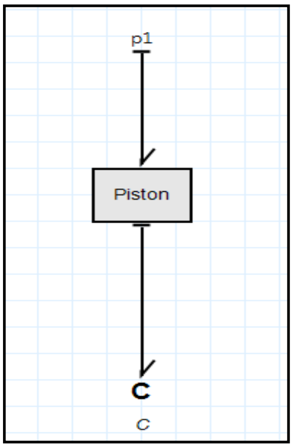

Fig. 6 Bond Graph model of low-pressure cylinder

The BG model for the cylinder is shown in Fig. 6, it receives $\dot{v}$ from the piston and computes pressure. The program used for the *C* element are

```
parameters
      real global n;
      real global Lpcvol;
      real global Ta;
      real global R;
      real Thigh = 383;
variables
      real global theta;
      real global Lpv1;
      real v2,C;
      real T2;
equations
      state = int(p.f);
      v2 = Lpv1 - state;
      if theta < pi then
            C = pa * v1^n;
      else
            C = pc * cvol^n;
      end;
      p.e = C/(v2)^n;
      if (p.e>pmax) then
            p.e = pmax;
      end;
      if (p.e<pmin) then
            p.e = pmin;
      end;
```

BG equations are based on (6) and (7). The p.e is the pressure generated at the surface of the piston is computed by polytrophic process p v^n = C. The `v2` is the instantaneous volume of the cylinder, `v1`is the total volume of the cylinder and `cvol` is the clearance volume of the cylinder. The `pmax` and `pmin` are maximum and minimum values of cylinder pressure. The maximum value is slightly greater than output pressure of compressor to open the outlet valve and minimum value is bit smaller than intake pressure or atmospheric pressure to open the inlet valve.

Simulation Result of Pressure in cylinder

Table 2 shows the parameters used during simulation of pressure. The maximum pressure in the compressor will be equal to P_{max} and minimum pressure in the compressor will be equal to $P_{min.}$ The compression and expansion of air follows the law pv^n = C.

TABLE 2 Parameters considered for Low pressure cylinder head

Parameters	Symbol	Value	Units
Minimum Pressure	P_{min}	$1.012(10^5)$	N/m^2
Maximum pressure	P_{max}	$1.914(10^5)$	N/m^2
Stroke	s_p	0.055	M
dia of the piston	d_p	0.065	M
polytrophic index	n	1.25	-
clearance volume	v_{cl}	3.5955e-5	m^3

Fig. 7 shows simulation result with respect to time and Fig. 8 shows PV diagram of the air during compression and expansion.The PV diagram in Fig. 8 clearly shows that during compression and expansion, the process is polytrophic. While inlet and exhaust valves are open, the process is constant pressure process.

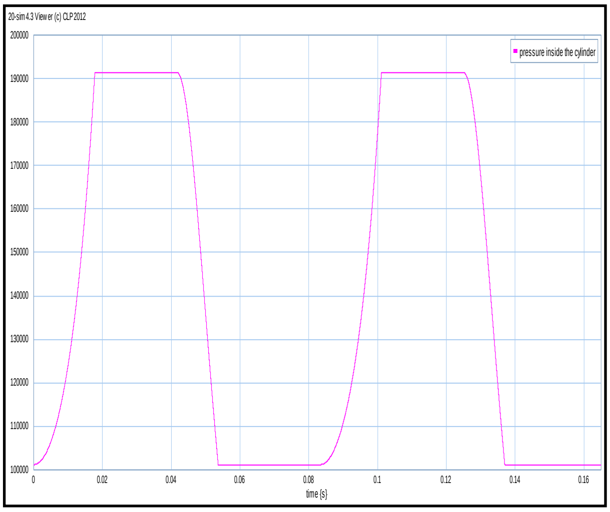

Fig. 7 Simulation result pressure in cylinder

The results are encouraging and in close tolerance of the values in the published literature.

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D. The BG Model of Valves

The valves are assumed to be treated as simple orifice. There are two pressure sources in both valves they are modeled as signal sources `Se` for inlet valve and `Se1` for an outlet valve as shown in Fig. 9.

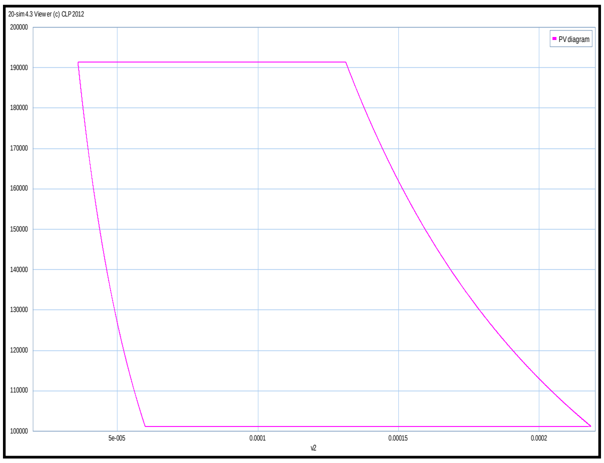

Fig. 8 PV diagram of air compressor

The Inlet valve opens when pressure inside the cylinder is below atmospheric the mass flow rate through the valve $\dot{m}$ is the function of valve area, upstream pressure P_u and downstream pressure P_d and it can be calculated using well known relation mentioned in (8). The upstream pressure is atmospheric pressure and downstream pressure is pressure inside the cylinder.

$$\dot{m} = \rho c_d A\sqrt{2(P_u - P_d)/\rho} \quad (8)$$

The outlet valve opens when pressure inside the cylinder is above the critical pressure. In the case of the outlet valve, upstream pressure is the pressure inside the cylinder and downstream pressure is the critical pressure.

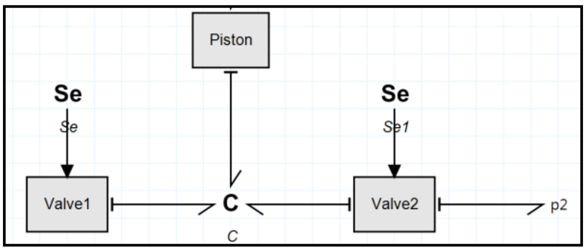

Fig 9 Bond graph model of inlet and Exhaust valve

The BG for the intake and exhaust valves is shown in Fig. 9. The *C* element is a 3 port element one port is pneumatic port which is connected to piston; it takes volume flow from pistons and computes pressure as explained. The other two ports are pseudopneumatic ports. The bond variables of these ports are pressure and massflow. The port which is connected to intake valve is having preferred effort out causality, and corresponding valve port has preferred flow out causality. The inlet valve compares signal pressure which is atmospheric and pressure of *C* element which is intake pressure of the cylinder, and when there is the pressure difference then, mass of air flows inside the cylinder. The outlet valve works in a similar manner as that of the inlet valve and in case of the outlet valve the comparing pressures are maximum pressure of cylinder and exhaust pressure of the cylinder.

Bond Graph program of *Valve1 (Inlet valve)*

```
parameters
      real rho =1.29;
      real Cd = 0.62;
      real Do = 0.013;
variables
      boolean inlet;
equations
      Ao = pi/4*(Do)^2;
      if p - p2.e > 0 then
            inlet = true;
      p2.f = rho*Cd*Ao ~
           *sqrt(2*g_n*(p-2.e));
      else
            inlet = false;
            p2.f = 0;
      end;
```

where p2.e is the pressure inside the cylinder, it acts as downstream pressure and p are the atmospheric pressure it acts as upstream pressure. The p2.f is massflow going inside the cylinder. When there is the pressure difference between inside and outside cylinder then only mass of air flows inside the cylinder otherwise massflow is zero. The Cd is coefficient of discharge of orifice and the Ao is the area of the orifice. Bond graph program for the *valve2* (outlet valve) are similar to inlet valve equations, only difference is, in case of the outlet valve upstream pressure is the pressure inside the cylinder,and downstream pressure is output pressure

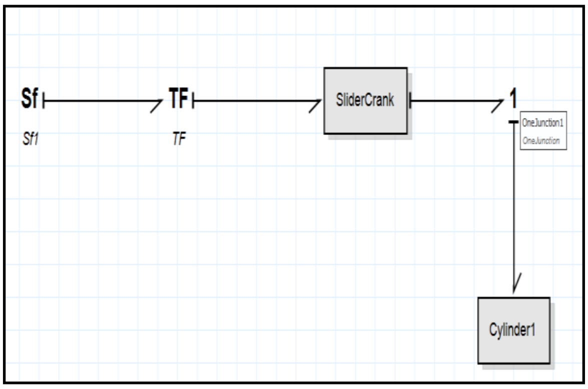

Fig.10 : Complete Bond Graph of Single Stage reciprocating air compressor

The complete BG model of the single stage air compressor is shown in Fig. 10.

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IV CONCLUSION AND FUTURE SCOPE

The BG modeling of single stage reciprocating air compressor is discussed starting with angular velocity of electrical motor as input, gradually illustrating the change in domain, respective effort and flow variablesthereof with proper casualty assignments. The significant outcomes of the work are

1. The rotational mechanical input is transferred with TF to reciprocating linear output. The each element like crank, in slider crank mechanism, has a model in this representation.
2. The piston is represented as a multi domain element, on one side it is linear mechanical and other side it has thermal domain.
3. The cylinder is represented as C element which is combination of thermal and pneumatic domains. The critical pressure is the parameter operating the inlet and outlet valves, which completes the representation.
4. The P-V diagram obtained as a result of simulation clearly confirms the accuracy of the representation. The Pressure-time diagrams also manifest the precision of the results.
5. Finally it is successfully demonstrated the way to implode the sub systems and combine to form the overall representation without any changes in any of parameter, casualty, etc.

Future Scope

With the elements developed so far, the next scope would be

1. To simulate two stage reciprocating air compressorsby adding next cylinder using developed model with some parameter change like diameter of cylinder, etc.
2. To add the intercooler based on constitutive equations of heat transfer.
3. To model the electrical motor and investigate the effect of adding a stage, adding intercooler on power consumed, with experimental verification.

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