Buckling Analysis of laminated Composite Cylindrical Shells Subjected to Axial Compressive Loads Using Finite Element Method

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Abstract

The Laminated cylindrical shells are being used in submarine, underground mines, aerospace applications and other civil engineering applications. Thin cylindrical shells and panels are more prone to fail in buckling rather than material failure. In this present study linear and non-linear buckling analysis of GFRP cylindrical shells under axial compression is carried out using general purpose finite element program (ANSYS). Non-linear buckling analysis involves the determination of the equilibrium path (or load-deflection curve) upto the limit point load by using the Newton-Raphson approach. Limit point loads evaluated for geometric imperfection magnitudes shows an excellent agreement with experimental reults [25]. The influence of composite cylindrical shell thickness, radius variation on buckling load and buckling mode has also investigated. Present study finds direct application to investigate the effect of geometric imperfections on other advanced grid-stiffened structures

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1. Introduction

Various fields of engineering such as civil, mechanical, aerospace and nuclear engineering fields the thin walled cylindrical shells finds wider applications as primary structural members. The stiffened and unstiffened shells made up of metallic and laminated composite materials (large diameter to thickness ratio) are extensively used in underwater, surface, air and space vehicles as well as in construction of pressure vessels, storage vessels, storage bins and liquid storage tanks. The geometric imperfections due to manufacturing processes takes dominant role in decreasing the buckling load of cylindrical shells. Buckling is often viewed as the controlling failure mode of these structures due to its relatively small thickness of these structural members. It is therefore essential that the buckling strength of the thin shells along with knowledge of its buckling has been the subject of many researchers in both analytical and experimental investigations.

The researchers [1-10] has been investigated the problem of cylindrical shell buckling subjected to axial compressive loads using approximate analytical methods as well as finite element methods. The classical buckling load which is calculated theoretically much higher than the actual buckling load of the cylindrical shell and a knock-down factor is introduced to evaluate a better approximation based on an extensive experimental investigation. The effect of bending stresses and pre-buckling deformations investigated by Fischer [11], Yamaki and Kodama [12] and emphasized that the effect of pre-buckling deformations is not a primary reason for the difference between the classical prediction and the experimental results. According to von Karman and Tsien [13], Donnell and Wan [14], Koiter et al. [15], Budiansky and Hutchinson [16] the initial geometric imperfections are the single dominant factor for contributing the discrepancy between theories and experiments on cylindrical shell buckling.
The form geometric imperfections and amplitude dependent on fabrication process and quality of cylindrical shells according to Arbocz and Hol [17]. Buckling of imperfect cylindrical shells thus remains a subject of active area of research with special emphasis on modeling of the real imperfections as well as of boundary conditions and load eccentricity if any. The buckling of cylindrical shell structures taking dimple as geometric imperfection pattern was investigated by the Shen and Li [18], and Schneider [19]. Frano and Forasassi [20], Prabu et al. [21] investigated the buckling behavior of imperfect thin cylindrical shells under lateral pressure by taking ovality as

imperfection sensitivity and observed that buckling load decreases with increase in imperfection magnitude. To the best of the author's knowledge a detailed and generalized approach of qualitative study on non-linear buckling and post-buckling behavior of cylindrical shells including the influence of geometric imperfection has been rarely found. Recently, Kobayashi et al. [22] employed a stabilization technique by using artificial damping to investigate the post-buckling behavior of perfect Yamaki cylinder subjected to axial compression and these researchers emphasized the difficulty of using conventional arc-length method when applied for the analysis of imperfect cylindrical shells. Spagnoli et al. [23] investigated the buckling behavior of laminated composite cylinders and a correlation study on theoretical vs. experimental end shortening behavior is discussed and a summary of knock-down factors as well as FE (finite element) reduction factors are reported.

Present study makes an attempt to accurately evaluate the limit point load of a composite (GFRP), imperfect cylindrical shell [22] by means of linear as well as non-linear buckling analysis approaches by using general purpose finite element software (ANSYS) [24]. A generalized procedure is established here which can be extended to any other modes of buckling and to any given choice of imperfection shapes can be modeled more accurately

2. Finite element formulation

An eight-noded isoparametric element is used with six degrees of freedom viz. u, v, w, θ_x, θ_y and θ_z at each node. The finite element discrimination process for geometrically non-linear analysis yields a set of simultaneous equations:

$$[K]\{U\} = \{F^a\} \qquad (1)$$

Where [K] is stiffness matrix, U is unknown degrees of freedom and F^a is vector of applied forces. By using von-Karman strain–displacement relations geometric non-linearity is considered where the moderately large rotations and displacements of the order of characteristic dimension of the problem are allowed. The stiffness matrix [K] itself is function of the unknown degrees of freedom for the non-linear analysis, which leads to system of non-linear equations. An iterative process of solving the non-linear equations and these can be written as (ANSYS version 13.0).

$$[K_i^T]\{\Delta U_i\} = \{F^a\} - \{F_i^{nr}\} \qquad (2)$$

$$U_{i+1} = U_i - \Delta U_i \qquad (3)$$

$[K_i^T]$ is Tangent Matrix, i representing the current equilibrium iteration and F_i^{nr} vector of restoring loads corresponding to the element internal loads. Eq. (2) presents a generalized system of simultaneous non-linear equations which needs to be solved for evaluating the equilibrium path of the thin cylindrical shell structure subjected to axial compressive load. The non-linear buckling load can be evaluated by performing either non-linear buckling or post-buckling analysis. Following summary explains a detailed procedure involved in these analysis.

(1) For the sake of simplicity the fundamental buckled mode shape has been chosen as the shape of imperfection after linear buckling analysis has been performed. Magnitude of imperfection is referred with reference to the thickness parameter of the cylindrical shell. It must be noted that the shape of imperfection can be given in the form of linear combination of buckled mode shapes or random imperfection or experimentally measured imperfection shape also can be imparted.

(2) To trace the equilibrium path (Figure 1) a non-linear analysis (non-linear buckling) has been performed by applying initial geometric imperfection. Non-linear buckling involves the application of Newton-Raphson method to solve Eq. 1

(3) Load-deflection curve obtained using the Newton-Rapson method represents the primary path

3. Results and Discussion

The geometry and material properties of the composite (E-glass/Epoxy) cylindrical shell subjected to axial load are outlined as follows (Table 1):
Radius of the composite cylindrical shell =70mm
Thickness of the cylindrical shell = 0.5mm
Length of the cylindrical shell = 280mm

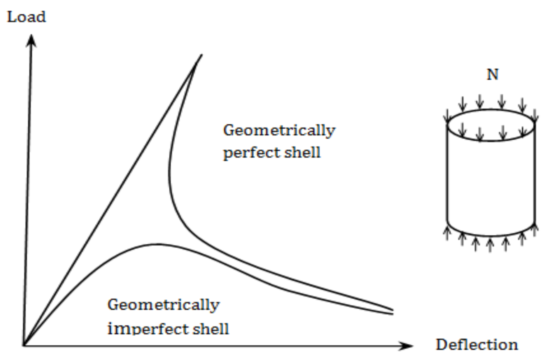

Figure: 1 General Buckling phenomenon and geometry of cylindrical shell.

Table 1

S.No.	Material properties (E-glass/Epoxy)		
	property	***direction***	***value***
1	Longitudinal modulus (GPa)	E_{11}	36
2	Transverse modulus(GPa)	E_{22}	5.8
3	Transverse modulus(GPa)	E_{33}	5.8
4	Shear modulus (GPa)	G_{12}	3.2
5	Poisson's ratio	ν_{12}	0.3

3.1 Mesh convergence study

Fig. 2 shows the mesh convergence study obtained from the linear buckling analysis (eigen buckling) and an optimum element size of 10mm is used for linear and nonlinear analysis results presented in this study

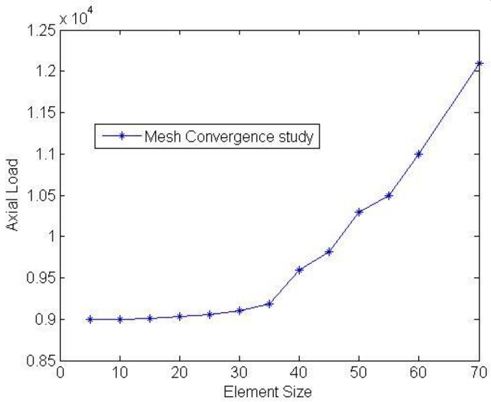

Figure **2:** Mesh convergence study

3.2 Eigen Value Buckling Analysis

The buckling strength of perfect cylindrical shell found using block lancoczos iteration scheme to extract to the load factors or eigen values. The first step in the parametric study consists of eigen value analyses for the above cylindrical shell configuration. This type of analysis has several limitations in shell buckling problems but can still provide some useful information, first as a preliminary assessment on buckling strength and, second, as a guide in selecting appropriate imperfection modes for non-linear analysis. The second step consists of incremental non-linear analysis including initial imperfection profiles. The eigenvectors pertaining to the lowest eigen value are used in order to create 'critical imperfection' profiles.

The mechanical properties of the Glass Fiber Reinforced Plastic (GFRP) [25] material are shown in Table 1. 0.25mm ply thickness and +45°/-45° ply orientation of the laminate is considered in linear and non-linear analysis.

3.3 Non-linear Buckling analysis

Non-linear analysis is accurate approach and the finite element analysis has capability of analyzing the actual structures with geometric imperfections. In this analysis both geometric and material non linearity's can be taken, because of thin shell structures are subjected to large deformations and also at some of imperfection locations on the structures the stresses may exceed elastic limit due to imperfections present in that locations. Newton-Raphson iteration scheme is used to solve system of equations in non linear equations. Nonlinear analysis is carried out by modeling first eigen buckled mode shapes as geometric irregularities on the non linear geometric model.

Figure 3 shows the comparison of non-linear buckling load and the experimental results [25] for the GFRP laminated composite cylindrical shell. It is clearly observed that the load obtained in nonlinear analysis in present study and the experimental results carried out by the referenced. Author [25] shows good agreement in predicting the primary equilibrium path as well as in predicting the limit point load of the laminated composite cylindrical shell.

Table.2

Experimental Results (25)			Non Linear Buckling Load (Present Study)		
sample	***End shortening (m)***	***Axial load (N)***	***Imperfection magnitude (w/t)***	***End shortenin g(m)***	***Axial load(N)***
1	0.00347	4806.8	ξ =0.689	0.00330	5083.5
2	0.00327	4577.2	ξ =0.690	0.00331	5083.5
3	0.00350	4846.8	ξ =0.699	0.00317	5010.3
4	0.00327	4718.7	ξ =0.70	0.00328	4937.8

Nonlinear analysis is carried out by modeling eigen buckled mode shapes as geometric irregularities on the non linear perfect geometric model. Fig. 3 shows the results obtained from the non-linear buckling analysis for various imperfection magnitudes (ξ=w*/t, w* is the maximum imperfection amplitude, t is the thickness of the cylindrical shell) considered in this study. Trend analysis of the same is carried out to assess the non linear buckling load for L/d ratio is equal to 2.0(where L and d are the length and diameter of the cylindrical shell respectively)

It is observed that the limit point buckling load decreases as the imperfection magnitude increases from zero to 0.70 of thickness. Figure 4. shows the linear and non-linear buckled mode shapes for l/d =2.0. It is also observed from the non-linear buckling analysis that the buckling load increases as the l/d ratio of the cylindrical shell decreases

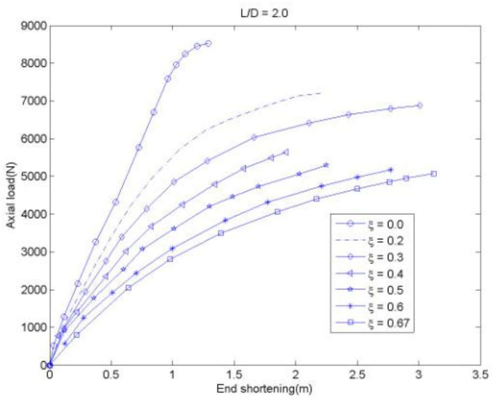

Figure 3. Imperfection sensitivity study

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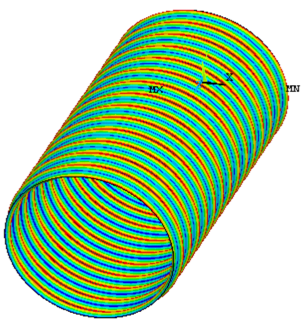
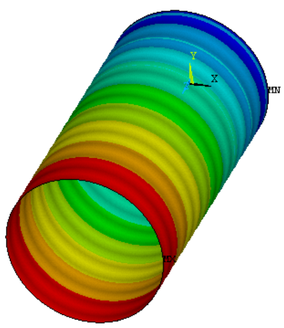
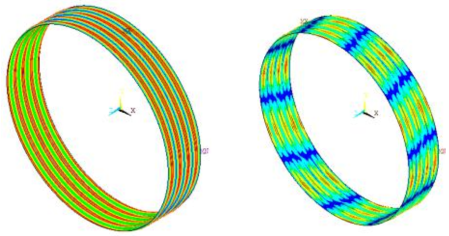


(a) Linear buckled mode shape



(b) Non-linear buckled mode shape

Figure 4.Buckled mode shapes (L/D =2.0)

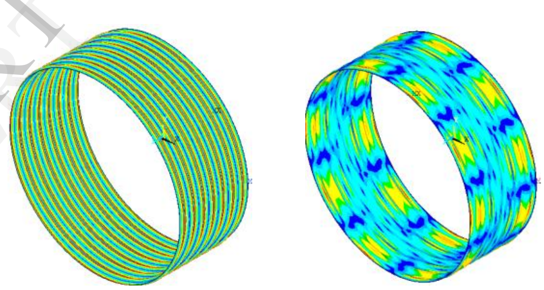


a) m =4, n = 0 b) m = 5, n = 10

Figure 5. Mode shapes for L/t = 560, r =700mm

3.4 Effect of radius

The influence of radius on buckling load and buckling modes is investigated, keeping L/t ratio constant. This nonlinear buckling analyses is performed for varying radii from 700 to 70 mm, keeping length to thickness ratio *(L/t)* constant equal to 560. Results of these analyses are shown in Table 3. The effect of cylinder radius on some significant buckling mode shapes is also shown in Figures 5 and 6.



a) m =13, n = 0 b) m = 13, n = 9

Figure 6. Mode shapes for L/t = 560, r =350mm

Table.3

L/t	*Radius(m)*	Imperfection magnitude (*ξ=w*/t*)	*Non-Linear Buckling Load (N)*
560	R = 0.07	ξ =0.70	4937.83
560	R = 0.35	ξ =0.70	7832.09
560	R = 0.70	ξ =0.70	8178.13

4. Conclusions

Non-linear buckling analysis of composite thin cylindrical shells subjected to axial compressive load is briefly investigated for first mode eigen imperfection amplitude. These approaches can be directly extended to any other mode of interest. For the sake of simplicity the shape of imperfection is chosen as the shape of the fundamental buckled mode shape. Non-linear buckling analysis uses Newton-Raphson approach to predict the primary equilibrium paths. Limit point loads obtained from this approach shows an excellent agreement with experimental results carried out earlier (25). This method finds direct application to investigate the effect of geometric imperfections on other advanced grid-stiffened structures.

The below are the conclusions derived from the present study carried out on the composite thin cylindrical shells under axial compression with different types of imperfect patterns taken to investigate the buckling load

1. The nonlinear buckling load with imperfection mag of tnitude of the order 0.7 of thickness agreed with the experimental buckling load.
2. As the magnitude of geometric imperfections increases the buckling load of cylindrical shell decreases
3. When the maximum amplitude of the imperfection is 1 of thickness the eigen buckling mode gives lowest bulking load.
4. As the l/d ratio of the composite cylindrical shell increases nonlinear buckling load decreases

For constant L/t ratio the nonlinear buckling load increases as the radius of the cylinder increases

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