International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 3 Issue 8, August - 2014

Capacity Analysis for MIMO-OFDM Systems based on various Forward Error Correction Techniques

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Abstract* - The data rate for the Multiple-Input Multiple-Output (MIMO) - Orthogonal Frequency Division Multiplexing (OFDM) systems can be increased by spatial multiplexing which uses Space-Time coding techniques. The spatial multiplexing provides increased spectral (bandwidth) efficiency. In this paper, vertical and horizontal encoding techniques of layered Space-Time codes are implemented. Their performance is analyzed by comparing the Bit Error Rate (BER) and Signal to Noise Ratio (SNR). This technique performs better than conventional methods. The powerful Forward Error Correction (FEC) codes like Turbo codes and Rate-Compatible Punctured Convolutional (RCPC) codes are compared. The Maximum likelihood (ML) receiver is used for detection of transmitted symbols. The simulation results show that Turbo codes gives enhanced performance than RCPC and optimal detection accuracy is given by ML receiver than others but with higher complexity. This paper provides the possibilities of increasing the data rate and data transmission with less error for MIMO-OFDM systems using spatial multiplexing and various FEC codes with Maximum Likelihood receiver.

Keywords - MIMO-OFDM; Spatial multiplexing; Space-Time codes; Adaptive Modulation and Coding; Link adaptation.

I. INTRODUCTION

The Advent of MIMO for recent wireless communication satisfies the growing demands like high data rate, throughput, capacity, link reliability and spectral efficiency. MIMO advantages are achieved by using the concept of increasing the number of antennas with same bandwidth and transmit powers [1], [2]. The MIMO design is widely classified into two ways like MIMO Spatial Multiplexing and MIMO Diversity. The MIMO diversity technique provides gain in the form of increased reliability whereas; Spatial Multiplexing gain is in the form of increased capacity (maximum achievable data rate). Earlier days, higher data rate was achieved by transmitting data at higher modulation rates. Now-a-days, Spatial Multiplexing techniques are used where the data is de-multiplexed which increases the maximum achievable data rate. The diversity techniques give more diversity gain compared to multiplexing gain [3], [4]. The Space-Time codes can provide additional diversity gain [5].

In wireless communication, the channel suffers from much impairment like fading, additive noise and interference from other users. The space-time codes [6], [7] are used to improve transmission over fading channels [8]. The inter symbol interference can be overcome by using OFDM systems and it is used mostly for broadband applications [9]. The OFDM is mostly used in MIMO channel because it turns a frequency-selective MIMO channel into a set of parallel frequency-flat MIMO channels. This makes multi-channel equalization particularly simple [2].

The link adaptation technique called "Adaptive Modulation and Coding (AMC)" is used which also increases the overall system capacity. The AMC automatically assigns the channel code rate and modulation levels to each user accordingly to match the average channel conditions [10]. The FEC is a way of adding redundant bits so that errors can be detected easily and corrected without the need for re-transmission but at the cost of increased bandwidth which can be overcome by puncturing. The Turbo codes are the most widely used FEC codes. If very strong error correction is required, two or more codes should work in parallel. In this paper, Turbo code is used which uses concatenated convolutional codes in parallel and also includes puncturing concept.

In real time communication, some streams are transmitted with higher important information than the others. In such cases, different code rates are needed for each transmission which is provided by puncturing. The puncturing reduces the number of redundant data to be transmitted by making use of puncturing matrix. There are various detection algorithms like maximum likelihood, MMSE detection etc. The maximum likelihood detection gives better performance compared to others and it is the most widely used receiver algorithm [11].

Two bounds are used for ML receiver to find the capacity of MIMO-OFDM system. Practically, the capacity of the MIMO-OFDM system can be analyzed by estimating the spectral efficiency. The vertical and horizontal encoding structures also called as vertical layered space-time code and horizontal layered space-time code are implemented. The maximum data rate (capacity) for above structures is analyzed from the simulation results. The capacity is directly related to SNR and SNR is related to BER, therefore, capacity is inversely mapped to BER. In vertical encoding structure, all the data streams or antennas are connected to a single channel encoder, whereas in horizontal encoding structure, each data stream is connected to separate channel encoder [12], [13].

The RCPC codes are implemented first and then compared with Turbo codes. Based on comparison, since Turbo codes gives highest error correction, rest of this paper uses Turbo FEC codes. The simulation results show that the data rate is increased by spatial multiplexing technique using ML receiver with Turbo codes.

II. SYSTEM MODEL

The MIMO-OFDM system with AMC is considered with N_c subcarriers, N_t transmitter antennas and N_r receiver antennas. The MCS are chosen by the receiver based on the current channel condition for each encoding block and feeds it back to the transmitter. The modulation levels and channel code rate are given by the MCS. The input bit is encoded vertically or horizontally as shown in Fig. 1 and Fig. 2. The Turbo code with octal polynomials (133,171) is used as FEC. The encoded value is bit–wise interleaved; QAM modulated and signal is converted to time domain by Inverse Fast Fourier Transform (IFFT). The inter-symbol interference is prevented by adding cyclic prefix. The wireless channel undergoes Rayleigh fading [2] and the channel model is given by Jakes model. At receiver reverse operation of the transmitter is performed and the received signal vector at k^{th} subcarrier is represented as,

$$Y_k = H_k x_k + n_k \qquad \text{for } k = 1,\dots, N_c \tag{1}$$

Where x_k is the transmitted symbol vector, n_k denotes the Additive White Gaussian Noise (AWGN) vector with zero mean and covariance matrix $\sigma_n^2 I_{Nr}$ and H_k is the Rayleigh fading channel vector and 2 x 2 MIMO channel is shown in Fig. 3. The optimum detection of ML receiver is obtained by comparing all possible combinations of symbols which could have been transmitted and is given as,

$$\bar{x} = \arg\min \| Y - Hx \| \tag{2}$$

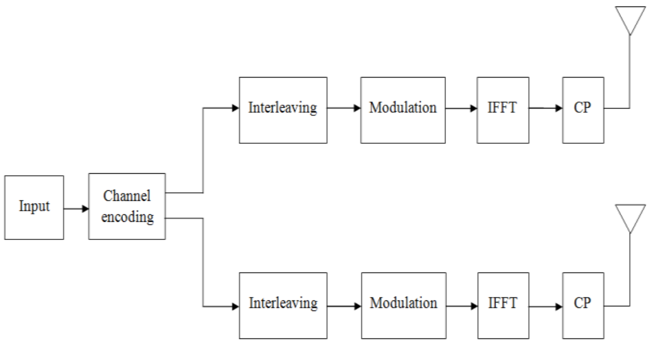

Fig. 1. Transmitter block diagram for Vertical encoding structure

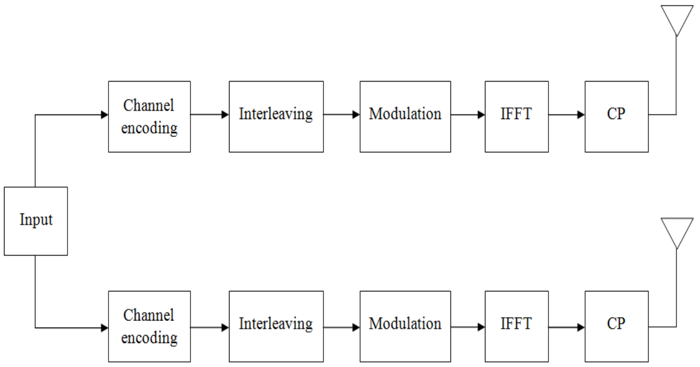

Fig. 2. Transmitter block diagram for Horizontal encoding structure

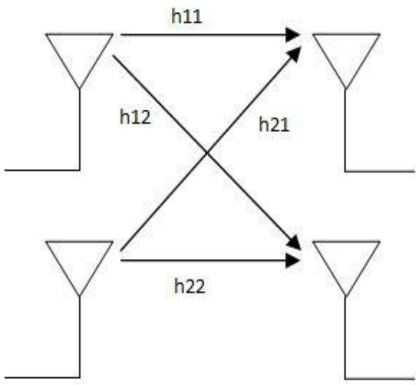

Fig. 3. 2 x 2 MIMO Channel

III. VERTICAL ENCODING STRUCTURE

In Vertical Encoding (VE) case, all the de-multiplexed data streams are given the same modulation level and the code rate meaning a single MCS across all streams. The ML receiver performance is calculated by using two capacity bounds of linear receivers. The upper bound capacity is estimated assuming the symbol interference is perfectly cancelled which is known as Perfect Interference Cancellation (PIC) where the transmitter is also aware about the channel condition. The lower bound is derived assuming that there is an error and it is removed by minimum mean square error (MMSE) receiver.

The post detection of per stream SNR (γ_n^{ML}) of the ML receiver [12] [13] is given as

$$\gamma_n^{ML} = (1+\gamma_n^{PIC})^{\alpha\beta}\,(1+\gamma_n^{MMSE})^{1-\alpha\beta} - 1. \tag{3}$$

where

$$\alpha = \frac{C_{Open} - C_{MMSE}}{C_{PIC} - C_{MMSE}},\ 0 \le \alpha \le 1 \tag{4}$$

$$\beta = \frac{C_{ML} - C_{MMSE}}{C_{Open} - C_{MMSE}},\ 0 \le \beta \le 1 \tag{5}$$

The capacities for PIC, MMSE, and Open-loop are given by

$$C_{PIC} = \sum_{n=1}^{Nt} \log_2(1+\gamma_n^{PIC}) \tag{6}$$

$$C_{OPEN} = \log_2 \det\,(I_{Nr} + \rho HH^H) \tag{7}$$

where $\rho = p/(I_{Nt}\,\sigma_n^2)$

$$C_{MMSE} = \sum_{n=1}^{Nt} \log_2(1+\gamma_n^{MMSE}). \quad (8)$$

The SNR under conditions of PIC and MMSE is given by

$$\gamma_n^{PIC} = \rho \, ||h_n||^2 \quad (9)$$

$$\gamma_n^{MMSE} = \frac{1}{[(I_{Nt} + \rho H^H H)^{-1}]_{nn}} - 1. \quad (10)$$

$$C_{ML} = \sum_{n=1}^{Nt} \log_2(1+\gamma_n^{ML}) \quad (11)$$

The capacity of ML is bounded as,

$$C_{MMSE} \leq C_{ML} \leq C_{OPEN} \leq C_{PIC} \quad (12)$$

The C_{ML} is the maximum achievable rate when ML receiver is used.

The optimal values of β for different MCS levels are generated and tabulated in Table.1 [12] and to avoid complex calculation; the β value is taken directly from table. The β depends on the channel code rate. Here I_{Nt} is the identity matrix, ρ is the average SNR, H is a Rayleigh fading channel ,H^H is the Hermitian transpose, n is the nth stream and *det* is the determinant.

IV. HORIZONTAL ENCODING STRUCTURE

In Horizontal Encoding (HE) case, all the de-multiplexed data streams are given different modulation levels and the code rates. The post detection of per stream SNR (γ_n^{ML}) of the ML receiver [12], [13] is obtained by modifying the equation (3) by

$$\gamma_n^{ML} = (1+\gamma_n^{PIC})^{\alpha(\beta+\Delta\beta n)} \; (1+\gamma_n^{MMSE})^{1-\alpha(\beta+\Delta\beta n)} - 1. \quad (13)$$

Since there are non-identical modulation levels, we have a tuning parameter $\Delta\beta_n$ for each stream to adjust the value of β. The $\Delta\beta_n$ is independent on channel code rate because detection performance is given by the minimum Euclidean distance of the symbol constellation. The tuning parameter $\Delta\beta_n$ for non-identical MCS levels are given in Table.2 [12].

V. CAPACITY AND SPECTRAL EFFICIENCY ESTIMATION

The capacity is used to express the spectral efficiency of the system. The capacity for MIMO system is derived by considering a time-invariant channel and the input-output relationship of the channel is given as,

$$y = H\,x + n \quad (14)$$

where y is the received signal, H is the channel and n is AWGN with zero mean and covariance $\sigma_n^2 I_{Nr}$.

By Shannon's channel coding theorem, capacity is given by mutual information,

$$C = \max I(x{:}y) \quad (15)$$

$$I(x{:}y) = H(y) - H(y/x) = H(y) - H(n) \quad (16)$$

$$H(y) = \log_2 \det(\pi e R_{yy}) \quad (17)$$

$$R_{yy} = E[yy^H] = HH^H R_{xx} + N_0 I_{Nr} \quad (18)$$

$$H(y/x) = H(n) = \log_2 \det(\pi e N_0 I_{Nr}) \quad (19)$$

where N_0 is the variance. On substituting equation (17) and (19) in (16), and assuming power p is given to the system, I(x:y) is expressed as,

$$I(x{:}y) = \log_2 \det\left(I_{Nr} + \frac{p\,HH^H R_{xx}}{N_0}\right) \quad (20)$$

p/N_0 is the average SNR 'ρ' and R_{xx} as unity, equation is simplified as,

$$I(x{:}y) = \log_2 \det(I_{Nr} + \rho HH^H) \quad (21)$$

For MIMO systems with many channels, power should be assigned equally to each channel. In order to assign equal power, SNR is divided by number of transmitter antennas.

$$I(x{:}y) = \log_2 \det\left(I_{Nr} + \frac{\rho\,HH^H}{N_t}\right) \quad (22)$$

Hence the capacity is given as,

$$C = \max \log_2 \det\left(I_{Nr} + \frac{\rho HH^H}{Nt}\right) \quad \text{bits/s/Hz} \quad (23)$$

In case of MIMO, the capacity depends on maximum number of transmitter antennas and expressed as,

$$C = N_t{*}\log_2 \det\left(I_{Nr} + \frac{\rho HH^H}{Nt}\right) \quad \text{bits/s/Hz} \quad (24)$$

SISO capacity is given by,

$$C = \log_2 \det(I_{Nr} + \rho HH^H) \quad \text{bits/s/Hz} \quad (25)$$

In terms of bandwidth,MIMO capacity is given as,

$$C = BW{*}N_t{*}\log_2 \det\left(I_{Nr} + \frac{\rho HH^H}{Nt}\right) \quad \text{bits/s/Hz} \quad (26)$$

And for SISO,

$$C = BW{*}\log_2 \det(I_{Nr} + \rho HH^H) \quad \text{bits/s/Hz} \quad (27)$$

where BW is the bandwidth. The MIMO capacity increases linearly with number of transmitter antennas.

The spectral efficiency refers to the information rate that can be transmitted over a given bandwidth in a specific communication system. It is expressed as,

$$T = (\log_2 M)\,(1 - BER) \quad (28)$$

where M is the modulation level. For MIMO-OFDM system with many data streams and each has its spectral efficiency. The overall spectral efficiency is given as,

$$T_{total} = \sum_{i=1}^{n} T_i \quad (29)$$

VI. SIMULATION RESULTS

In this section, the simulation results of vertical and horizontal encoding technique for 2 x 2 MIMO-OFDM systems are analyzed. In Fig. 4, capacity of MIMO systems with different number of antennas are compared. It clearly shows that the capacity increases linearly with the increase of transmitter antennas. Also, capacity of MIMO system increases with bandwidth and is shown in Fig. 5. The Fig. 6 shows the comparison of Turbo and RCPC FEC codes for vertical encoding structure and it proves that Turbo codes gives less BER compared to RCPC codes. It means capacity can be indirectly increased by using Turbo codes by avoiding retransmission as in earlier days. The BER is reduced due to channel coding by maximizing the possibility of detecting and receiving the corrupted data by adding redundancy to the transmitted data. It is concluded that when code rate is less, parity bits added are more so BER is reduced compared to higher order code rate from Fig. 7. The Fig. 8 concludes that when modulation level is low, the distance between symbol constellations is more, so there is no noise or corruption, hence BER is less. Whereas when modulation level increases, the distance decreases and BER is increased. But higher modulation level has an advantage of transmitting more bits. In Fig. 9 and Fig. 10 the spectral efficiency comparison for vertical encoding MIMO-OFDM system for various MCS is given. As modulation level decreases, the spectral efficiency increases and it also depends on the code rate. Fig. 11 gives the comparison of FEC codes for horizontal encoding structure of 2 x 2 MIMO-OFDM systems whose conclusion are similar to Fig. 6. Analyzes of the horizontal encoding structure for various non-identical MCS are shown in Fig. 12 and Fig. 13. The Fig. 14 gives the spectral efficiency of horizontal encoding structure for identical code rate.

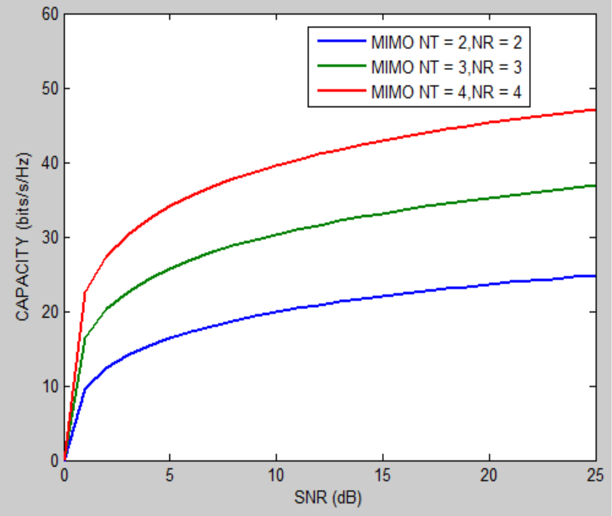

Fig. 4. MIMO Capacity with different number of antennas

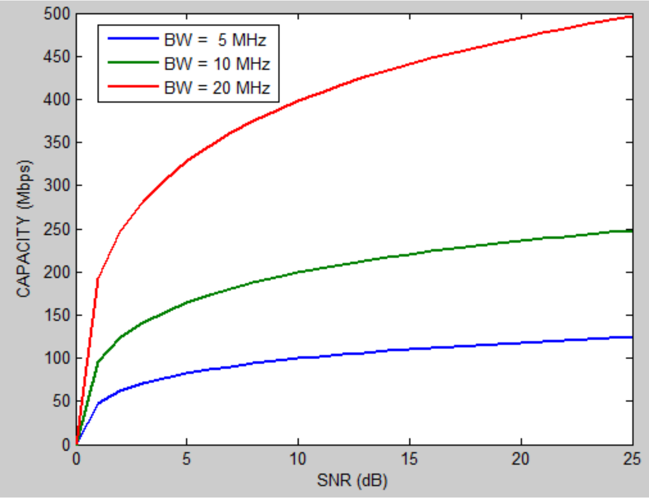

Fig. 5. MIMO Capacity with different bandwidth for 2x2 systems

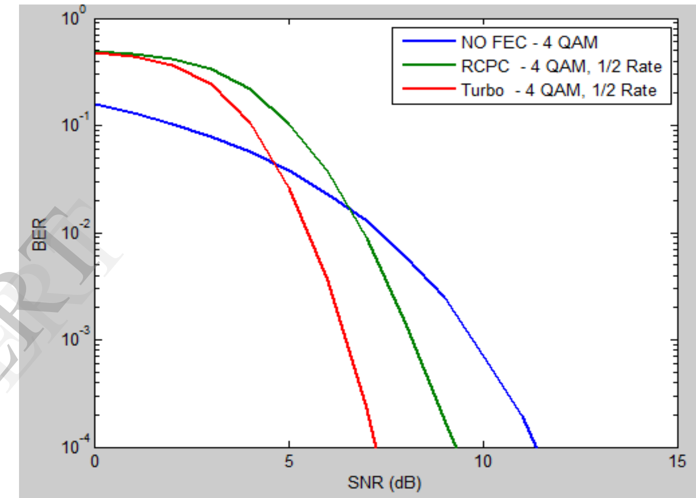

Fig. 6. FEC Comparisons for VE systems

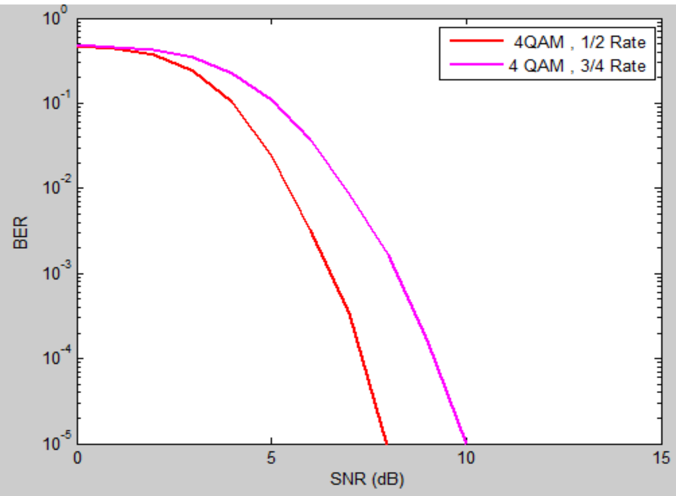

Fig. 7. Non-identical code rates for VE systems

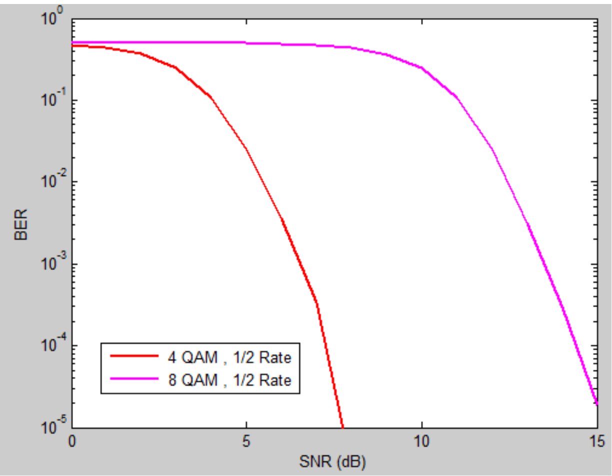

Fig. 8. Non-identical modulation levels for VE systems

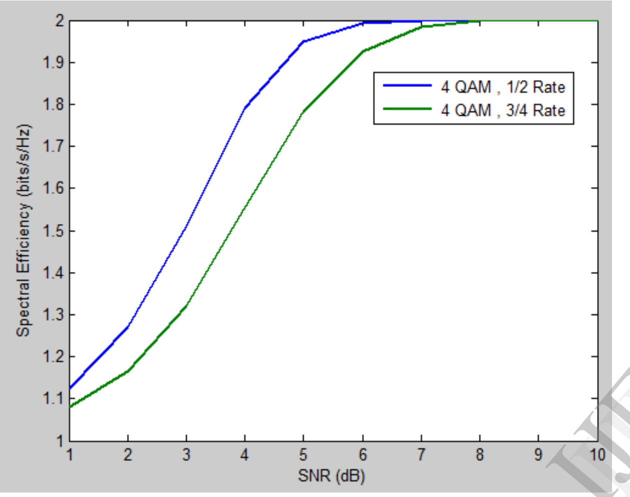

Fig. 9. Spectral efficiency for VE systems with non- identical code rate and 4 QAM

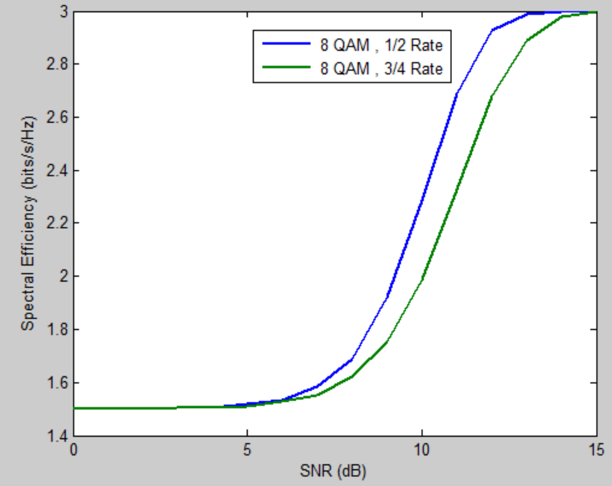

Fig. 10. Spectral efficiency for VE systems with non-identical code rate and 8 QAM

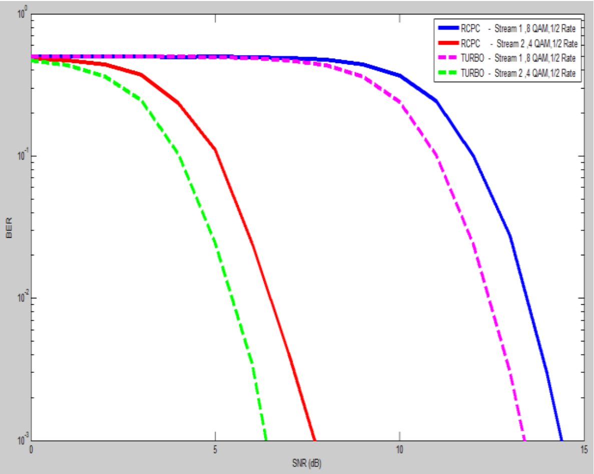

Fig.11. FEC Comparisons for HE systems

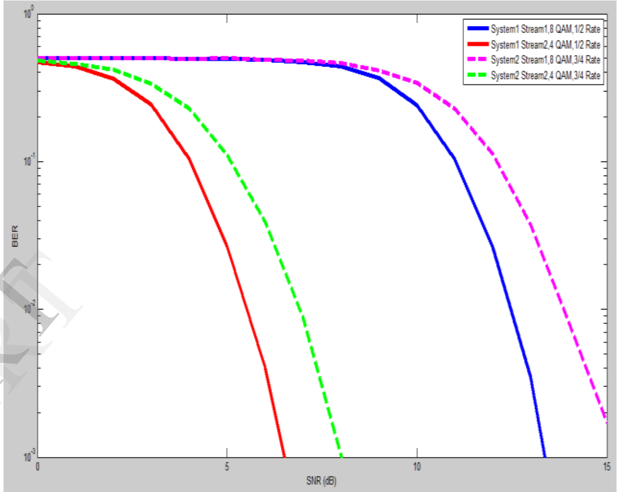

Fig. 12. Non-identical MCS for HE systems

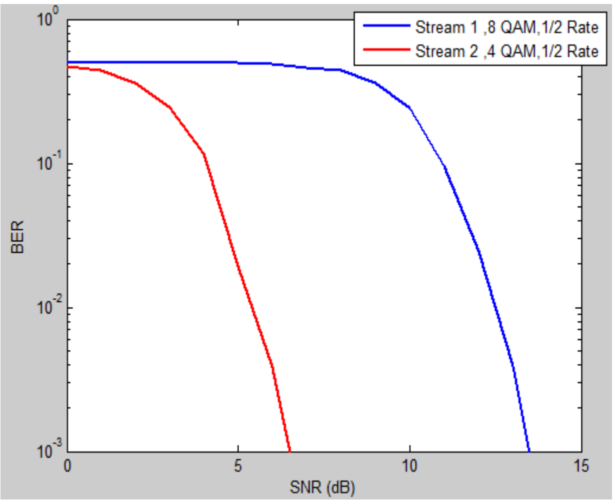

Fig. 13. Non-identical modulation levels for HE systems

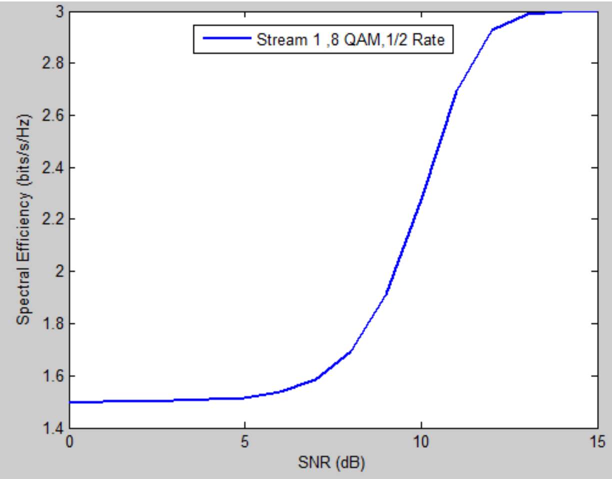

Fig. 14. (a) Stream 1

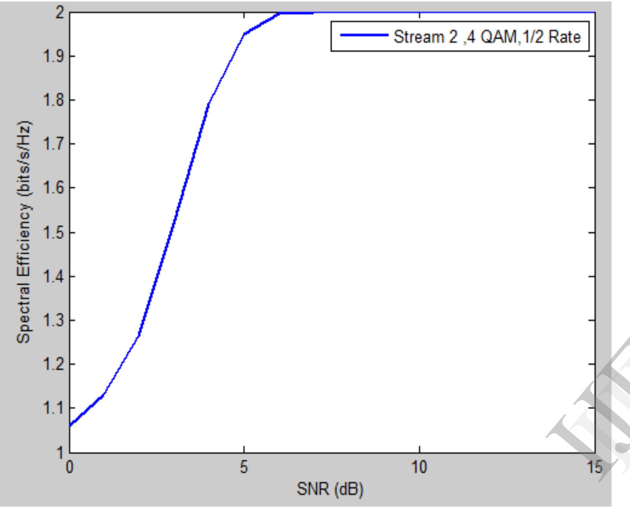

Fig. 14. (b) Stream 2

Fig. 14. Spectral efficiency for 2 x 2 HE systems with Non-identical code rate and 4 QAM

VII. CONCLUSION

In this paper, MIMO-OFDM systems' data rate is increased by spatial multiplexing techniques which are better than conventional methods. The capacity increases with increase in SNR and decrease in BER. The spectral efficiency is estimated to analyze the capacity performance for both vertical and horizontal technique. The Turbo codes outperform the RCPC codes. From simulation, it is concluded that, using vertical and horizontal encoding structures with ML receiver along with Turbo codes can give improved data rate (capacity) for MIMO-OFDM systems.

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