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Certain Aspects of Fuzzy $\alpha-$ Compactness

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ABSTRACT

In this paper , we study several aspects of fuzzy α – compactness due to T. E. Gantner et al. [5] in fuzzy topological spaces and also obtain its several other properties .

Keywords : Fuzzy topological spaces , α – compactness .

1. INTRODUCTION

The concept of fuzzy sets and fuzzy set operations was first introduced by L. A. Zadeh in his classical papers [10] in the year 1965 describing fuzziness mathematically first time . Compactness occupies a very important place in fuzzy topological spaces . The purpose of this paper is to study the concept due to T. E. Gantner et al. in more detail and to obtain several other features .

2. PRELIMINARIES

We briefly touch upon the terminological concepts and some definitions , which are needed in the sequel . The following are essential in our study and can be found in the paper referred to.

2.1 **Definition** (10) **:** Let X be a non-empty set and I is the closed unit interval [0, 1]. A fuzzy set in X is a function u : X $\rightarrow$ I which assigns to every element x $\in$ X. u(x) denotes a degree or the grade of membership of x. The set of all fuzzy sets in X is denoted by I^X . A member of I^X may also be a called fuzzy subset of X .

2.2 Definition (10) **:** Let X be a non-empty set and A $\subseteq$ X . Then the characteristic function 1_A(x) : X $\rightarrow$ {0 , 1} defined by 1_A(x) = $\begin{cases} 1 \ \ if \ \ x \in A \\ 0 \ \ if \ \ x \notin A \end{cases}$

Thus we can consider any subset of a set X as a fuzzy set whose range is {0 , 1}.

2.3 Definition (9) **:** A fuzzy set is empty iff its grade of membership is identically zero . It is denoted by 0 or ϕ .

2.4 Definition [9] **:** A fuzzy set is whole iff its grade of membership is identically one in X . It is denoted by 1 or X .

2.5 Definition [3] **:** Let u and v be two fuzzy sets in X. Then we define

(i) u = v iff u(x) = v(x) for all $x \in X$

(ii) $u \subseteq v$ iff $u(x) \leq v(x)$ for all $x \in X$

(iii) $\lambda = u \cup v$ iff $\lambda(x) = (u \cup v)(x) = \max[u(x), v(x)]$ for all $x \in X$

(iv) $\mu = u \cap v$ iff $\mu(x) = (u \cap v)(x) = \min[u(x), v(x)]$ for all $x \in X$

(v) $\gamma = u^c$ iff $\gamma(x) = 1 - u(x)$ for all $x \in X$.

2.6 Definition [3] **:** In general , if { u_i : $i \in J$ } is family of fuzzy sets in X , then union $\cup u_i$ and intersection $\cap u_i$ are defined by

$\cup u_i(x) = \sup\{ u_i(x) : i \in J \text{ and } x \in X \}$

$\cap u_i(x) = \inf\{ u_i(x) : i \in J \text{ and } x \in X \}$, where J is an index set .

2.7 Definition [3] **:** Let $f : X \to Y$ be a mapping and u be a fuzzy set in X. Then the image of u , written $f(u)$, is a fuzzy set in Y whose membership function is given by

$$f(u)(y) = \begin{cases} \sup\{u(x) : x \in f^{-1}(y)\} & if \ f^{-1}(y) \neq \phi \\ 0 & if \ f^{-1}(y) = \phi \end{cases} .$$

2.8 Definition [3] **:** Let $f : X \to Y$ be a mapping and v be a fuzzy set in Y. Then the inverse of v, written $f^{-1}(v)$, is a fuzzy set in X whose membership function is given by $(f^{-1}(v))(x) = v(f(x))$.

2.9 De-Morgan's laws [10] **:** De-Morgan's Laws valid for fuzzy sets in X i.e. if u and v are any fuzzy sets in X, then

(i) $1 - (u \cup v) = (1 - u) \cap (1 - v)$

(ii) $1 - (u \cap v) = (1 - u) \cup (1 - v)$

For any fuzzy set in u in X , $u \cap (1 - u)$ need not be zero and $u \cup (1 - u)$ need not be one .

2.10 Definition [3] **:** Let X be a non-empty set and t $\subseteq I^X$ i.e. t is a collection of fuzzy set in X. Then t is called a fuzzy topology on X if

(i) 0 , 1 $\in$ t

(ii) $u_i \in$ t for each i $\in$ J , then $\bigcup_i u_i \in$ t

(iii) u , v $\in$ t , then u $\cap$ v $\in$ t

The pair (X , t) is called a fuzzy topological space and in short, fts. Every member of t is called a t-open fuzzy set. A fuzzy set is t-closed iff its complements is t-open. In the sequel, when no confusion is likely to arise, we shall call a t-open (t-closed) fuzzy set simply an open (closed) fuzzy set .

2.11 Definition [3] **:** Let (X , t) and (Y , s) be two fuzzy topological spaces. A mapping f : (X , t) $\rightarrow$ (Y , s) is called an fuzzy continuous iff the inverse of each s-open fuzzy set is t-open .

2.12 Definition [9] **:** Let (X , t) be an fts and A $\subseteq$ X. Then the collection t_A = { u|A : u $\in$ t } = { u $\cap$ A : u $\in$ t } is fuzzy topology on A, called the subspace fuzzy topology on A and the pair (A , t_A) is referred to as a fuzzy subspace of (X , t) .

2.13 Definition [8] **:** An fts (X , t) is said to be fuzzy Hausdorfff iff for all x , y $\in$ X , x $\neq$ y, there exist u , v $\in$ t such that u(x) = 1 , v(y) = 1 and u $\subseteq$ 1 – v .

2.14 Definition [8] **:** An fts (X , t) is said to be fuzzy regular iff for each x $\in$ X and u $\in t^c$ with u(x) = 0 , there exist v , w $\in$ t such that u(x) = 1 , u $\subseteq$ w and v $\subseteq$ 1 – w .

2.15 Definition [4] **:** Let (A , t_A) and (B , s_B) be fuzzy subspaces of fts's (X , t) and (Y , s) respectively and f is a mapping from (X , t) to (Y , s), then we say that f is a mapping from (A , t_A) to (B , s_B) if f(A) $\subseteq$ B .

2.16 Definition [4] **:** Let (A, t_A) and (B, s_B) be fuzzy subspaces of fts's (X, t) and (Y, s) respectively. Then a mapping f : $(A, t_A) \rightarrow (B, s_B)$ is relatively fuzzy continuous iff for each v$\in s_B$, the intersection f^{-1}(v) $\cap$ A $\in t_A$.

2.17 Definition [1] **:** Let $\lambda \in I^X$ and $\mu \in I^Y$. Then ($\lambda \times \mu$) is a fuzzy set in X $\times$Y for which ($\lambda \times \mu$) (x , y) = min { λ(x) , μ(y) } , for every (x , y) $\in$ X $\times$Y .

2.18 Definition : Let (X, t) be an fts and $\alpha \in$ I . A collection M of fuzzy sets is called an α – shading (res. α^* – shading) of X if for each x $\in$ X there exists a u $\in$ M such that u(x) > α (res. u(x) $\geq \alpha$) . A subcollection of an α – shading (res. α^* – shading) of X which is also an α – shading (res. α^* – shading) is called an α – subshading (res. α^* – subshading) of X .

2.19 Definition [5] **:** An fts (X, t) is said to be α – compact (res. α^* – compact) if each α – shading (res. α^* – shading) of X by open fuzzy sets has a finite α – subshading (res. α^* – subshading) where $\alpha \in$ I .

2.20 Definition [6] **:** Let (X, t) be an fts and 0 $\leq \alpha$ < 1 , then the family t_α = { α(u) : u $\in$ t } of all subsets of X of the form α(u) = { x $\in$ X : u(x) > α } is called α – level sets , forms a topology on X and is called the α – level topology on X and the pair (X, t_α) is called α – level topological space .

3. Characterizations of fuzzy α – compactness .

Now we obtain some tangible properties of fuzzy α – compact spaces .

3.1 Theorem : Let 0 $\leq \alpha <$ 1. An fts (X, t) is α – compact iff for every family { F_i } of α – level closed subsets of X , $\bigcap_{i \in J} F_i = \phi$ implies { F_i } contains a finite subfamily { F_{i_k} } (k $\in J_n$) with $\bigcap_{k=1}^{n} F_{i_k} = \phi$.

Proof : Let (X, t) be α – compact . Suppose M = { F_i : i $\in$J } be a family α – level closed subsets of X with $\bigcap_{i \in J} F_i = \phi$. Then , since for each F_i , there exists a $\mu_i \in t^c$ such that $F_i = \alpha(\mu_i)$, we have M = { $\alpha(\mu_i)$: i $\in$J }. Then by De – Morgan's law X = $\phi^c = \left(\bigcap_{i \in J} F_i \right)^c = \bigcup_{i \in J} F_i^c$. Then the family H = { μ_i^c : i$\in$ J }is an open α – shading of (X, t). To see this , let x $\in$X . Since M is a family of α – level closed subsets of X , there is an $F_{i_0} \in$ M such that x $\in F_{i_0}$. But $F_{i_0} = \alpha(\mu_{i_0})$, for some $\mu_{i_0} \in t^c$. Since (X, t) is α – compact , there exist $\mu_{i_k}^c \in$ H (k $\in J_n$) such that X = $\bigcup_{i \in J} \alpha(\mu_{i_k}^c) = \bigcup_{i \in J} F_{i_k}^c$. Then by De – Morgan's law $\phi = X^c = \left(\bigcup_{i \in J} F_{i_k} \right)^c = \bigcap_{i \in J} F_{i_k}^c$.

Conversely , suppose $\bigcap_{i \in J} F_i = \phi$ and M = { μ_i : i$\in$J } be an open α – shading of (X, t) . Then by De – Morgan's law $\phi = X^c = \left(\bigcap_{i \in J} F_i \right)^c = \bigcup_{i \in J} F_i^c$. Then the family H = { $\alpha(\mu_i)$: i$\in$ J } is a α – level open subsets of X , where $F_i = \alpha(\mu_i^c)$. For let x $\in$ X . Then there exists a $\mu_{i_k} \in$ M (k $\in J_n$) such that μ_{i_k}(x) > α . Hence (X, t) is α – compact .

3.2 Theorem : Let $0 \le \alpha < 1$. Let (X, t) be an fts and (X, t_α) be a α – level topological space . Let f : $(X, t) \to (X, t_\alpha)$ be continuous and onto . If (X, t) is α – compact , then (X, t_α) is compact topological space .

Proof : Let M = { U_i : i$\in$ J } be an open cover of (X, t_α) . Then , since for each U_i , there exists a $g_i \in$t such that $U_i = \alpha(g_i)$, we have M = { $\alpha(g_i)$: i$\in$ J } . Then the family W = { g_i : i$\in$ J } is an α – shading of (X, t) . Since f is continuous , then f^{-1} (M) = { $f^{-1}(U_i)$: $U_i \in t_\alpha$ } is an open α – shading of (X, t) . To see this , let x $\in$X . Since M is an open cover of (X, t_α) , then there is an $U_{i_0} \in$ M such that x $\in U_{i_0}$. But $U_{i_0} = \alpha(g_{i_0})$ for some $g_{i_0} \in$ t . Therefore x $\in \alpha(g_{i_0})$ which implies that g_{i_0}(x) > α . Since f is continuous and onto , then U_{i_0}(f(x)) > α which implies that $f^{-1}(U_{i_0})$(x) > α . By α – compactness of (X, t) , W has a finite α – subshading , say { g_{i_k} } (k $\in J_n$) such that

$f^{-1}(g_{i_k})(x) > \alpha$ or $g_{i_k}(f(x)) > \alpha$ for some x $\in$ X . Thus { U_{i_k} } or { $\alpha(g_{i_k})$ } (k $\in J_n$) forms a finite subcover of M . Hence (X, t_α) is compact topological space .

3.3 Theorem : Let (X, t) and (Y, s) be two fuzzy topological spaces and let f : $(X, t) \rightarrow (Y, s)$ be a continuous surjection . Let A be an α – compact subset of (X, t) . Then f(A) is also α – compact of (Y, s) .

Proof : Assume that f(X) = Y . Let { $u_i : u_i \in$ s } be an open α – shading of f(A) . Since f is continuous , then $f^{-1}(u_i) \in$ t . For if x $\in$ A , then f(x) $\in$ f(A) as A is α – compact subset of (X, t) . Thus we see that $f^{-1}(u_i)(x) > \alpha$ and so $f^{-1}(u_i)$ is an open α – shading of A . Since A is α – compact , then { $f^{-1}(u_i)$ } has a finite α – subshading , say { $f^{-1}(u_{i_k})$ } (k $\in J_n$) . Now if y $\in$ f(A) , then y = f(x) for some x $\in$ A . Then there exists $u_{i_k} \in \{u_i\}$ such that $f^{-1}(u_{i_k})(x) > \alpha$ which implies that $u_{i_k}(f(x)) > \alpha$ or $u_{i_k}(y) > \alpha$. Thus { u_i } has a finite α – subshading { u_{i_k} } (k $\in J_n$) . Hence f(A) is α – compact .

3.4 Definition : The mappings π_x : X $\times$Y $\rightarrow$ X such that π_x(x , y) = x for all (x , y) $\in$ X $\times$Y and π_y : X $\times$Y $\rightarrow$ Y such that π_y(x , y) = y for all (x , y) $\in$ X $\times$Y are called projection mappings or simply projection of X $\times$Y on X and Y respectively .

3.5 Theorem : Let (X, t) and (Y, s) be two fuzzy topological spaces . Then the product space $(X \times Y, t \times s)$ is α – compact iff (X, t) and (Y, s) are α – compact , where $0 \le \alpha < 1$.

Proof : First suppose that $(X \times Y, \delta)$, where δ = { $g_i \times h_i$: $g_i \in$ t and $h_i \in$ s } is α – compact . Now we can define a fuzzy projection mappings π_x : $(X \times Y, \delta) \rightarrow (X, t)$ such that π_x(x , y) = x for all (x , y) $\in$ X $\times$Y and π_y : $(X \times Y, \delta) \rightarrow (Y, s)$ such that π_y(x , y) = y for all (x , y) $\in$ X $\times$Y which we know are continuous . Hence (X, t) and (Y, s) are continuous images of $(X \times Y, \delta)$ which are therefore α – compact when $(X \times Y, \delta)$ is given to be α – compact .

Conversely, let (X, t) and (Y, s) be α – compact. Let δ = { $g_i \times h_i$: $g_i \in$ t and $h_i \in$ s }, where g_i and h_i are open fuzzy sets and { g_i : i∈ J } is an α – shading of (X, t) and { h_i : i∈ J } is an α – shading of (Y, s). That is g_i(x) > α for all x ∈ X, h_i(y) > α for all y ∈ Y. We see that ($g_i \times h_i$)(x , y) = min { g_i(x) , h_i(y) } > α . As (X, t) and (Y, s) are α – compact, there exist $g_{i_k} \in$ t such that g_{i_k}(x) > α and $h_{i_k} \in$ s such that h_{i_k}(y) > α respectively. Hence we have δ = { $g_i \times h_i$: $g_i \in$ t and $h_i \in$ s } has a finite α – subshading, say { $g_{i_k} \times h_{i_k}$ } (k ∈ J_n). Thus $(X \times Y, \delta)$ is α – compact.

3.6 Theorem : Let (X, t) be a fuzzy Hausdorff space and A be an α – compact ($0 \le \alpha < 1$) subset of (X, t). Suppose x ∈ A^c, then there exist u , v ∈ t such that u(x) = 1 , A ⊆ v^{-1}(0 , 1] and u ⊆ 1 – v .

Proof : Let y ∈ A . Since x ∉ A (x ∈ A^c), then clearly x ≠ y . As (X, t) is fuzzy Hausdorff , then there exist u_y , v_y ∈ t such that u_y(x) = 1 , v_y(y) = 1 and u_y ⊆ 1 – v_y . Let us take $\alpha \in I_1$ such that v_y(y) > α > 0 . Thus we see that { v_y : y ∈ A } is an α – shading of A . Since A is α – compact in (X, t), so it has a finite α – subshading , say { v_{y_k} : y_k ∈ A } (k ∈ J_n) . Now, let v = $v_{y_1} \cup v_{y_2} \cup \ldots\ldots \cup v_{y_n}$ and u = $u_{y_1} \cap u_{y_2} \cap \ldots\ldots \cap u_{y_n}$. Thus we see that v and u are open fuzzy sets , as they are the union and finite intersection of open fuzzy sets respectively i.e. v , u ∈ t . Moreover , A ⊆ v^{-1}(0 , 1] and u(x) = 1 , as u_{y_k} (x) = 1 for each k .

Finally , we claim that u ⊆ 1 – v . As u_y ⊆ 1 – v_y implies that u ⊆ 1 – v_y . Since u_{y_k} (x) ≤ 1 – v_{y_k} (x) for each k , then u ⊆ 1 – v . If not , then there exists x ∈ X such that u_y(x) $\not\le$ 1 – v_y(x) . We have u_y(x) ≤ u_{y_k} (x) for all k . Then for some k , u_{y_k} (x) $\not\le$ 1 – v_{y_k} (x) . But this is a contradiction as u_{y_k} ⊆ 1 – v_{y_k} for all k . Hence u ⊆ 1 – v .

3.7 Theorem : Let (X, t) be a fuzzy Hausdorff space and A , B be disjoint α – compact ($0 \le \alpha < 1$) subsets of (X, t). Then there exist u , v ∈ t such that A ⊆ u^{-1}(0 , 1] , B ⊆ v^{-1}(0 , 1] and u ⊆ 1 – v.

Proof : Let y ∈ A . Then y∉B , as A and B are disjoint . Since B is α – compact , then by theorem (3.6) , there exist u_y , v_y ∈ t such that u_y(y) = 1 , B ⊆ v_y^{-1}(0 , 1] and u_y ⊆ 1 – v_y . Let us take $\alpha \in I_1$ such that v_y(y) > α > 0 . As u_y(y) = 1 , then we observe that{ u_y : y ∈ A } is an α – shading of A .

Since A is α – compact in (X, t), so it has a finite α – subshading , say { u_{y_k} : $y_k \in$ A } (k $\in J_n$) . Furthermore , since B is α – compact , so B has a finite α – subshading , say { v_{y_k} : $y_k \in$ B } (k $\in J_n$) as B $\subseteq v_{y_k}^{-1}$(0 , 1] for each k . Now , let u = $u_{y_1} \cup u_{y_2} \cup \ldots\ldots \cup u_{y_n}$ and v = $v_{y_1} \cap v_{y_2} \cap \ldots.. \cap v_{y_n}$. Thus we see that A $\subseteq u^{-1}$(0 , 1] and B $\subseteq v^{-1}$(0 , 1] . Hence u and v are open fuzzy sets , as they are the union and finite intersection of open fuzzy sets respectively i.e. u , v $\in$ t .

Finally , we have to that u $\subseteq$ 1 – v . First we observe that $u_{y_k} \subseteq 1 - v_{y_k}$ for each k , implies that $u_{y_k} \subseteq$ 1 – v for each k and it is clearly shows that u $\subseteq$ 1 – v .

3.8 Theorem : Let (X, t) be an fts and A $\subseteq$ X .

(i) If $0 \le \alpha < 1$ and if A is α – compact , then A is closed in X .

(ii) If 0 < $\alpha \le$ 1 and if A is α^* – compact , then A is closed in X .

Proof : (i) : Let x $\in A^c$. We have to show that , there exist u $\in$ t such that u(x) = 1 and u $\subseteq A^p$, where A^p is the characteristic function of A^c . Indeed , for each y $\in$ A , there exist u_y , $v_y \in$ t such that u_y(x) = 1 , v_y(y) = 1 and $u_y \subseteq 1 - v_y$. Let us take $\alpha \in I_1$ such that v_y(y) > α > 0 . Thus we see that { v_y: y $\in$ A } is an α – shading of A . Since A is α – compact in (X, t) , so it has a finite α – subshading , say { v_{y_k} : $y_k \in$ A } (k $\in J_n$) . Now , let u = $u_{y_1} \cap u_{y_2} \cap \ldots\ldots \cap u_{y_n}$. Thus we see that u_y(x) =1 and u $\subseteq 1 - v_{y_k}$ for each k and it is clear that u $\subseteq$ 1 – v . For , each z $\in$ A , there exists a k such that v_{y_k} (z) > $\alpha \ge$ 0 and so u(z) = 0 . Hence u $\subseteq A^p$. Therefore , A^c is open in X . Thus A is closed in X .

(ii) The proof is similar .

3.9 Theorem : Let (X, t) be a fuzzy regular space and A be an α – compact subset of (X, t) . Suppose x $\in$ A and u $\in t^c$ with u(x) = 0 . Then there exist v , w $\in$ t such that v(x) = 1 , u $\subseteq$ w , A $\subseteq v^{-1}$(0 , 1] and v $\subseteq$ 1 – w .

Proof : Suppose x $\in$ A and u $\in t^c$ we have u(x) = 0 . As (X, t) is fuzzy regular , then there exist v_x , $w_x \in$ t such that v_x(x) = 1 , $u_x \subseteq w_x$ and $v_x \subseteq 1 - w_x$. Let us take $\alpha \in I_1$ such that v_x(x) > α > 0 . Thus we observe that { v_x: x $\in$ A } is an open α – shading of A . Since A is α – compact in (X, t) , then it has a finite α – subshading , say { v_{x_k} : $x_k \in$ A } (k $\in J_n$) . Let v = $v_{x_1} \cup v_{x_2} \cup$ ….. $\cup v_{x_n}$ and w = $w_{x_1} \cap w_{x_2} \cap$ ….. $\cap w_{x_n}$. Thus we see that v and w are open fuzzy sets , as they are the union and finite intersection of open fuzzy sets respectively i.e. v , w $\in$ t . Furthermore , u $\subseteq$ w , A $\subseteq$ v^{-1}(0 , 1] and v(x) = 1.

Finally , we have to show that v $\subseteq$ 1 – w . As $v_{x_k} \subseteq 1 - w_{x_k}$ for each k implies that $v_{x_k} \subseteq$ 1 – w for each k and hence it is clear that v $\subseteq$ 1 – w .

3.10 Theorem : Let (X, t) be a fuzzy regular space and A , B be disjoint α – compact subsets of (X, t) . For each x $\in$ X and u $\in t^c$ with u(x) = 0 , there exist v , w $\in$ t such that A $\subseteq$ v^{-1}(0 , 1] , B $\subseteq$ w^{-1}(0 , 1] and v $\subseteq$ 1 – w .

Proof : Suppose for each x $\in$ X and u $\in t^c$ we have u(x) = 0 . Let x $\in$A . Then x $\notin$ B , as A and B are disjoint . As B is α – compact , then by theorem (3.9) , there exist v_x , $w_x \in$ t such that v_x(x) = 1 , B $\subseteq$ w_x^{-1}(0 , 1] and $v_x \subseteq 1 - w_x$. Let us take $\alpha \in I_1$ such that v_x(x) > α > 0 . As v_x(x) = 1 , then we see that { v_x : x $\in$ A } is an open α – shading of A . Since A is α – compact in (X, t) , then it has a finite α – subshading , say { v_{x_k} : $x_k \in$ A } (k $\in J_n$) . Further more , as B is α – compact , so it has a finite α – subshading , say { w_{x_k} : $x_k \in$ B } (k $\in J_n$) , as B $\subseteq$ w_x^{-1}(0 , 1] for each k . Let v = $v_{x_1} \cup v_{x_2} \cup$ ….. $\cup v_{x_n}$ and w = $w_{x_1} \cap w_{x_2} \cap$ ….. $\cap w_{x_n}$. Thus we see that A $\subseteq$ v^{-1}(0 , 1] and B $\subseteq$ w^{-1}(0 , 1] . Hence v and w are open fuzzy sets , as they are the union and finite intersection of open fuzzy sets respectively i.e. v , w $\in$ t .

Lastly , we have to show that v $\subseteq$ 1 – w . First , we observe that $v_{x_k} \subseteq 1 - w_{x_k}$ for each k implies that $v_{x_k} \subseteq$ 1 – w for each k and hence it is clear that v $\subseteq$ 1 – w .

3.11 Theorem : Let (A, t_A) and (B, s_B) be fuzzy subspaces of fts's (X, t) and (Y, s) respectively with (A, t_A) is α – compact . Let f : $(A, t_A) \to (B, s_B)$ be relatively fuzzy continuous , one – one and onto . Then (B, s_B) is α – compact .

Proof : Let { v_i : $v_i \in s_B$ } be an open α – shading of (B, s_B) for every i $\in$ J . As f is fuzzy continuous , then $f^{-1}(v_i) \in$ t . By definition of subspace fuzzy topology , there exist $u_i \in$ s such that $v_i = u_i \cap$ B . We see that for every x $\in$ X , $f^{-1}(v_i)$ (x) = $f^{-1}(u_i \cap$ B) (x) > α and so { $f^{-1}(u_i \cap$ B) } is an open α – shading of (A, t_A) , i $\in$ J . Since (A, t_A) is α – compact , then { $f^{-1}(u_i \cap$ B) } has a finite α – subshading , say { $f^{-1}(u_{i_k} \cap$ B) } (k $\in J_n$) . Now , if y $\in$ Y , then y = f(x) for some x $\in$ X . Then there exists $v_{i_k} \in$ { v_i } such that $f^{-1}(v_{i_k})$ (x) > α implies that $f^{-1}(u_{i_k} \cap$ B) (x) > α . So ($u_{i_k} \cap$ B) f(x) > α or ($u_{i_k} \cap$ B) (y) > α . Hence we observe that { v_{i_k} } (k $\in J_n$) is a finite α – subshading of { v_i } . Thus (B, s_B) is α – compact .

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