International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 6, June - 2013

Channel Capacity Comparison of MIMO Systems with Rician Distributions, Rayleigh Distributions and Nakagami-m

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Abstract

This paper represents the MIMO channel capacity over Rician fading, Rayleigh fading and Nakagami fading channel. Here Rician fading model employs a zero-mean stochastic sinusoid as the line-of-sight component. This paper offers analyses and simulations to the behavior of MIMO system and its expected capacity for various channel distribution under flat fading. Several types of distributions (Rician, Rayleigh and Nakagami) are considered with different parameters to generate the channel matrix and determine the capacity for several cases of antenna numbers in both transmitter and receiver sides. For Nakagami-m fading the join eigenvalue of W=HH, where H is the channel matrix, is derived in a closed form in the 2 × 2 case and for integervalues of m .

Index Terms—Fading distributions, Rayleigh distribution, Rician distribution, Nakagami-*m* distribution, Eigenvalue distribution, MIMO channels.

1. Introduction

MULTIPLE-ANTENNA wireless terminals, which are used along with special signal-processing techniques to achieve diversity and multiplexing benefits, characterize multiple-input–multiple-output (MIMO) wireless technology. MIMO technology exploits the space dimensions, in addition to the time and frequency dimensions, to deliver data rates and a quality of service unmatched otherwise with comparable spectral resources. A MIMO channel is represented by a channel matrix, whose elements are channel gains between transmitter–receiver antenna pairs. Thus, mathematical tools such as the random matrix theory help the analysis. Many differenttechniques have been proposed for the modelling and simulation of mobile radio channels.

2. Theory

The general MIMO system is shown in Fig. 1 with N_T transmitter antennas and N_R receiver antennas. The signal model represented as:

$$r = Hx + n \qquad (1)$$

where r is (N_R x 1) received signal vector, x is (N_T x 1) transmitted signal vector, n is (N_T x 1) complex additive white Gaussian noise (AWGN) vector with variance σ, and H is the (N_R x N_T) channel matrix. The channel matrix H represents the effect of the medium on the transmitter –receiver links. The channel matrix H can be represented as,

$$H=\begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_{R1}} & \cdots & h_{N_R N_T} \end{bmatrix} \qquad (2)$$

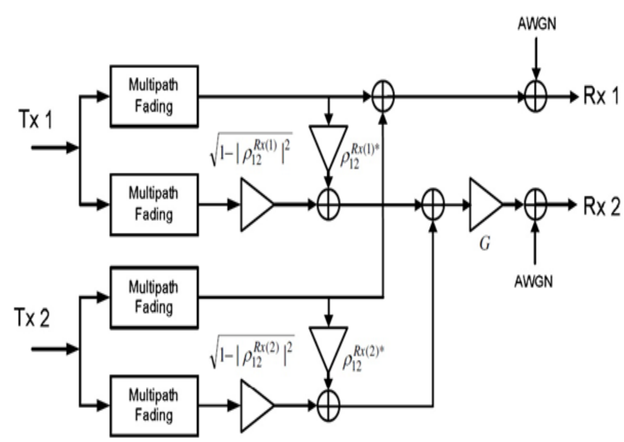

Figure1. General MIMO system model

Channel matrix may offer K equivalent parallel sub channels with different mean gains, where

$$K = \text{rank}\,(HH^H) \leq \min(N_T N_R) \qquad (3)$$

Singular value decomposition (SVD) simplification can be used to demonstrate the effect of channelmatrix

H on the capacity. Then, channel matrix H can be expressed as:

$$H=UBV^H \quad (4)$$

With the columns of the unitary matrix U (N_T x N_R) contains the eigenvectors of HH^H and the columns of the unitary matrix V (N_T x N_T) contains the eigenvectors of H^HH. The diagonal matrix B (N_R x N_T) has non-negative, real valued elements (called singular values) equal to the square roots of the Eigen values λ of HH^H.

Assuming that the channel is known at both T_X and R_X (full or prefect channel sensing information CSI) then the maximum normalized capacity with respect to bandwidth (in term of b/s/Hz spectral efficiency) of parallel sub channels equals :

$$C=\sum_{i=1}^{k} log_2(1+\lambda_i \frac{p_i}{\sigma_N^2}) \quad (5)$$

where p_i is the power allocated to each sub channel *i* and can be determined to maximize the capacity using water filling theorem such that each sub channel was filled up to a common level D :

$$\frac{1}{\lambda_1} + P_1 + \ldots\ldots\ldots\ldots\ldots + \frac{1}{\lambda_k} + P_k = D \quad (6)$$

Or

$$P_i = D - \frac{1}{\lambda_i} \quad (7)$$

Such that it satisfies the following condition that sums of all sub channels power equal to the total transmitted power or :

$$\sum_{i=1}^{k} P_i = P_{TX} \quad (8)$$

And if $\frac{1}{\lambda_i}$> D then P_i is set to zero.

A brief overview of the random distributions used in this work is as following:

RICIAN DISTRIBUTION:

The Rician distribution is appropriate to use when the receiver's position is on a line of sight (LOS) with respect to the transmitter, thus there will be an LOS signal component in the received signal due to the multipath. The density function for this distribution is given by:

$$f(x)= \frac{x}{b^2} e^{(-\frac{(x^2+s^2)}{2b^2})} I_0(\frac{xs}{b^2}) \quad (9)$$

Where I_0 the zero-order is modified Bessel function of the first kind, s (s ≥ 0) non-centrality parameter and b (b > 0) scale parameter. The Rician distribution is used to generate the channel matrix and determine the related capacity for the system:

$$H_{Rician}= \begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R1} & \cdots & h_{N_RN_T} \end{bmatrix} \quad (10)$$

RAYLEIGH DISTRIBUTION:

Rayleigh distributions are used to model scattered signals that reach a receiver by multiple paths.The Rayleigh distribution is a special case of Weibull distribution. The distribution function of this Weibull distribution is given by:

$$f(x|b)=\frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(\frac{x}{\beta})^\alpha} \quad , x>0 \qquad (11)$$

Weibull distribution with α=2 and β=√2b where b is the scale parameter of Rayleigh distribution which probability density function is given by

$$f(x|b) = \frac{x}{b^2} e^{(-\frac{x^2}{2b^2})} \quad (12)$$

This Rayleigh distribution is used to generate the channel matrix and determine the related capacity for the system:

$$H_{Rayleigh}= \begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R1} & \cdots & h_{N_RN_T} \end{bmatrix} \quad (13)$$

NAKAGAMI-M DISTRIBUTION:

The Nakagami-m distribution is another important distribution used in communication field to model the statistical fading of the multipath scenarios and was developed from experimental measurements.The probability density function for this Nakagami-m is given by:

$$f(x)=\frac{2m^m x^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{mx^2}{\Omega}} = \frac{2x^{2m-1}}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m e^{-\frac{mx^2}{\Omega}} \quad (14)$$

Where Ω is the second moment and represent the scale parameter, m ($m \geq 0.5$) is known as the Nakagami fading parameter or shape parameter, and $\Gamma(.)$ is the standard Gamma function. The Nakagami-m distribution covers a wide range of fading conditions; when m=0.5 it is a one-sided Gaussian distribution and when m=1, it is a Rayleigh distribution and when m < 1, the Nakagami model applies a fading scenario that is more severe than Rayleigh fading .

The Nakagami-m distribution is used to generate the channel matrix and determine the related capacity for the system:

$$H_{Nakagami} = \begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_{R1}} & \cdots & h_{N_{RN_T}} \end{bmatrix} \quad (15)$$

3. Simulation Results

In this paper, MATLAB m-file and Simulink is used to verify the model and simulate the effects of several types of distributions (Rician, Rayleigh and Nakagami-m) for a MIMO system under flat fading to generate the channel matrix.

The simulation is done for several pairs of $N_R$and$N_T$as detailed in Table below:

Case	Number of Transmitter antenna(N_T)	Number of Receiver antenna (N_R)
1st	1	1
2nd	2	2
3rd	3	3
4th	4	4

A. RICIAN DISTRIBUTION

The first distribution considered is Rice distribution with three different sets of non-centrality parameter s and parameter σ. The capacity of the system (in term of b/s/Hz), for each set of the Rice distribution parameters, is calculated for each case in Table 1 over awide range of SNR (-10 dB to 30 dB). Each of the eight cases is represented with capacitycurves using different colors and special marker symbols. The first set of parameters is unity non-centrality parameter (s = 1) and unity scale parameter (b = 1). The achieved results are shown in Fig. 2.

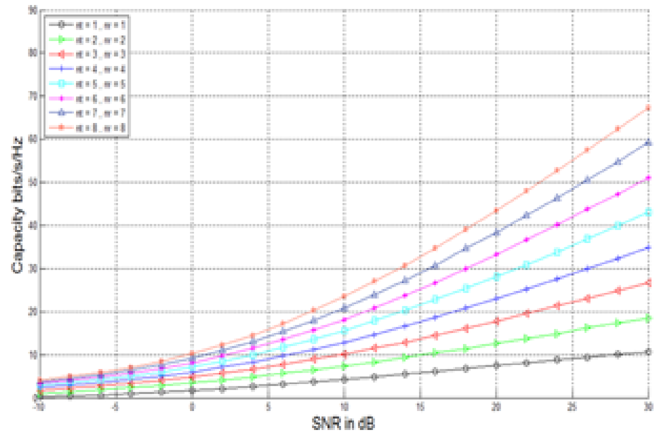

Figure 2. The channel capacity with Rician distribution (s = 1, b = 1)

From the inspection of the Fig .2 , and for the 1st curve ($N_T = 1$, $N_R = 1$), it's obvious that the capacity is increased as signal to noise ratio (SNR) increases with respect to eq. (5) which is relate to the generating channel matrix H by Rician distribution as in eq. (9).

For the 2nd case ($N_T = 2$, $N_R= 2$), the capacity is improved for the same values of SNR comparing to the1st one because of increasing number of antennas in both transmitter and receiver sides.

The 3rd case ($N_T = 3$ and $N_R = 3$) shows that the capacity is increased for the same values of SNR comparing to the first and 2nd case. The capacity increasingcorresponds to the H_{Rician}in approximating exponential manner.

The 4th case ($N_T = 4$ and $N_R = 4$) shows that the capacity is increased for the same values of SNR comparing to previous cases in more approximating exponential behavior.

B. RAYLEIGH Distribution

The capacity of the system (in term of b/s/Hz), for each set of the Weibull distribution parameters is calculated for each case in Table 1 over a wide range ofSNR (-10 dB to 30 dB).The results of Fig. 3, illustrates variation of capacity with number of

ISSN: 2278-0181
Vol. 2 Issue 6, June - 2013

employed antennas. The capacity is increasing function to the number of antennas in both transmitter andreceiver sides and manner similar to that of Rician distribution.

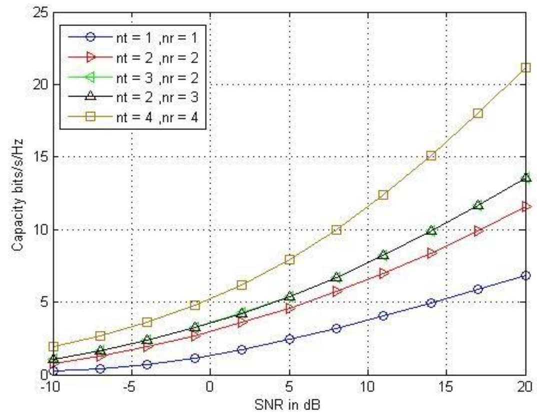

Fig. 3 The channel capacity of Rayleigh

Comparing with results in Fig. 2, the capacity with Rayleigh distribution is lower in value comparing to that with Rician distribution (s =1, b = 1).

C. NAKAGAMI-M DISTRIBUTION

As for both Rician and Rayleigh distributions, the capacity of the system (in term of b/s/Hz) is calculated for each case in Table 1 over a wide range of SNR (-10 dB to 30 dB). The first set of evaluation parameters is the scale parameter (Ω = 1), m shape parameter (m = 1). The obtained results are depicted in Fig. 4. However, comparing the results with that in Fig.3, it seen that the capacity of first case (N_T = 1, N_R = 1) with Nakagami-m is greater than that with Rayleigh distribution for the same value of SNR. While, the capacity for the (2nd up to 4th cases) are lower than those with Rayleigh distribution at the same SNR.

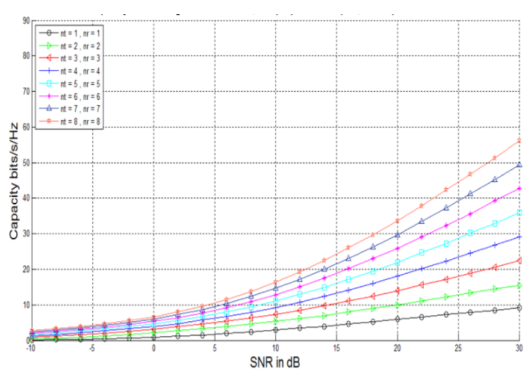

Fig.4 The channel capacity with Nakagami-m distribution (Ω = 1, m = 1)

The second set of parameters is the scale parameter (Ω = 2) and shape parameter (m = 2). The achieved results are illustrated in Fig. 5.

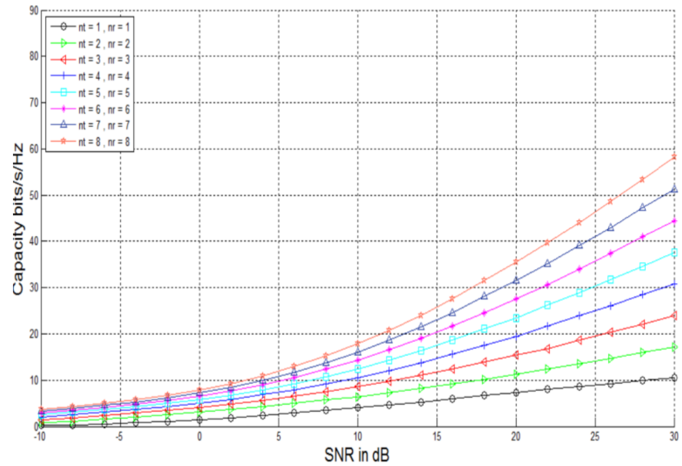

Fig.5.The channel capacity with Nakagami-m distribution (Ω = 2, m = 2)

Fig. 5 shows that the capacity is increasing function to the number of antennas in both transmitter and receiver sides, as that in Fig. 4, but with a slightly increase in the capacity for the same SNR and number of antenna pairs.

4. Conclusions

The obtained results give an inspection to the influence of the distribution selection over the capacity of multi-input multi-output MIMO system estimation and led to better understanding of the effect of each distribution and how it can be used to approximate different environments. The change of the evaluation parameters of each distribution, for the same number of antenna pair at receiver and transmitter and SNR,led to different value of capacity since its effect the generating of H matrix. Also, the investigating of more channel distributions is benefit led to better modelling of channel for different operation scenarios and various environments.

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