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Commutative Multi Anti L – Fuzzy Subgroups

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Abstract

In this paper, we define the algebraic structures of multi-anti fuzzy subgroup and some related properties are investigated. The purpose of this study is to implement the fuzzy set theory and group theory in multi-anti fuzzy subgroups. In this paper the concepts of L – Fuzzy subgroup and anti L – Fuzzy subgroups and some results involving them are generalized to the L – Fuzzy case where L is an arbitrary Lattice with 0 and 1. Commutativity for Multi Anti L – Fuzzy subgroup is introduced and some necessary and sufficient conditions for a Multi Anti L – Fuzzy subgroup to be commutative are derived.

Keywords

L – fuzzy subgroup, Multi L – Fuzzy subgroups, Anti L – Fuzzy subgroup, Multi Anti L – Fuzzy subgroup, Normal L – Fuzzy subgroup, Commutative L – Fuzzy subgroup, Commutative Multi Anti L – Fuzzy subgroup.

1. Introduction

Applying the concept of Fuzzy sets introduced by Zadeh [11] to group theory Rosenfeld [7] defined Fuzzy subgroup of a given group and derived some of their properties.

Das [4] characterized fuzzy subgroups by their level subgroups. The concept of anti – fuzzy subgroup was introduced by Biswas [3]. The Concept of Commutativity L-fuzzy subgroups was introduced by Souriar Sebastian and S.Babu Sundar [10]. In all these studies, the closed unit interval [0, 1] is taken as the Membership lattice.

In this paper, we extend these concepts to the Multi Anti L – Fuzzy [7] case, where L is an arbitrary Lattice and derive some more properties. We also introduce commutative for Multi Anti L – Fuzzy subgroups and obtain some characterizations.

2. Preliminaries

Throughout this paper G denotes an arbitrary Multiplicative group with "e" is an identity element and L denotes an arbitrary Lattice with least element 0 and greatest element 1. The join and meet operations in L are denoted by $\vee$ and $\wedge$ respectively. A function A: G $\rightarrow$ L is called and Multi L – Fuzzy subset of G. If H $\subseteq$ G then Ψ_H denotes the characteristic function of H.

2.1 Definition [11]

Let X be any nonempty set. A fuzzy set μ of X is $\mu : X \rightarrow [0, 1]$.

2.2 Definition [11]

A L-Fuzzy subset A of X is a mapping from X into L, where L is a complete lattice satisfying the infinite meet distributive law. if L is the unit interval [0,1] of real numbers, there are the usual fuzzy subset of X.

A L-fuzzy subset A: X→L is said to be non-empty, if it is not the constant map which assumes the values 0 of L.

2. 3 Definition [5]

A L-fuzzy subset A of G is said to be a L-fuzzy group of G, if for all x,y $\in$ G

i. $A(xy) \geq A(x) \wedge A(y)$
ii. $A(x^{-1}) = A(x)$.

2. 4 Definition [3]

A L-fuzzy subset A of G is said to be a anti L-fuzzy group of G, if for all x,y $\in$ G

i. $A(xy) \leq A(x) \vee A(y)$

ii. $A(x^{-1}) = A(x)$.

3. Some properties of multi Anti L- fuzzy subgroups

In this section, we discuss some of the properties of multi Anti L- fuzzy subgroups.

3.1 Definition [9]

Let X be a non – empty set. A Multi L – fuzzy set A in X is defined as a set of ordered sequences. $A = \{ (x, \mu_1(x), \mu_2(x), ..., \mu_i(x), ...) : x \in X\}$, where $\mu_i : X \rightarrow L$ for all i.

3.2 Definition [5]

A Multi L – Fuzzy subset A of G is called an Multi L – Fuzzy subgroup (MLFS) of G if for every x,y $\in$ G,

i. $A(xy) \geq A(x) \wedge A(y)$

ii. $A(x^{-1}) = A(x)$

Remark

It can be proved that if A is an MLFS of G then $A(e) \geq A(x)$ for all x,y $\in$G. Also a L-Fuzzy subset A of G is an MLFS of G iff $A(xy^{-1}) \geq A(x) \wedge A(y)$ for all x, y $\in$G.

3.3 Definition [10]

A Multi anti L – Fuzzy subset A of G is called an Multi anti L – Fuzzy subgroup (MALFS) of G if for every x,y $\in$ G,

i. $A(xy) \leq A(x) \vee A(y)$

ii. $A(x^{-1}) = A(x)$.

Remark

It can be proved that if A is an MALFS of G then $A(e) \leq A(x)$ for all $x,y \in G$. Also a L-Fuzzy subset Aof G is an MALFS of G iff $A(xy^{-1}) \leq A(x) \vee A(y)$ for all $x, y \in G$.

3.1 Theorem

(a) For any non-empty subset H of G, the characteristic function of H, Ψ_H is an MLFS of G iff H is a subgroup of G

(b) If A is an MALFS of G then $A(e) \leq A(x)$ for every $x \in G$.

(c) Multi L – Fuzzy subset A is an MALFS of G iff $A(x\,y^{-1}) \geq A(X) \vee A(Y)$ for every $x,y \in G$.

(d) An Multi L – Fuzzy subset $A \neq \bar{o}$ of G is an MLFS of G iff

$A_a = \{ x \in G ; A(x) \geq a \}$ is a subgroup of G, for every $a \in L$ for which

$0 < a \leq A(e)$. The subgroup A_a is called the level subgroup of A determined by a.

Proof

Their proofs are straight forward.

3.4 Definition [10]

If c: L $\rightarrow$ L is an order reversing involution satisfying De Morgan Law and A^c denotes c(a) for every $a \in L$, then for every $a \in L$ then for any Multi Anti L – Fuzzy subset_A of G, $A^c : G \rightarrow L$ defined by $A^c(x) = (A(x))^c$ for every $x \in G$ is called the c-Complement of A.

3.5 Definition[8]

Let $A = (\mu_1, \mu_2, ..., \mu_k)$ be a multi-fuzzy set of dimension k and let μ_i' be the fuzzy complement of the ordinary fuzzy set μ_i for i = 1, 2, ..., k. The Multi-fuzzy

Complement of the multi-fuzzy set A is a multi-fuzzy set (μ_1', ..., μ_k') and it is denoted by c(A) or A' or A^c.

That is, $c(A) = \{(x, c(\mu_1(x)), ..., c(\mu_k(x))) : x \in X\} = \{(x, 1 - \mu_1(x), ..., 1 - \mu_k(x)) : x \in X\}$, where c is the fuzzy complement operation.

3.2 Theorem

A is a Multi L – Fuzzy subgroup of G iff A^c is a Multi anti L – Fuzzy subgroup of G.

Proof

Suppose A is a multi-fuzzy subgroup of G. Then for all $x, y \in G$,

$$A(xy) \geq \min \{A(x), A(y)\}$$

$$\Leftrightarrow \quad 1 - A^c(xy) \geq \min \{(1 - A^c(x)), (1 - A^c(y))\}$$

$$\Leftrightarrow \quad A^c(xy) \leq 1 - \min \{(1 - A^c(x)), (1 - A^c(y))\}$$

$$\Leftrightarrow \quad A^c(xy) \leq \max \{A^c(x), A^c(y)\}.$$

We have, $A(x) = A(x^{-1})$ for all x in G

$$\Leftrightarrow \quad 1 - A^c(x) = 1 - A^c(x^{-1})$$

Therefore $A^c(x) = A^c(x^{-1})$.

Hence A^c is a multi-anti L- fuzzy subgroup of G.

Corollary

If L has an order reversing involution satisfying De Morgan Law and H is a non – empty subset of G. Then Ψ_H is an Multi anti L – Fuzzy subgroup (MALFS) of G iff H is the set complement of a subgroup of G.

Proof

Ψ_H is an MALFS of G.

$\Psi^c_H = \Psi_{G-H}$ is an MLFS of G.

$\Leftrightarrow$ G – H is a subgroup of G.

3.6 Definition [9]

Let X be a non – empty set. A Multi L – Fuzzy set A in X is defined as a set of ordered sequences. A = { (x, $\mu_1(x)$, $\mu_2(x)$, ..., $\mu_i(x)$, ...) : x $\in$ X}, where $\mu_i : X \to [0, 1]$ for all i.

Remark

i. If the sequences of the membership functions have only k-terms (finite number of terms), k is called the dimension of A.
ii. The set of all multi-fuzzy sets in X of dimension k is denoted by $M^kFS(X)$.
iii. The multi fuzzy membership function μ_A is a function from X to $[0, 1]^k$ such that for all x in X, $\mu_A(x) = (\mu_1(x), \mu_2(x), ..., \mu_k(x))$.
iv. For the sake of simplicity, we denote the multi-fuzzy set

$$A = \{(x, \mu_1(x), \mu_2(x), ..., \mu_k(x)) : x \in X\} \text{ as } A = (\mu_1, \mu_2, ..., \mu_k).$$

3.7 Definition [9]

Let k be a positive integer and let A and B in $M^kFS(X)$, where A= (μ_1, μ_2, ..., μ_k) and B = (ν_1, ν_2..., ν_k), then we have the following relations and operations:

i. $A \subseteq B$ if and only if $\mu_i \le \nu_i$, for all i = 1, 2, ..., k;
ii. A = B if and only if $\mu_i = \nu_i$, for all i = 1, 2, ..., k;
iii. $A \cup B = (\mu_1 \cup \nu_1, ..., \mu_k \cup \nu_k) = \{(x, \max(\mu_1(x), \nu_1(x)), ..., \max(\mu_k(x), \nu_k(x))) : x \in X\}$;
iv. $A \cap B = (\mu_1 \cap \nu_1, ..., \mu_k \cap \nu_k) = \{(x, \min(\mu_1(x), \nu_1(x)), ..., \min(\mu_k(x), \nu_k(x))) : x \in X\}$;
v. $A + B = (\mu_1+\nu_1, ..., \mu_k+\nu_k) = \{(x, \mu_1(x) + \nu_1(x) - \mu_1(x)\nu_1(x), ..., \mu_k(x) + \nu_k(x) - \mu_k(x)\nu_k(x)) : x \in X\}$.

3.8 Definition [8]

Let A be a fuzzy set on a group G. Then A is said to be a fuzzy subgroup of G if for all x, y $\in$G,

i. $A(xy) \ge \min \{A(x), A(y)\}$
ii. $A(x^{-1}) = A(x)$.

3.9 Definition [8]

A multi-fuzzy set A of a group G is called a multi-fuzzy subgroup of G if for all x, y $\in$G ,

i. $A(xy) \geq \min\{A(x), A(y)\}$
ii. $A(x^{-1}) = A(x)$.

3.10 Definition [8]

A multi-fuzzy set A of a group G is called a multi-anti fuzzy subgroup of G if for all x, y $\in$ G,

i. $A(xy) \leq \max\{A(x), A(y)\}$
ii. $A(x^{-1}) = A(x)$

3.11 Definition [8]

Let A and B be any two multi-fuzzy sets of a non-empty set X. Then for all x$\in$ X,

i. $A \subseteq B$ iff $A(x) \leq B(x)$,
ii. $A = B$ iff $A(x) = B(x)$,
iii. $A\cup B(x) = \max\{A(x), B(x)\}$,
iv. $A\cap B(x) = \min\{A(x), B(x)\}$.

3.12 Definition [8]

Let A and B be any two multi-fuzzy sets of a non-empty set X. Then

i. $A\cup A = A$, $A\cap A = A$,
ii. $A \subseteq A\cup B$, $B \subseteq A\cup B$, $A\cap B \subseteq A$ and $A\cap B \subseteq B$,
iii. $A \subseteq B$ iff $A\cup B = B$,
iv. $A \subseteq B$ iff $A\cap B = A$.

3.3 Theorem

Let 'A' be a multi-anti fuzzy subgroup of a group G and 'e' is the identity element of G. Then

i. $A(x) \geq A(e)$ for all $x \in G$.
ii. The subset $H = \{x \in G \,/\, A(x) = A(e)\}$ is a subgroup of G.

Proof

i. Let $x \in G$.

$$A(x) = \max\{A(x), A(x)\}$$
$$= \max\{A(x), A(x^{-1})\}$$
$$\leq A(xx^{-1})$$
$$= A(e).$$

Therefore, $A(x) \geq A(e)$, for all $x \in G$.

ii. Let $H = \{x \in G \,/\, A(x) = A(e)\}$

Clearly H is non-empty as $e \in H$.

Let $x, y \in H$. Then, $A(x) = A(y) = A(e)$

$$A(xy^{-1}) \leq \max\{A(x), A(y^{-1})\}$$
$$= \max\{A(x), A(y)\}$$
$$= \max\{A(e), A(e)\}$$
$$= A(e)$$

That is, $A(xy^{-1}) \leq A(e)$ and obviously $A(xy^{-1}) \geq A(e)$ by i.

Hence, $A(xy^{-1}) = A(e)$ and $xy^{-1} \in H$.

Clearly, H is a subgroup of G.

3.4 Theorem

Let 'A' be any multi-anti fuzzy subgroup of a group G with identity 'e'. Then $A(xy^{-1}) = A(e) \Rightarrow A(x) = A(y)$ for all x ,y in G.

Proof

Given A is a multi-anti fuzzy subgroup of G and $A(xy^{-1}) = A(e)$.

Then for all x , y in G ,

$$
\begin{aligned}
A(x) &= A(x(y^{-1}y)) \\
&= A((xy^{-1})y) \\
&\leq \max \{ A(xy^{-1}), A(y)\} \\
&= \max \{ A(e) , A(y)\} \\
&= A(y).
\end{aligned}
$$

That is, $A(x) \leq A(y)$.

Now, $A(y) = A(y^{-1})$, since A is a multi-anti fuzzy subgroup of G.

$$
\begin{aligned}
&= A(ey^{-1}) \\
&= A((x^{-1}x)y^{-1}) \\
&= A(x^{-1}(x y^{-1})) \\
&\leq \max \{ A(x^{-1}) , A(x y^{-1})\} \\
&= \max \{A(x) , A(e)\} \\
&= A(x).
\end{aligned}
$$

That is, $A(y) \leq A(x)$.

Hence, $A(x) = A(y)$.

3.5 Theorem

A is a multi-anti fuzzy subgroup of a group G if and only $A(x y^{-1}) \leq \max\{A(x), A(y)\}$, for all x, y in G .

Proof

Let A be a multi-anti fuzzy subgroup of a group G. Then for all x ,y in G ,

$$A(x y) \leq \max\{A(x), A(y)\}$$

and $$A(x) = A(x^{-1}).$$

Now , $$A(x y^{-1}) \leq \max\{A(x), A(y^{-1})\}.$$

$$= \max\{A(x), A(y)\}$$

$$\Leftrightarrow \quad A(x y^{-1}) \leq \max\{A(x), A(y)\}.$$

3.6Theorem

If A is an Multi L – Fuzzy subset of G, then the following are equivalent.

i. A is both an MLFS and MALFS of G.
ii. A is constant.

Proof

$i \Rightarrow ii$

If A is both an MLFS and MALFS G, then we have

$A(e) \leq A(x) \leq A(e)$ for every $X \in G$,

Hence $A(x) = A(e)$ for all $x \in G$ and therefore, A is constant. The converse is trivial.

3.7 Theorem

If L is a chain and A is an MALFS of G then A (xy) = A(yx) = A(x) $\vee$A(y) for every x, y $\in$ G with A (x) $\neq$ A (y).

Proof

Since L is a chain A(x) $\neq$ A(y) without loss of generality. We assume that A(x) < A(y). Then,

$$\begin{aligned} A(xy) &\leq A(x) \vee A(y) \\ &= A(x) \\ &= A(xyy^{-1}) \\ &\leq A(xy) \vee A(y) \end{aligned}$$

Since A(x) < A(y). we have A(xy) = A(xy) $\vee$ A(y).

Thus, A(xy) $\geq$ A(x) $\geq$ A(xy) and hence by the anti – circularity law for lattices, A (xy) = A(x).

In a similar way, we can prove that A(yx) = A(x).

Hence, A(xy)= A (yx) = A(x) = A(x) $\vee$A(y).

3.8 Theorem

If L is A chain and A is an MALFS of G, then the following are equivalent.

i. A(xy) = A(x) $\vee$ A(y) whenever A(x) = A(y),

ii. A is a constant.

Proof

In view of theorem 3.7, Implies that A(xy) = A(x) $\vee$ A(y) for every x, y $\in$ G.

Putting y = x^{-1} we get A(x) = A(e) for every x $\in$ G.Hence i $\Rightarrow$ ii.

The converse is obvious.

Remark

Theorem 3.7 - 3.8 imply that if L is a Chain then the equality in the first axiom of the definitions of MLFS and MALFS hold only for constants.

3.13 Definition [10]

A lattice L is said to be without zero meets $a \wedge b > 0$ for every $a,b \in L$ such that $a > 0$ and $b > 0$.

Every chain is a lattice without zero meets. If A is an Multi Anti L – Fuzzy subset of G, then the support of A is defined as the set supp (A) = { $x \in G$;A(x)>0}.

3.9 Theorem

Let L be a lattice without zero meets If A($\neq 0$) is an MALFS of G then supp (A) is a subgroup of G. Further, all subgroups of G can be realized as the support of some MALFS of G.

Proof

Let $x,y \in$ supp (A). Then A(x) > 0 and A(y) > 0. Since L is a lattice without zero meets

$A(xy^{-1}) > A(x) \wedge A(y) > 0$.Hence $xy^{-1} \in$ supp (A) and therefore supp (A) is a subgroup of G.
Now let H be any subgroup of G. Fix $a \in L$ such that a > 0. Define $A : G \rightarrow L$ by

$$A(x) = \begin{cases} a & \text{if } x \in H \\ 0 & \text{otherwise} \end{cases}$$

Then A is an MALFS of G and supp (A) = H.

3.1 Example

Let G = {1,-1,i,-i} where i = $\sqrt{-1}$. This is a group under usual multiplication of complex numbers. Let = [0,1]. Then L is a lattice without zero meets. Define $A:G \rightarrow L$ by A(1) = ½, A(-1) = 1 A(i) = A(-i) = 0. Then A is an multi L-Fuzzy subset of G and supp (A) = {1,-1} is a subgroup of G. But A is not an MLFS of G, since A(-1,-1) = A(1) = ½ and $A(-1) \wedge A(-1) = 1$ and hence A(-1-1) < $(A(-1) \wedge A(-1)$.

4. Properties of Commutative Multi Anti L – Fuzzy subgroups

Throughout this section we assume that L is an Lattice withput zero meets. If A($\neq \bar{o}$) is an MALFS of G , then the restriction of A to supp(A). we shall denote A $|$supp (A)$|$ also by A.

4.1Definition [10]

An MALFS A of G is said to be commutative if $A_{xy} = A_{yx}$ for every x,y$\in$G with A(x) > 0 and A(y) > 0. Where A_{xy} denotes the restriction of A (considered as a function) to the singletion subset {xy} of G.

It may be noted that commutativity of A requires xy and yx to coincide whenever x,y$\in$ supp (A). Observe that this definition actually generalizes the notion of commutativity of ordinary subgroups. That is, for any non-empty subset H of G, Ψ_H is a commutative MALFS of G iff H is a commutative subgroup of G.

4.1Theorem

Let A($\neq 0$) be an MALFS of G. Then the following are equivalent.

(i)A is a commutative MALFS of G

(ii)Supp (A) is a commutative subgroup of G.

(iii)The level subgroups A_a are commutative subgroups of G, for every a $\in$L with 0 < a$\leq$ A(e).

Proof

(i) $\Rightarrow$ (ii)

Let A be a commutative MALFS of G. By theorem(3.9), supp (A) is a subgroup of G. Let x,y$\in$ supp (A). Since A is commutative. $A_{xy} = A_{yx} \Rightarrow xy = yx$.

(ii) $\Rightarrow$ (iii)

Assume that supp (A) is a commutative subgroup of G. Let a$\in$L such that 0 < a$\leq$ A(e). By theorem (3.1) (d) A_a is a subgroup of G. Let x,y$\in A_a$.

Then A(x) $\geq$a and A(y) $\geq$ a. Since a > 0, we have x,y$\in$supp (A) and hence xy = yx

(iii) $\Rightarrow$ (i)

Assume (iii) let $x,y \in$ G such that $A(x) > 0$ and $A(y) > 0$. Let $A(x) = a_1$ and $A(y)= a_2$. Then $x \in A_{a1}$ and $y \in_{a2}$. Put $a = a_1 \wedge a_2$. Since L is without zero meets, $a > 0$. Also $a \leq a_1$, $a \leq a_{2.}$ So that $A_a \supseteq A_{a1}$ and $A_a \supseteq A_{a2}$. Therefore $x,y \in A_a$. But A_a is a commutative subgroup of G. Hence xy = yx and therefore $A_{xy\,=}\,A_{yx.}$

4.2 Theorem:

If A is a commutative MALFS of G and supp (A) is a normal subgroup of G. Then A is a normal MALFS of G.

Proof:

Let $x,y \in$G. We have three different cases.

Case (i)

$x,y \in$ supp (A). Then by Definition [4.1] $A_{xy\,=}\,A_{yx.}$ Hence A(xy) = A(yx).

Case (ii)

$X \in$ supp (A) and $y \notin$ supp (A). Then both xy and yx does not belong to supp (A).

Hence A(xy) = A(yx) = 0.

Case (iii)

$x,y \notin$ supp (A). Then xy and yx may or may not belong to supp (A). Since supp (A) is normal subgroup of G, either xy and yx both belongs to supp (A) or both does not belong to supp (A).

a) Xy, yx $\notin$ supp (A), then A(xy) = A(yx) =0
b) If xy, yx $\in$ supp (A) then by Theorem (4.2) xy = yx and hence A(xy) = A(yx).

4.1Example

Let L = [0,1] and G be any non – commutative group. Since G is a normal subgroup of itself, $\Psi_G$is normal MALFS of G, But supp(Ψ_G)= G is not commutative. HenceΨ_G is not commutative MALFS of G.

4.3Theorem

For any group G the following are equivalent.

(i) G is commutative

(ii) All MALFSs of G are commutative

(iii) Ψ_G is a commutative MALFS of G.

Proof:

(i)$\Rightarrow$(ii)

Let G be a commutative group and A be any MALFS of G. For any x,y $\in$ G xy = yx and hence $A_{xy} = A_{yx}$. Hence any MALFS of G is commutative.

(ii) $\Rightarrow$ (iii)

Trivial, since Ψ_G itself is an MALFS of G.

(iii) $\Rightarrow$ (i)

Let Ψ_G be a commutative MALFS of G.

Then by theorem (4.1) supp (Ψ_G) = G is commutative.

Let G_i (i = 1,2 …. n) be groups. G=$\prod_{i=1}^{n} G_i$ be their product and $\Pi_i : G \to G_i$ be the projections defined by$\Pi_i(x_1,x_2,\ldots\ldots x_n) = x_i$ The direct product of Multi L- Fuzzy subsets A_i of G_i (i=1,2…. n) is defined as the Multi Anti L – Fuzzy subset of A = $\prod_{i=1}^{n} A_i$ of G given by

$A(x) = \wedge \{ A_i(\Pi_i(x)) : i = 1,2,3,\ldots\ldots\ldots n \}$

4.4 Theorem

If A_i is a (commutative) MALFS of G_i for each i= 1,2 …. n then $\prod_{i=1}^{n} A_i$ is a (commutative) MALFS of $\prod_{i=1}^{n} G_i$.

Proof:

Let A = $\prod_{i=1}^{n} A_i$ and G = $\prod_{i=1}^{n} G_i$ for x = ($x_1, x_2 \ldots. X_n$), y = ($y_1, y_2 \ldots. Y_n$) $\in$ G.

We have

$$A(xy^{-1}) = \vee\{A_i(x_iy_i^{-1}) : i=1,2,\ldots\ldots n\}$$

$$\leq \vee \{A_i(x_i) \vee A_i(y_i^{-1}) : i=1,2,\ldots\ldots n\}$$

$$= (\vee_I A_i(x_i)) \vee (\vee_I A_i(y_i^{-1}))$$

$$= A(x) \vee A(y)$$

Hence by theorem 3.1 (c) A is an MLFS of G.

Now let A_i be the commutative MALFS\s and $x,y \in$ supp (A). Then

$A(x) = \vee \{ A_i(x_i) : i = 1,2,3\ldots\ldots n \} > 0. A_i(x_i) > 0 \ \forall \ i = 1,2,3\ldots\ldots n.$

Similarly $A_i(y_i) > 0 \ \forall$ i= 1,2 … n Hence $x_i, y_i \in$ supp (A_i) $\forall$ i= 1,2 … n. Since each A_i is a commutative MALFS by Theorem 4.2 $x_i y_i = y_I x_i$ for all i= 1,2 … n. Hence xy = yx. Thus supp (A) is a commutative and hence A is a commutative MALFS of G.

The converse of the above proposition is not true.

References

[1] M. Akgul, Some properties of fuzzy groups, J. Math. Anal. Appl. 133 (1988) 93-100.

[2] G. Birkhoff, Lattice Theory (AMS Colloquium Publications, Providence, 1973)

[3] R. Biswas, Fuzzy subgroups and anti fuzzy subgroups, Fuzzy Sets and Systems 35 (1990) 121 – 124.

[4] P.S. Das, Fuzzy groups and level subgroups, J. Math. Anal. Appl. 84 (1981) 264 – 269.

[5] J.A. Goguen, L – Fuzzy sets, J. Math. Anal. Appl. 18 (1967) 145 – 174.

[6] T.W. Hungerford, Algebra (Springer, New York, 1984).

[7] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971) 512 – 517.

[8] R.Muthuraj and S.Balamurugan, Multi-Anti Fuzzy group and its Lower Level Subgroups

(Communicated)

[9] S.Sabu and T.V.Ramakrishnan, Multi-fuzzy sets, International Mathematical Forum, 50 (2010), 2471-2476

[10] Souriar Sebastian,S.Babu Sundar , Commutative L-Fuzzy Subgroups Fuzzy Sets and Systems 68(1994) 115-121.

[11] I.A. Zadeh, Fuzzy sets, Inform. Control. 8 (1965) 338 – 353.