

# Comparative Analysis of Mathematical Model and Software Investigation on Emission of Bio-Diesel in Internal Combustion Engine

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**Abstract**—Major problem faced by the entire world is depletion of energy source and increasing energy demand. Due to reduction of energy source an alternate way have to be found to accommodate the energy demand. One of the best ways to accommodate this energy gap is Bio-Diesel. There are numerous Bio Diesel has been founded until now, all the Bio Diesel blends have to be run on engine and emission analysis been done using large amount of manpower and time. This work suggested a mathematical model for emission parameters like CO, CO<sub>2</sub>, HC, NO<sub>x</sub>, and smoke. These mathematical models are checked for fidelity criteria's. Moreover, comparative analysis of mathematical model on emission of bio-diesel in internal combustion engine has been done with Software generated (MINITAB) model and found that the mathematical model is predicting the values correctly at all part loads and software generated model predicts for MCR load by the fidelity criteria Average Percentage Deviation (APD).

**Keywords:** Bio-diesel; Minitab; regression; method of least squares; emission.

## I. INTRODUCTION

World transport growth fuel consumption rate is increasing every year. At the same time fuel availability decreases due to continuous extraction of oil from well. Biofuel is a one of the solutions for the future fuel demands. Most of the researchers concentrate to extract the bio fuel from vegetables. The major drawback of biofuel research is the volume of oil collection. Whereas in Indian biodiesel research from nonedible oil source like Jatropha and Karanja for internal combustion engine powered by diesel fuel. The non edible biodiesel production in india is more than 20 million tons in an year. [1-3]. Vivek and AK Gupta, Fangrui Maa and Milford detail mentioned about Karanja oil extraction of oil from seeds and transesterification oil (biodiesel production) is a major process and lengthy process in biodiesel research [4-5].

Chen Zheng analyzed the data by using mathematical model and artificial neural networks. Minitab software analysis mainly analyzed diesel engine parameters [6]. Win et al. analyzed diesel engine diesel engine working parameters like noise, emission and fuel consumption by using mintab software [7]. Ganapathy et al analyzed various engine design datas like operation and combustion parameters [8]. Anand and Karthikeyan analyzed the engine parameters such as

efficiency and combustion [9]. R.Ganapathy and P. Gakkhar reported the optimized injection parameters and exhaust gas recirculation method in the diesel engine [10]. In this paper mathematical model for investigating on emission of biodiesel in internal combustion engine is proposed. It is also compared with the experimental data and the minitab software data.

## II. EXPERIMENTAL ANALYSIS

### A. Experimental Setup

The engine used in this work is Kirloskar made, four-stroke, single cylinder, direct-injection, water-cooled, constant speed (1500 rpm), and naturally aspirated VCR engine. Engine cylinder a bore of 87 mm and stroke of 110 mm; the compression ratio of 15:1 to 18:1 and the manufacturer's recommended injection timing and injection pressure of 23° crank angle before TDC and 210 bar, respectively. The combustion chamber is direct injection type with a bowl-in piston design. This work has been done with single compression ratio of 17:1. Figure 1 shows the engine setup.

### B. Data Acquisition System

The cylinder pressure at each crank angle is measured and stored by a digital data acquisition system. It consists of a Kistler water-cooled flush mounted piezoelectric pressure transducer in conjunction with Kistler charge amplifier for converting the electric charge into voltage. It could measure and store up to 200 cycles engine pressure histories. The measured data can be analyzed online or stored for post – processing. The test shown in fig 1.

## III. MATHEMATICAL INVESTIGATION

The simulation or optimization of a thermal system is the first step of modeling the characteristics of the equipment or processes. The simulation and optimization operations always use the data in an equation form. The conversion of experimental data into equation form is called mathematical modeling. Engineers may have a variety of reasons for wanting to develop equations, but the crucial one in the design of thermal systems are,

- i. To facilitate the process of system simulation

ii. To develop a mathematical statement for optimization.



Fig 1.Experimental setup

A. Criteria for Fidelity of Representation

In order to measure the effectiveness of this proposed mathematical model, the mathematical fitting needs the constant, co-efficient and criteria for the closeness of the mathematical model. In this work criteria are,

- Sum of deviations squared (SDS).
- Average Percent Absolute Deviation (APD).
- Goodness of Fit (GOF).[11]

SDS criteria is estimated by the following equation

$$SDS = \sum_{i=1}^n (y_i - Y_i)^2 \tag{1.1}$$

APD criteria is estimated by the following equation

$$APD = \frac{n}{100} \sum_{i=1}^n \sqrt{\left(\frac{Y_i - y_i}{Y_i}\right)^2} \tag{1.2}$$

The third criterion GOF is estimated by the following equation

$$GOF, \% = 100 \left(1 - \frac{SDS}{G}\right)^{0.5} \tag{1.3}$$

Where G= sum of the squares of the deviations of  $Y_i$  from the mean value of  $Y$ .

$$G = \sum_{i=1}^n (y_i - Y_{mean})^2 \tag{1.4}$$

Various Mathematical Models:

There are various types of equations available;

- Polynomials
- Polynomials with negative exponents
- Exponential Equations
- Gompertz Equation
- Combination Forms

Two methods of solving the equations;

- Method Of Least Squares
- Lagrange Interpolation

In this work, the “method of least squares” is used.

B. Minitab Software

Minitab is statistical software used by industrial experts for statistical data analysis. Minitab can also be used as a optimization tool in four designs (Factorial, Response surface method, mixture and Taguchi designs). It is one of the effective tools to analyze the trend of data, pattern of data and manipulate the data by creating the mathematical model.

Some of the statistical data analyses have been done by the Minitab software are ANOVA table, Regression

analysis like ordinary least square, weighted least square, two stage least square, Non-linear least square, Logistic Regression and many other. In this work mathematical model developed by ordinary least square regression analysis. Minitab 14 is used to develop a mathematical model.

IV. MATHEMATICAL MODEL USING METHOD OF LEAST SQUARES

A. Mathematical model for diesel engine emission

The mathematical model equation for hydrocarbon is given below. Mathematical model has been developed by using cubical polynomial equation with two independent variables like load (L) and different concentration of biodiesel (D).

Mathematical Model for hydrocarbon:

$$HC = (41.45833 - 0.8 * L + 0.006354 * L^2) + (-0.055 + 0.0125 * L - 0.00014 * L^2) * D + (0.002267 - 0.0001 * L + 1.33E-06 * L^2) D^2 \tag{1.5}$$

Mathematical Model for carbon monoxide:

$$CO = (0.104583 - 0.00233 * L + 0.0000219 * L^2) + (-0.00133 + 0.00006 * L - 0.00000069 * L^2) * D + (0.0000157 - 0.000067 * L + 0.0000000075 * L^2) * D^2 \tag{1.6}$$

Mathematical Model for Oxides of nitrogen:

$$NO_x = (-100.333 + 11.25833 * L - 0.04708 * L^2) + (0.5475 + 0.028 * L + 0.000231 * L^2) * D + (-0.00137 - 0.00065 * L + 0.000000833 * L^2) * D^2 \tag{1.7}$$

Mathematical Model for smoke intensity:

$$HSU = (38.9125 + 0.459167 * L - 0.00191 * L^2) + (-1.3065 + 0.0354 * L - 0.00024 * L^2) * D + (0.01072 - 0.00039 * L + 0.000003 * L^2) * D^2 \tag{1.8}$$

Mathematical Model for Carbon dioxide:

$$CO_2 = (1.89583 + 0.05583 * L - 3E-05 * L^2) + (-0.003 + 0.0006 * L - 7E-06 * L^2) * D + (6.7E-05 - 9E-06 * L + 1E-07 * L^2) * D^2 \tag{1.9}$$

Mathematical Model for Oxygen:

$$O_2 = (18.15542 - 0.08908 * L + 0.000191 * L^2) + (-0.01108 - 4E-05 * L + 3.19E-06 * L^2) * D + (1.63E-05 + 6.93E-06 * L - 8.7E-08 * L^2) * D^2 \tag{1.10}$$

V. MATHEMATICAL MODEL USING MINITAB SOFTWARE

Minitab software uses linear Regression method to form a mathematical model. The following are the equations are to analyze the engine emission parameters. All the pollutant equations are generated by Minitab software.

$$NO_x = 69.48 + 6.54 L - 1.1776 D \tag{1.11}$$

$$SMOKE = 28.476 + 0.4912 L - 0.1752 D \tag{1.12}$$

$$O_2 = 17.1616 - 0.05994 L + 0.004896 D \tag{1.13}$$

$$CO_2 = 2.536 + 0.0444 L - 0.00552 D \tag{1.14}$$

$$HC = 20.34 - 0.011 L + 0.2832 D \tag{1.15}$$

$$CO = 0.0518 - 9e-005 L + 0.000104 D \tag{1.16}$$

VI. RESULTS AND DISCUSSION

Numerical study on emission is carried out by the above mentioned mathematical models (using method of least squares and regression analysis using Minitab software). The same is compared with the experimental data. The comparison is reduced to five bio diesel blends (B0, B25, B50, B75 AND B100) and five loading conditions (20%, 40%, 60%, 80% and 100%). In this work, fidelity for criteria is considered for comparison of experimental with two mathematical models by Average Percent Absolute Deviation (APD). The data have been plotted in figure 2-7. Experimental result consider as base result for both mathematical model as well as software analysis. Deviation always consider from experimental result. The variation of mathematical model APD and software analysis APD calculated and it is also clearly mentioned in graph. All the graph positive side mentioned mathematical variation and negative side mentioned software investigation values variation. Below cited the entire graphs clearly bring up the level of deviation from experimental results. The graph values which is near to zero (X axis reference line) that values very closer to experimental result.

It is clear from fig.2 that the mathematical results are very closer then the software results in most of the load. It is also seen that DAPD is nearly zero values in many load except 40%, from this result mathematical model analysis of NOx is more suitable than software analysis. It is found that average (APD) deviation between mathematical results and software results is  $\pm 0.00813\%$ .

Fig 3 clear indicated from fig.3 that the mathematical results are very closer then the software results in most of the load. It is found that deviation between mathematical results and software results is  $\pm 0.00124\%$ . It is clear from fig.4 that the mathematical results are very closer then the software results in most of the load. It is found that deviation between mathematical results and software results is  $\pm 0.0024\%$ .

It is clear from fig.5 that the mathematical results are very closer then the software results in most of the load. It is found that deviation between mathematical results and software results is  $\pm 0.00524\%$ . It is clear from fig.6 that the mathematical results are very closer then the software results in most of the load. It is found that deviation between mathematical results and software results is  $\pm 0.01914\%$ .

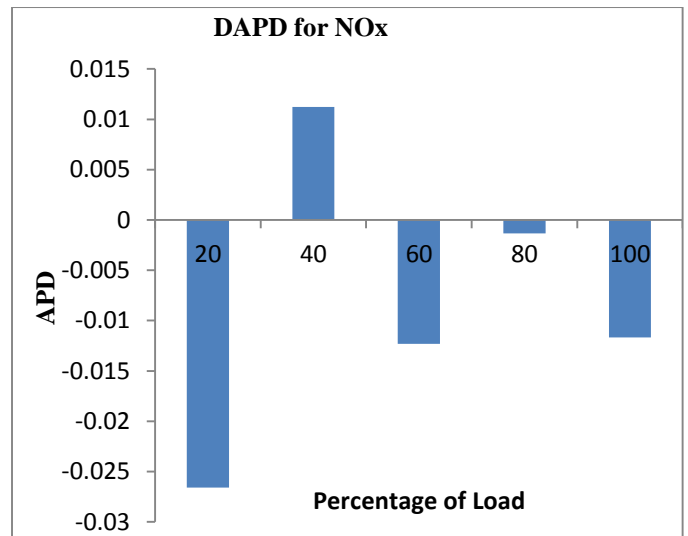


Fig 2. Mathematical and software APD difference for NO<sub>x</sub>

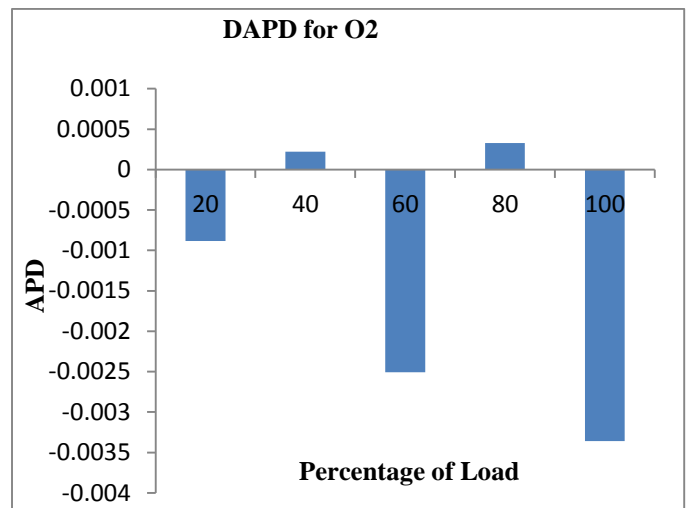


Fig3. Mathematical and software APD difference for O<sub>2</sub>

It is clear from fig.7 that the mathematical results are very closer then the software results in most of the load. It is found that deviation between mathematical results and software results is  $\pm 0.00186\%$ .

It is found that the average percentage deviation for manual mathematical model is lesser than the Minitab software generated model. It is because of, the Minitab software consider the equation for mathematical model is of polynomials of Single order equation, but the manual mathematical model considered in this work is polynomials of third order equation. Therefore the error has been squared in manual mathematical model than the software model.

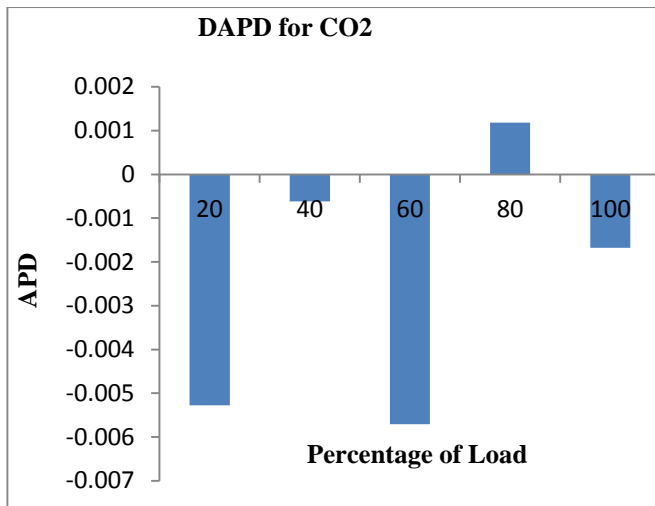


Fig4. Mathematical and software APD difference for CO<sub>2</sub>

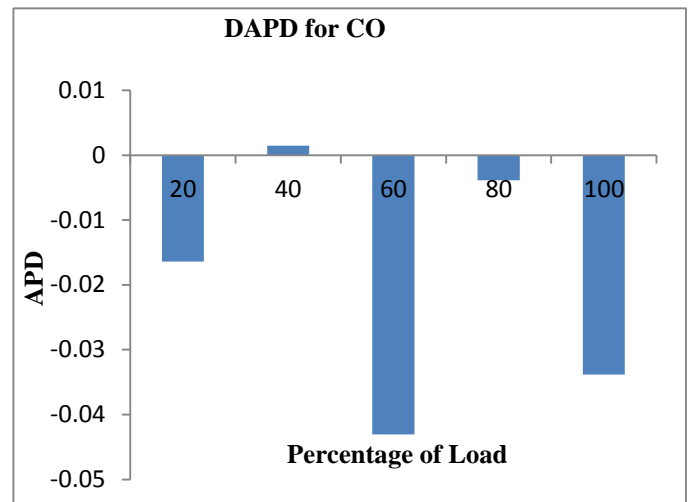


Fig 6. Mathematical and software APD difference for CO

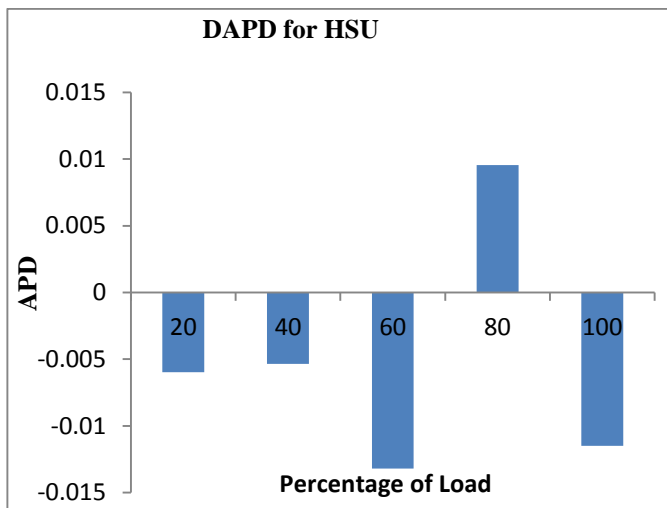


Fig 5. Mathematical and software APD difference for Smoke

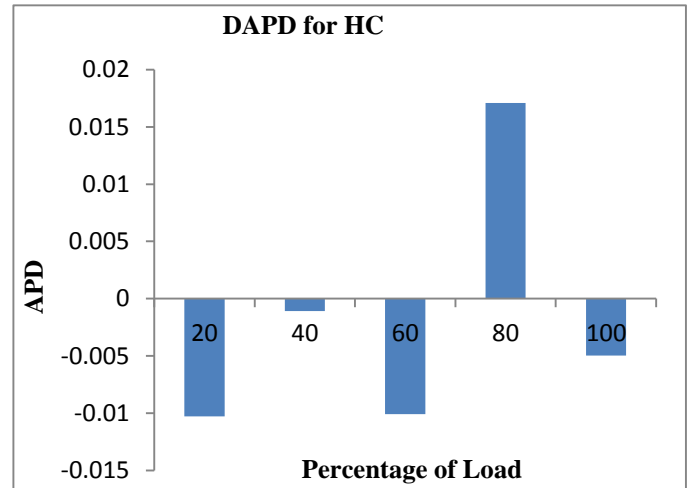


Fig 7. Mathematical and software APD difference for HC

VII. CONCLUSION

In this work, comparative analysis of mathematical model and software investigation on emission of bio-diesel in internal combustion engine is proposed. The following are the conclusion of the proposed work.

1. Mathematical model is found to be more suitable for NO<sub>x</sub> analysis at 20%, 60%, 80% and 100% loads, and manual mathematical model found to be accurate than the software generated mathematical model.
2. It is studied that the CO analysis 40% load gives very closer to experimental result compare with software result.
3. The MCR (Maximum Continues Rating) i.e., the 80% load for HSU, CO<sub>2</sub>, HC produced maximum variation in the mathematical model than software results. Therefore the software generated mathematical model is found to be accurate for HSU, CO<sub>2</sub>, HC at 80% load.

4. It is also found that the O<sub>2</sub>, two loads (40% and 80%) produced minimum variation in mathematical model. At the same time three loads (20%, 60% and 100%) produced maximum variation in software analysis
5. It is concluded that the Mathematical model seems to be accurate at many load and maximum pollutant analysis in internal combustion engine. Therefore, Mathematical model more suitable for internal combustion engine emission analysis compare with software analysis.

NOMENCLATURE

D	Percentage of diesel
L	Percentage of load
HC	Hydrocarbon
CO	Carbon monoxide
CO <sub>2</sub>	Carbon dioxide
O <sub>2</sub>	Oxygen
NO <sub>x</sub>	Oxides of nitrogen
SDS	Sum of deviations squared
APD	Average Percent Absolute Deviation
DAPD	Difference of mathematical and software APD
GOF	Goodness of Fit
y <sub>i</sub>	value of the dependent variable computed from the equation.

$Y_i$	value of the dependent variable computed from simulated.
$n$	Total number of data points.
MR	Mathematical Result
ER	Experimental Result

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