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Comparative Design of two-way Slab using Deterministic Partial Factors and Partial Factors of Safety Obtained through Calibration

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Abstract: **Code calibration is another level of structural reliability which yields a close result with FORM. Load on a structure are stochastic in nature and as such the partial factors of safety for design must be determined through reliability method. The code calibration of the two-way slab yielded 21.4% reduction in partial factors of safety for dead load and 37.5% increase for live-load. When the calibrated partial factors were used in design, an economical area of reinforcement in the order of 16.7% was obtained compared with that of the deterministic safety factor. It is therefore concluded that the new partial factors of safety is suitable for application.**

Keywords - Code calibration, Two-way solid slabs, partial factors of safety, FORM, structural reliability.

1.0 INTRODUCTION

The intent of a design code is to provide a minimum safety level. Current codes use deterministic formulas Abejide,1997, Afolayan, 1992; however, the optimum design will require the consideration of structural reliability as an acceptance criterion. Depending on the approach to reliability, there are four levels of design codes (Madsen, Krenk and Lind, 1986):

LEVEL I codes use deterministic design formulas. The safety margin is introduced through central factors (ratio of design resistance to design load) or partial factors (load and resistance factors).

LEVEL II codes define the design acceptance criterion in terms of "closeness" of actual reliability index for a design to target reliability index or other safety related parameters.

LEVEL III code requires a full reliability analysis to quantify the probability of failure of the structure under various loading scenarios. The acceptance criterion is defined in terms of the closeness of the actual reliability index to the optimum reliability level.

LEVEL IV codes use the total expected cost of the design as the optimization criterion. The acceptable design maximizes the utility function, which describes the difference between the benefits costs associated with a particular design.

In practice, the current design codes are based on a level I code philosophy in which calibration of partial factors of safety is based (Baker,1976). However, in the new developed level I codes, the design parameters are derived using level II methods. At present, level III and level IV methods are used mainly in advanced research or in the design of critical structures.

A structural design code is basically a set of requirements to be satisfied by a class of structures to be designed in a jurisdictional area. These requirements include values and/or determine design load and resistance. Therefore, the development of the code involves not only determination of safety factors, but also verification of the nominal (design) values of load and resistance as well as analytical procedures (Andrzej and Kelvin; 2000).

2.0 Calibration of partial factors of safety for Level I Code.

Code calibration is another level of structural reliability which yield close results with FORM. The limit state of the two-way slab in involving R, dead load effect D and live-load effect is given by:

$$G(R, D, L) = R - (D + L) \ldots\ldots\ldots\ldots\ldots (1)$$

A possible corresponding design equation to this limit state equation in load-resistance factor design (LRFD) format for the two-way solid slab is:

$$\varphi R_n \geq \gamma_D D_n + \gamma_L L_n \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

R_n, D_n *and* L_n are nominal values of the loadings while φ, γ_D *and* γ_L are ,design factors for resistance, dead and live load respectively. Target safety index of $\beta_T = 3$ was adopted for live and dead load combination (Ellingwood, 1982).

The procedures for the calibration of partial factors of safety are itemized below:

I. Formulate the limit state function and the design equation. Determine the probability distribution and appropriate parameters for as many random variables as possible (i.e $X_i (i = 1,2,3 \ldots \ldots .. n)$). It is assumed that the coefficient of variation and standard deviation for all random variables are known.

II. Obtain an initial design point$\{x_i^*\}$ by assuming values for (n-1) of the random variables X_i. Solve the limit state equation at $g = 0$ to obtain a value for the remaining random variables.

III. For each of the design values x_i^* corresponding to a non-normal distribution. Determine the equivalent mean, μ_x^e and standard deviation, σ_x^e.

IV. Determine the partial derivatives of the limit state function with respect to the reduced variates using a column vector $\{G\}$ as a the vector whose elements are these partial derivatives:

$$\{G\} = \begin{Bmatrix} G_1 \\ G_2 \\ . \\ . \\ . \\ G_n \end{Bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Where, $\{G\} = -\frac{\partial g}{\partial Z}$ ……………………(4)
Evaluated at design point.

V. Calculate the column vector $\{\alpha\}$ using

$$\{\alpha\} = \frac{\{\rho\}\{G\}}{\sqrt{\{G\}^T\{\rho\}\{G\}}} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Where, $\{\rho\}$ is the matrix of correlation coefficients.

VI. Determine a new design point in reduced variates for (n-1) of the variable using:

$$z_i^* = \alpha_i \beta_i \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Where, β_i is the target reliability index.

VII. Determine the corresponding design point values in original coordinates for the n-1 values in step VI using;

$$x_i^* = \mu_{xi}^e + z_{xi}^e \ldots\ldots\ldots\ldots\ldots (7)$$

VIII. Determine the value of the remaining random variable by solving the limit state function g=0. Update the relationship of the two unknown mean using;

$$\mu_x = \frac{x_i^*}{1+\alpha_i \beta V_x} \ldots\ldots\ldots\ldots\ldots (8)$$

IX. Repeat step III-VIII until $\{\alpha\}$ converges.

X. Calculate the partial factors of safety using;

$$\gamma_i = \frac{x_i}{\mu_i} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

For extreme type I variables, the parameters *'u'* and *'a'* of the distribution are given by:

$$a = \sqrt{\frac{\pi^2}{6\sigma_L^2}} = \frac{\pi}{\sqrt{6}\left(V_L \mu_L\right)} \ldots\ldots\ldots (10)$$

$$u = \mu_L - \frac{0.5772}{a} \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$F_L\left(l^*\right) = \exp\left[-\exp\left(-a\left(l^* - u\right)\right)\right] \ldots\ldots (12)$$

$$f_L\left(l^*\right) = a\left\{\exp\left(-a\left(l^* - u\right)\right)\right\}\exp\left[-\exp\left(-a\left(l^* - u\right)\right)\right] \ldots (13)$$

The equivalent parameters of L are obtained thus:

$$z = \Phi^{-1}(p) = -t + \frac{C_0 + C_1 + C_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} \ldots\ldots\ldots (14)$$

$$z = \Phi^{-1}(p^*) \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Thus,
The equivalent normal parameters for L are:

$$\sigma_L = \frac{1}{f_L\left(l^*\right)} \phi\left[\Phi^{-1}\left(F_L\left(l^*\right)\right)\right] \ldots\ldots (16)$$

$$\mu_L = l^* - \sigma_L\left[\Phi^{-1}\left(F_L\left(l^*\right)\right)\right] \ldots\ldots (17)$$

Determining the equivalent normal parameters for R
Since the coefficient of variation is less than 20%

$$\sigma^e{}_R \approx \gamma^* V_R, \ldots\ldots\ldots\ldots\ldots (18)$$

$$\mu^e{}_R \approx \gamma^* \left[1 - In\left(\frac{\gamma^*}{\mu_R}\right)\right] \ldots\ldots (19)$$

3.0 MATERIALS AND METHOD

Research materials: Code calibration of partial factors of safety for the two-way solid slab was carried out using statistical data obtained from Joint Committee of structural safety. The mean, standard deviation, distribution functions were obtained from the probabilistic model code.

Methods: The level I code and a target safety index of β_T= 3(Ellinwood, 1982) was adopted for live-load and dead-load combination and used to calibrate the two-way slab.

Table1: Resistance models for concrete elements

Model Type	Distribution	Mean	COV(%)
Resistance models concrete (static)	LN	1.14	13
Bending moment capacity (solid weight element)	LN	1.12	12
Bending moment capacity (light weight element)	LN	1.4	25
Shear capacity	LN	1.0	10
Connection capacity			

Table 2: Statistical parameters for load components

Load component	Bias factor'	Coefficient of variation
Dead load		
Factor-made components	1.03	0.08
Cast-in-place components	1.05	0.10
Asphalt wearing surface	1.00	0.25
Live load and dynamic load	1.0 – 1.8	0.18-0.25

Table 3: Probabilistic models of basic variables.

Basic variable	Symbol	Name of basic variable	Distribution type	Units
Material properties	A_s f_c f_y	Reinforcement area concrete strength Yield strength	DET LN LN	m^2 N/mm^2 N/mm^2
Geometrical data	L D	Span of beam Effective height	DET N	M M
Action	G Q	Permanent load Imposed load	N Extreme type 1	KN/m^2 KN/m^2
Moment	R	Resistive load effect on beam	LN	KNm

Table 4: Statistical parameters for resistance

Material	Limit state	Bias factor	Coefficient of variation
Steel	Moment	1.12	0.100
	Shear	1.14	0.105
Reinforced concrete Light weight RC	Moment Moment	1.14 1.12	0.130 0.120
	Shear	1.20	0.155

4.0 RESULTS AND DISCUSSION

4.1 Code calibration

Code calibration were carried out using statistical data obtained from tables 1, 2,3and 4 respectively. The stochastic random variables are:

R is lognormal $V_R = 13\%$
$\lambda_R = 1.14$

L is extreme type I $V_L = 25\%$
$\lambda_L = 1.0$

D is Normal $V_D = 10\%$
$\lambda_D = 1.05$

Obtaining an initial design Point, $d^* = \mu_D$,

$$l^* = 0.5\mu_D$$

Assuming live to dead load ratio $\frac{\mu_L}{\mu_D}$ = 0.5, 1.0 and 3.0.

Therefore, the design factors are:

$$\phi = \frac{\gamma^*}{R_r} = \frac{\gamma^*}{\mu_R/\lambda_R} = \lambda_R \frac{\gamma^*}{\mu_R}$$

$$= \frac{1.911\mu_D \times 1.14}{2.56\mu_D} = 0.85$$

$$\gamma_D = \lambda_D \frac{d^*}{\mu_D} = (1.05) \times \frac{1.09\mu_D}{\mu_D} = 1.15$$

$$\gamma_L = \lambda_L \frac{\ell^*}{\mu_L} = (1.0) \times \frac{0.8165\mu_D}{0.5\mu_D} = 1.63$$

The procedures are continued for $\frac{\mu_L}{\mu_D} = 1.0,3$ until α-values converge. The summaries are presented in table 5.

Therefore, the design factors for $\frac{\mu_L}{\mu_D} = 1.0$ are calculated thus:

$$\phi = \frac{\gamma^*}{R_r} = \frac{\gamma^*}{\mu_R/\lambda_R} = \lambda_R \frac{\gamma^*}{\mu_R} =$$

$$\frac{4.05\mu_D \times 1.14}{4.61\mu_D} = 1.00$$

Therefore, the design factors for $\frac{\mu_L}{\mu_D} = 3$ are:

$$\phi = \frac{\gamma^*}{R_r} = \frac{\gamma^*}{\mu_R/\lambda_R} = \lambda_R \frac{\gamma^*}{\mu_R}$$

$$= \frac{6.921\mu_D \times 1.14}{8.429\mu_D} = 0.85$$

$$\gamma_D = \lambda_D \frac{d^*}{\mu_D} = (1.05) \times \frac{1.01\mu_D}{\mu_D} = 1.07$$

$$\gamma_L = \lambda_L \frac{\ell^*}{\mu_L} = (1.0) \times \frac{3.04\mu_D}{\mu_D} = 3.04$$

$$\gamma_D = \lambda_D \frac{d^*}{\mu_D} = (1.05) \times \frac{1.014\mu_D}{\mu_D} = 1.07$$

$$\gamma_L = \lambda_L \frac{\ell^*}{\mu_L} = (1.0) \times \frac{5.900\mu_D}{3\mu_D} = 1.967$$

Table 5: Summary of code calibration

μ_L/μ_D	Φ	γ_D	γ_L
0.50	0.85	1.15	1.63
1.00	1.00	1.07	3.04
3.00	0.85	1.07	1.97

4.2.1 Testing results from Code Calibration

The result from code calibration was tested using a two-way slab150mm thick, with all edges continuous. If l_y= 5.5m and l_x= 4.2m and compared with deterministic safety factors taking f_y= 410N/mm^2 and f_{cu}=25N/mm^2. The result is presented in table 6 below;

Table 6: Comparison between deterministic design and calibrated partial factors.

Equations	DET. DESIGN $1.4G_K + 1.6 Q_K$	CODE CALIBRATION $1.1G_K + 2.2 Q_K$	% difference
Design load (KN/m), w	13.620	10.560	22.5
Mmt,M=βwl^2 (KNm)	11.05	8.570	22.5
K-value	0.029	0.022	24.1
I_a, lever arm factor	0.950	0.950	0.00
Z, Lever arm (mm)	117.8	117.8	0.00
A_s, Area of steel reqd, (mm^2)			
Mid-span (Short span)	263	142	
Cont. edge (Short span)	361	187	
Mid-span (Long span)	173	103	
Cont. edge (Long span)	231	137	
A_s provided(mm^2)	377 (Y12@300c/c)	314 (Y10@ 200)	16.7
A_{smin} (mm^2)	195	195	-
Average Safety index, β using FORM	3.36	3.87	15.2

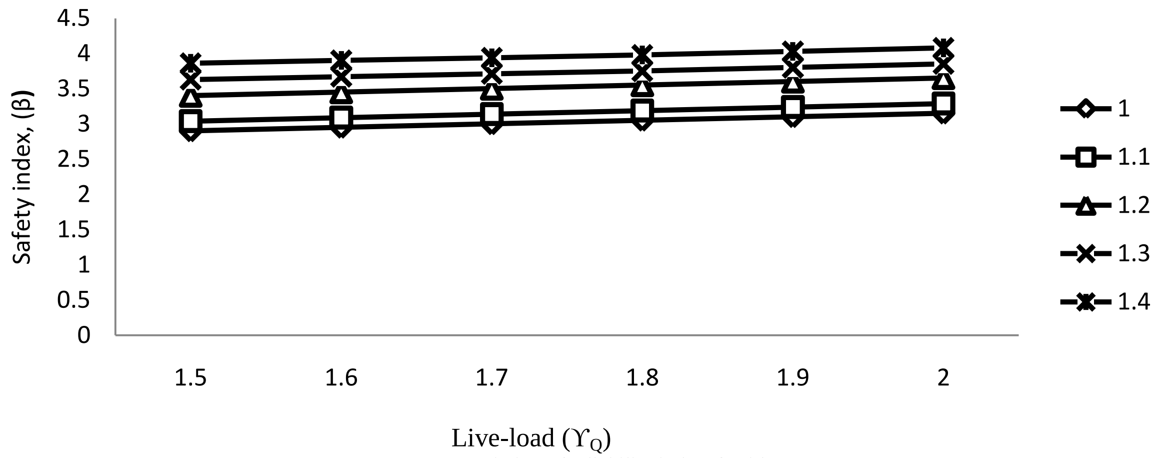

Figure 1: Variation of reliability index, β with γ_Q *at constant* γ_G

Table 7: Variation of reliability index, β against γ_Q at constant γ_G

γ_G	γ_Q					
	1.5	1.6	1.7	1.8	1.9	2.0
1.0	2.90	2.95	3.00	3.05	3.10	3.15
1.1	3.04	3.09	3.14	3.19	3.24	3.29
1.2	3.40	3.45	3.50	3.55	3.60	3.65
1.3	3.63	3.67	3.71	3.75	3.80	3.85
1.4	3.86	3.90	3.94	3.98	4.03	4.08

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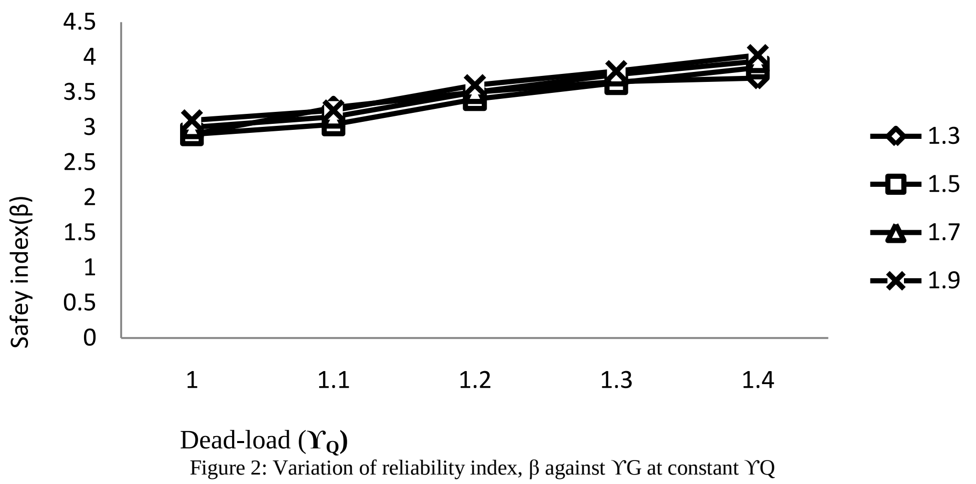

Figure 2: Variation of reliability index, β against ϒG at constant ϒQ

Table 8: Variation of reliability index, β against γ_G at constant γ_Q

γ_Q	γ_G				
	1.0	1.1	1.2	1.3	1.4
1.3	2.90	3.28	3.50	3.65	3.70
1.5	2.90	3.04	3.40	3.63	3.85
1.7	3.00	3.15	3.50	3.75	3.94
1.9	3.10	3.24	3.60	3.80	4.03

5.0 DISCUSSION OF RESULTS

Code calibration revealed a reduction in the partial factor of safety for dead load and increased factors of safety for live load. This is because two-way slab normally encounter load not bargained for in the course of use.

Figure 1 and 2 shows the variation of reliability index as a function of safety factors of imposed and permanent loads using FORM. Safety indices are fairly consistent with increased partial factors of safety for both permanent and imposed load.

While tables 7 and 8 shows the variation of safety index with partial factors of safety for permanent and imposed load respectively.

Taking the mean values of φ, γ_D and γ_L and keeping two significant figures, the new proposed resistance for the design of the two-way solid slab is $\varphi = 0.9,\ \gamma_G = 1.1,\ \gamma_Q = 2.2$

6.0 CONCLUSION AND RECOMMENDATIONS

6.1 Conclusion

Code calibration is generally performed for a given class of structures, materials and/or loads in such a way that the reliability measured by the first order reliability index β or the annual probability of failure estimated on the basis of the structures designed using the calibrated partial factors of safety are as close as possible to the reliability indices obtained through FORM.

Therefore, having considered worst situation of live to dead load ratio, through code calibration, the new partial factors of safety for the two-way solid slab are given in the form:

$\varphi = 0.9, \gamma_G = 1.1, \gamma_Q = 2.2$

When replaced by the old ones $\gamma_G = 1.4$ and $\gamma_Q = 1.6$ (BS 8110), a reliable and more economical section with an increased structural safety will be achieved.

6.2 Recommendations

Code calibration is another level of structural reliability which yields close results with FORM. Therefore, the new partial factors is recommended for practicing engineers and tutors in all engineering firm.

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