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Comparison of Transfer Mass Matrix Method with Finite Element Method for Modal Analysis of beams

Abhishek Kumar Sahu
M.Tech Scholar
Department of Civil Engineering
National Institute of Technology
Agartala, India

Dr. Surajit Das
Assistant Professor
Department of Civil Engineering
National Institute of Technology
Agartala,India

***Abstract*—In the present work two different methods for finding natural frequencies of beams having crack are compared. Both the methods are used for modal analysis of cracked beam having transverse cracks. In the present analysis, it is found that the present method of finding the natural frequencies is more accurate than transfer mass matrix method. Kausar H. Barad, in his paper has performed experiment on a cantilever beam having cracks at different locations and obtained natural frequencies corresponding to each crack. The same problem has been analysed by the transfer mass matrix method proposed by D.P Patil and present finite element formulation derived from formulations proposed by A.D Dimarogonas and Uttam Kumar Mishra. The error in natural frequencies obtained by present finite element formulation is less as compared to the transfer mass matrix method. Variation of graph for natural frequencies obtained by present finite element formulation is similar to the experimental one whereas it is not similar for that obtained by transfer mass matrix method.**

Keywords—Damage Detection, Structural Health Monitoring, Modal Analysis, Cracks, Beams.

I. INTRODUCTION (*Heading 1*)

Structures such as buildings, trusses etc. are subjected to damage due to varying loads. In buildings, beams get damaged earlier than columns. A large number of researchers have developed numerous ways to find out natural frequencies of beams subjected to transverse cracks. Transfer Matrix Method is one of such ways. Patil and Maiti (2003) have developed a method for detection of multiple open cracks in a slender Euler-Bernoulli beam based on frequency measurements. This method is based on approach given by Hu and liang. It has transverse vibration modelling through transfer matrix method and represents crack by rotational spring. The beam is virtually divided into a number of segments and each one is considered to be associated with the damage parameter. These parameters are determined from changes in natural frequencies and are used to determine crack location and crack size. This method eliminates the need of symbolic computation envisaged by Hu and liang to obtain mode shapes of corresponding uncracked beams. Calculations are done for two simultaneous cracks of size 10% and more of section depth. The differences between actual and predicted crack location and sizes are less than 10% and 15% respectively. The no. of segments in which beam is divided limits the maximum number of cracks that can be handled. The objective of the work done in paper is to present a method for modelling transverse vibration of beams with multiple open cracks by combining the transfer matrix method and rotational spring based representation of a crack and approximate approach of Hu and liang. This method uses symbolic computation to obtain mode shape of corresponding uncracked beam. Simply supported beam, cantilever beam, beams on elastic foundation and multiple pin supports are examined. Kausar H Barad et.al. (2012) have used the natural frequency to find the crack depth and crack location in the beam like structure. In the present paper, crack in the beam structure is considered as a rotational spring. Characteristic equation based on boundary conditions of the beam is derived. Relationship between crack depth and stiffness is derived and graph is plotted between normalized crack depth and crack location for the first two frequencies. Shen and Taylor (1990) have shown an identification procedure to determine crack location and size from dynamic measurements. The procedure is based on minimization of either mean square or max measure of difference between measurement data (natural frequencies and mode shapes) and corresponding predictions obtained from computational model. Necessary conditions are used for formulation. Method is tested for crack in simply supported Euler-Bernoulli beam. Sensitivity of solution of damage identification to the values of parameters characterizing damage is discussed. Christides and Barr (1983) have derived differential equation and associated boundary conditions for nominally uniform Bernoulli Euler beam containing one or more pairs of symmetric cracks. Reduction of one spatial dimension is achieved using integration over cross-section after plausible stress, strain, displacement. Momentum fields are chosen. Perturbation in stresses induced by crack is incorporated through local function assuming an exponential decay with distance from crack. It includes a parameter evaluated by experimental tests. Experiments on beams containing cuts to simulate cracks are described. Change in first natural frequency with crack depth is matched closely by theoretical predictions. The theory can be extended to beams having non symmetry cracks and has coupling between various forms of motion such as bending and torsion. Such couplings are significant in the regions where both frequencies of predominantly bending and predominantly wavelengths are approximately same. This theory can be applied to beams applied to flexural vibration with crack on one side only In the present paper, natural frequencies are calculated by transfer mass matrix method and present finite element method for the experimental problem analysed by Kausar H Barad. The results of the two are compared with the experimental results and it is observed that the natural frequencies obtained by former one deviates more from the experimental frequencies in comparison to the latter one.

II. PRESENT FINITE ELEMENT FORMULATION AND TRANSFER MASS MATRIX METHOD

A. Finite Element Formulation

When crack is induced in a beam, then its flexibility is increased. So, first we calculate the additional flexibility induced in it. Then it is added up with the flexibility matrix of intact beam element. The inverse of the overall flexibility matrix thus obtained is multiplied with the transformation matrix to obtain the required stiffness matrix of the cracked beam element. This stiffness matrix is assembled along with the stiffness matrices of the intact beam element and thereafter the natural frequencies are calculated from the equation K – ω^2M=0, where K= Assembled stiffness matrix of the beam, M=Assembled mass matrix and ω= Natural frequency (rad/sec). According to Dimarogonas et.al. (1983) and Tada et.al. (2000) the additional stain energy due to existence of crack can be expressed as

$$\Pi_C = \int_{A_C} G dA_C \tag{1}$$

Where, G = the strain energy release rate, and
A_C = the effective cracked area.

$$G = \frac{1}{E'}\left[(\sum_{n=1}^{2} K_{In})^2 + (\sum_{n=1}^{2} K_{IIn})^2 + k(\sum_{n=1}^{2} K_{IIIn})^2\right] \tag{2}$$

Where, E' = E for plane stess
E' = E/1-ν^2 for plane strain
k = 1 + ν
ν=Poisson's ratio
E=Young's Modulus of elasticity.

K_I, K_{II} and K_{III} = stress intensity factors for sliding, tearing and opening type cracks respectively. Neglecting effect of axial force and for open cracks above equation can be written as

$$G = \frac{1}{E'}\left[(K_{I1} + K_{I2})^2 + K_{II1}^2\right] \tag{3}$$

The expressions for stress intensity factors from earlier studies are given by Uttam Kumar Mishra (2014) as follows

$$K_{I1} = \frac{6P_1L_C}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right) \tag{4}$$

$$K_{I2} = \frac{6P_2}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right) \tag{5}$$

torsional motion and their corresponding

$$K_{II1} = \frac{P_2}{bh}\sqrt{\pi\xi}F_{II}\left(\frac{\xi}{h}\right) \tag{6}$$

From definition, the elements of the overall additional flexibility matrix C_{ij} can be

$$C_{ij} = \frac{\partial \delta_i}{\partial P_j} = \frac{\partial^2 \Pi_C}{\partial P_i \partial P_j} \quad , (\text{I, j=1,2}) \tag{7}$$

Substituting Eq (4),(5),(6) into Eq (3), then into Eq (1) and Eq (7) subsequently we get,

$$C_{ij} = \frac{b}{E'}\frac{\partial^2}{\partial P_i \partial P_j}\int\left[\left\{\frac{6P_1L_c}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right) + \frac{6P_1}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right)\right\}^2 + \left\{\frac{P_1}{bh}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right)\right\}^2\right]d\xi \tag{8}$$

Substituting i,j (1,2) values, we get

$$C_{11} = \frac{2\pi}{E'b}\left[\frac{36L_C^2}{h^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx + \int_0^{\frac{a}{h}} xF_{II}^2(x)dx\right] \tag{9}$$

$$C_{12} = \frac{72\pi L_c}{E'bh^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx = C_{21} \tag{10}$$

$$C_{22} = \frac{72\pi}{E'bh^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx \tag{11}$$

$$F_I(s) = \sqrt{\frac{\tan\left(\frac{\pi s}{2}\right)}{\left(\frac{\pi s}{2}\right)}}\left[\frac{0.923 + 0.199\left(1 - \sin\left(\frac{\pi s}{2}\right)\right)^4}{\cos\left(\frac{\pi s}{2}\right)}\right] \tag{12}$$

$$F_{II}(s) = \frac{1.122 - 0.561s + 0.085s^2 + 0.180s^3}{\sqrt{1-s}} \tag{13}$$

$$C_{total} = \begin{bmatrix} \frac{L_e^3}{3EI} + C_{11} & \frac{L_e^2}{2EI} + C_{12} \\ \frac{L_e^2}{2EI} + C_{21} & \frac{L_e}{EI} + C_{22} \end{bmatrix} \tag{14}$$

The stiffness matrix K_{crack} of a cracked beam element can be obtained as $K_{crack} = LC_{tot}^{-1}L^T$, Where, L is the transformation matrix for equilibrium condition

$$L = \begin{bmatrix} -1 & 0 \\ -L_e & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{15}$$

Here, equation 1, 2and 3 are coefficients of additional flexibility matrix, a= crack depth, h= total depth of the beam, L_c= distance of crack from right node of beam element. E'=E/(1-ν^2), where ν= Poisson's ratio, E= modulus of elasticity.

B. *TransferMass Matrix Method*

D. P. Patil and S. K. Maiti (2003) have used transfer mass matrix method to find out the natural frequency of a cantilever and simply supported beams. Let there be uniform beam with n cracks located at $\xi = x/L = \beta_1, \beta_2, \beta_3, \beta_4, \ldots B_n$. Each crack is represented by a rotational spring with stiffness given by

$$\overline{K_i} = \frac{Ebh^2}{72\pi f(r_i)} \tag{16}$$

Where, $\overline{K_i}$ is the equivalent spring stiffness for crack i, d is depth and b is width of cross section $r_i(=a/h)$ is nondimensional crack size and a_i is size of the crack. $f(r_i)$, is called flexibility function and is given by

$$f(r_i) = 0.6384(r_i)^2-1.035(r_i)^3+3.7201(r_i)^4-5.1773(r_i)^5+7.553(r_i)^6-7.3324(r_i)^7+2.4909(r_i)^8 \tag{17}$$

For an Euler Bernoulli beam the governing equation of motion is

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 y(x,t)}{\partial x^2}\right]+\rho A\frac{\partial^2 y(x,t)}{\partial t^2}=0 \tag{18}$$

Through separation of variables y(x,t) = Z(x) cos(ωt), the mode shape equation is obtained:

$$EI\frac{d^4Z}{dx^4}-\rho A\omega_i^2 Z=0 \text{ or } \frac{d^4Z}{dx^4}-p^4Z=0 \tag{19}$$

Where λ=pL, $\beta=L_1/L$, e=2 β-1, and K = $\frac{\overline{KL}}{EI}$.

Equation 13 and 14 can be used to find out natural frequencies for a single crack in a cantilever and simply supported beam.

Where ρ is mass density (kg/m^3), A, cross-sectional area (m^2), ω_i , mode i natural frequency (rad/s) , E, modulus of elasticity (N/m^2), I, area moment of inertia (m^4), and

$$P^4 = \frac{\rho A\omega^2}{EI} \tag{20}$$

The general solution of eq (25) can be written as

$Z(x) = C_1[\cos(px) + \cosh(px)] + C_2[\cos(px) - \cosh(px)] + C_3[\sin(px) + \sinh(px)] + C_4[\sin(px) - \sinh(px)]$------------(13)

Using this relation we calculate displacement Z, slope, bending moment, and shear force at two ends i and i -1 of an arbitrary segment. At crack location there is jump in slope. By forming transfer matrix we derive the following expression for cantilever and simply supported beam as :

$$4(1+\cosh\lambda\cos\lambda)+\lambda/K\{\sinh\lambda(\cos\lambda+\cos\lambda e)-\sin\lambda(\cosh\lambda+\cosh\lambda e)+2\cosh(\lambda\beta)\sin(\lambda\beta)-2\cos(\lambda\beta)\sinh(\lambda\beta)-2\sin[\lambda(1-\beta)]\cosh[\lambda(1-\beta)]+2\cos[\lambda(1-\beta)]\sinh[\lambda(1-\beta)]\}=0 \tag{21}$$

And

$$4\sin\lambda\sinh\lambda+\lambda/K\{\sinh\lambda(\cos\lambda-\cos\lambda e)-\sin\lambda(\cosh\lambda-\cosh\lambda e)\}=0 \tag{22}$$

III. Comparison Of Present Method Of Finding Natural Frequency With The Transfer Mass Matrix Method

Problem Description: A cantilever with crack depth ratio of 0.5 with cracks located at various locations is taken. It has following properties and is divided into 8 elements for comparison with the natural frequencies obtained by transfer matrix method with cracks considered as rotational spring model.

Length, L = 0.78m
Breadth, b = 0.04m
Height, h = 0.01m
Mass density, ρ = 7860 kg/m^3.
Young's Modulus, E = 210 GPa

A.Convergence Study

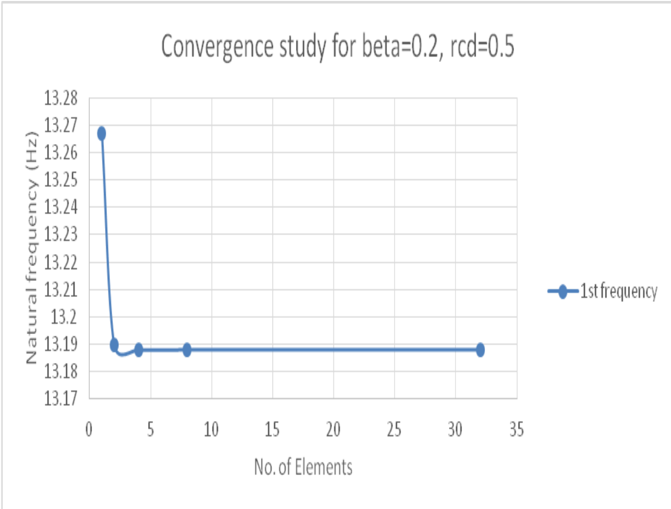

Figure 1: Convergence Study of single cracked cantilever beam

B. *Comparison of methods*

		Experimental	FEM	
Crack location	Crack depth	1st Frequency	1st Frequency	Error
No Crack	No Crack	13.45	13.726	-2.052
0.1	0.5	12.8	12.983	-1.43
0.2	0.5	13.02	13.188	-1.29
0.3	0.5	13.15	13.361	-1.6046
0.4	0.5	13.28	13.5	-1.657
0.5	0.5	13.37	13.598	-1.705
0.6	0.5	13.4	13.667	-1.9925
0.7	0.5	13.43	13.704	-2.04

Table 1: Comparison of 1st Natural Frequency (Hz) for single cracked cantilever beam.

Experimental			FEM		D.P.Patil et.al.	
Crack Location	Crack Depth	2nd frequency	2nd frequency	Error	2nd frequency	Error
No Crack	No Crack	84.3	86.291	-2.362	79.84	5.2872
0.1	0.5	82.89	84.397	-1.818	81.36	1.842
0.2	0.5	84.3	85.981	-1.994	82.7	1.898
0.3	0.5	83.3	85.398	-2.519	83.84	-0.6469
0.4	0.5	82.87	83.804	-1.127	84.748	-2.266
0.5	0.5	81.82	83.64	-2.224	85.42	-4.4
0.6	0.5	81.93	83.007	-1.314	85.86	-4.8
0.7	0.5	82.5	84.207	-2.07	86.103	-4.367

Table 2: Comparison of 2ndNatural Frequency (Hz) for single cracked cantilever beam.

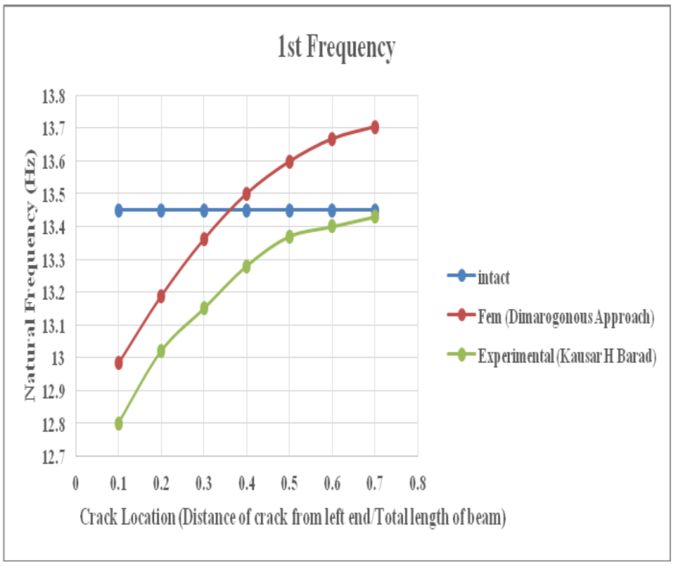

Figure 2: Comparison of 1st Natural Frequency with experimental results for single cracked cantilever beam.

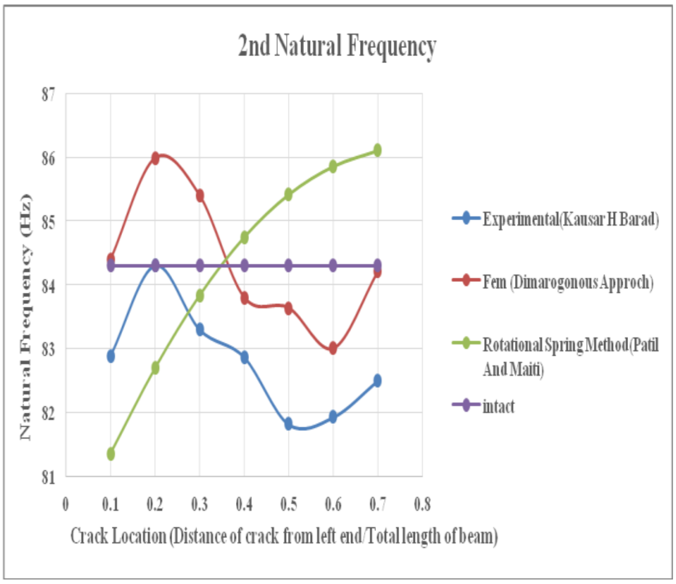

Figure 3: Comparison of 2nd Natural Frequency obtained by two different methods with experimental results.

IV. Results And Discussions

From the table 2, it is observed that the errors obtained by transfer mass matrix method of obtaining frequencies as proposed by D.P. Patil is more with respect to experimental frequencies of Kausar H. Barad. It is lesser for frequencies obtained by present finite element method. Also, from figure 3, the variation of natural frequency with crack location for the case of present finite element method is similar to the experimental one, whereas it is not so for the graph of natural frequency variation obtained by transfer mass matrix method. Hence, present method of modal analysis of cracked beams is preferable over transfer mass matrix method of modal analysis.

V. Conclusion

From the results obtained above it can be concluded that

- 1st frequency increases as the crack shifts away from the fixed end of a cantilever beam.
- Due to crack, the natural frequencies of the beam decreases with respect to the undamaged beam.
- Difference between frequencies obtained by transfer mass matrix method is more in comparison to the frequencies obtained by present finite element method.
- Variation of natural frequencies obtained by finite element method is similar to that of experimental one whereas it is not so for those obtained by transfer mass matrix method.

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